<u>Numeracy Review:</u>

<u>R.1 – Numeracy, Division, Factors, Primes, Pythagorean Theorem</u>

This booklet belongs to:_____________________________Block: ______

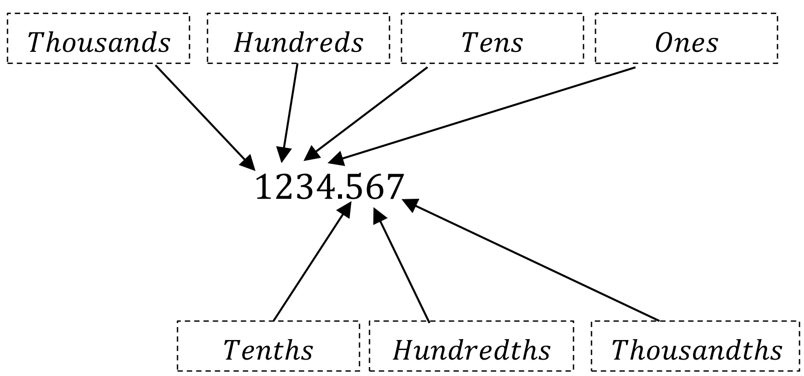
<u>Understanding Numbers</u>

- We need to look at numbers as what they are, don't use slang.
- 2017 It's not 20 17; it is two thousand and seventeen.

We often take for granted our number sense. If you **can't read it properly** or know what the position system is, how could you possibly understand it? It would be like trying to spell without knowing what the letters mean.

- First, we need to remember the **PLACE HOLDER SYSTEM**

What value does a number in a certain position represent?



<u>Example 1:</u> Convert to numbers or words

i) Forty-Two
ii) Seven Hundred, twenty-three and five tenths
iii) 123.56
iv) 53.1234

<u>Solution 1:</u>

i) 42
ii) 723.5
iii) One Hundred, twenty-three and fifty-six one hundredths
iv) Fifty-three and one thousand, two hundred, thirty-four ten thousandths

The Number System

- As we grow up, we learn about numbers
- What changes as we grow are the types of numbers we understand
- The vocabulary begins these numbers is part of everyday mathematical speech
- Think of the different number systems as the Russian Doll metaphor
    - All the numbers in the smallest system exist in the next system up
    - But the upper system contains numbers that the smaller ones do not

Below are the different systems from smallest to largest (See the diagram for a visual)

Natural Numbers: $\{1, 2, 3, \ldots\}$

Whole Numbers: $\{0, 1, 2, 3, \ldots\}$

Integers: $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers: All numbers that can be written as fractions with a non-zero denominator

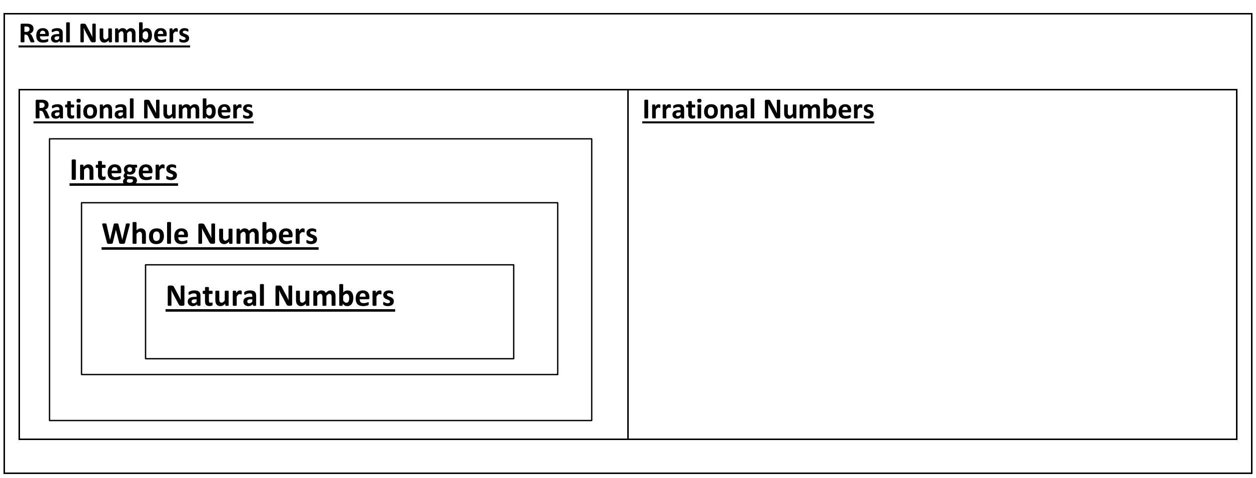
$-3,\ \ 0,\ \ 5,\ \ -\frac{2}{3},\ \ \frac{10}{7},\ \ 2.35,\ \ -2.\overline{35},\ \ \sqrt{4},\ \ -\sqrt{9},\ \ \sqrt{\frac{9}{16}}$

Note: Every terminating and repeating decimal can be written as a fraction

Irrational Numbers: All the numbers that cannot be written as a fraction

$\sqrt{2}, \pi,\ \ 2.134564 \ldots$

Real Numbers: All the systems combined



Divisibility Rules – Where to Even Begin When Trying to Divide

Divisibility Rules		
Divisible by	Conditions	Examples
2	Even Number (Last Digit: 0, 2, 4, 6, 8)	42, 156, 234 678
3	Sum of the digits is divisible by 3	15, 243, 3561
5	Ends in a 0 or a 5	10, 45, 3450
6	Number is divisible by 2 and 3	36, 516, 2316
9	Sum of the digits is divisible by 9	18, 243, 5481
10	Ends in a 0	40, 760, 120 560

Factors – All about Division (See Short Videos for a Refresher)

- Understanding the concept of **FACTORS** is really important
- As mentioned in the Vocabulary:
    - A FACTOR is **one of the pieces that multiplies with another to achieve** a PRODUCT

Example:

$\mathbf{2}$ and $\mathbf{3}$ are **factors** of $\mathbf{6}$, because $\mathbf{2 \cdot 3 = 6}$

- Any two numbers that multiply together are factors of the result

Example 2: What are the factors of 24?

Solution 2:

- We just need to think about what **numbers multiply together** to get 24**…**

$\mathbf{1 \cdot 24}$ $\mathbf{2 \cdot 12}$ $\mathbf{3 \cdot 8}$ $\mathbf{4 \cdot 6}$

- Do you **notice anything?**

- As the **factors get larger,** their **pairs get smaller**
- As **one factor increases,** the **other decreases**
- There is a **specific reason** for this and it is connected to Square Roots and Primes**.**

This trend continues with all numbers, they can have lots of factors or only a few

Example 3: What are the factors of 4, 15, and 36

Solution 3: There can be a number of different factor combinations

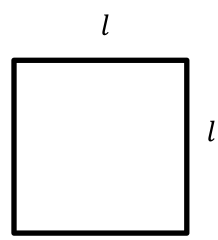
$$4 \rightarrow 1 \cdot 4 \quad \text{or} \quad 2 \cdot 2$$

$$15 \rightarrow 1 \cdot 15 \quad \text{or} \quad 3 \cdot 5$$

$$36 \rightarrow 1 \cdot 36 \quad \text{or} \quad 2 \cdot 18 \quad \text{or} \quad 3 \cdot 12 \quad \text{or} \quad 4 \cdot 9 \quad \text{or} \quad 6 \cdot 6$$

- Factors can also be variables (we will see this more later)
- Before we look further, let's look at Perfect Squares and Square Roots and connect it to factors.

Perfect Squares



- If you think about a square what comes to mind?
  - Two equal sides
- And what is the Area of a square?
  - $Length * Length$ or l^2
- So, what that means is that when we **SQUARE** a number, we **multiply** it by itself
  - That way it has **two factors**, and they are **identical**

Example:

$$1 * 1 = 1 = 1^2$$

$$2 * 2 = 4 = 2^2$$

$$3 * 3 = 9 = 3^2$$

$$9 * 9 = 81 = 9^2$$

$$241 * 241 = 58\,081 = 241^2$$

- Think about **Squaring** a number as another **one of our operations**
  - Like Multiplying, Dividing, Adding, and Subtracting
- Each operation has an **inverse**: **Multiply/Divide** and **Add/Subtract**
- **Squaring** a number has an **inverse** too: **SQUARE/SQUARE ROOT**

Think about it like a **zipper**: One operation zips up the other unzips

Square Root

- The **square rooting** of a number is **the unzipping of the squaring** process

If $2 \cdot 2 = 2^2 = 4$ then, $\sqrt{4} = 2$

If $3 \cdot 3 = 3^2 = 9$ then, $\sqrt{9} = 3$

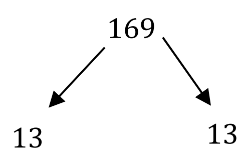
If $r \cdot r = r^2$ then, $\sqrt{r^2} = r$

- If we **break the number down to its factors** we can see if we **make 2 identical factors**

Example 1: What is the Square Root $\sqrt{169}$

Solution 1:

$\sqrt{169} = 13$

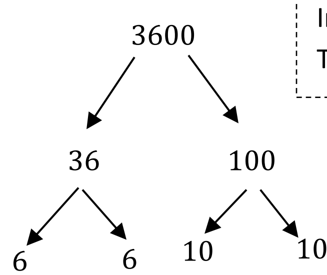


Since the **two factors** of 169 are 13 and 13, we have **two identical factors**. This makes the **Square Root** of $\mathbf{169}$, $\mathbf{13}$

Example 2: What is the Square Root $\sqrt{3600}$

Solution 2:

$\sqrt{3600} = 60$



We can **arrange the factors**: $6 \cdot 6 \cdot 10 \cdot 10$
Into **two groups**: $6 \cdot 10 \cdot 6 \cdot 10$
This give us **two identical factors**: $60 \cdot 60$

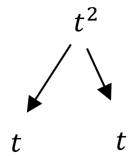
<u>Example 3:</u> What is the Square Root $\sqrt{t^2}$

Even though we don't know what t is, we have **two identical factors**: $\boldsymbol{t \text{ and } t}$

If you **square root any square**, you get the number you were squaring in the first place

<u>Solution 3:</u>

$\sqrt{t^2} = t$



<u>How does this connect to factors?</u>

- The square root is the exact middle point of factor pair relationships
- When we find a factor on one side it automatically gives its pair on the other side.

	36			48	
1	·	36	1	·	48
2	·	18	2	·	24
3	·	12	3	·	16
4	·	9	4	·	12
6	·	6	6	·	8
			$\sqrt{48} \approx 6.93$	·	$\sqrt{48} \approx 6.93$

<u>Prime Numbers and Prime Factors</u>

What is a Prime Number?

- They are numbers that **can only be divided by two numbers, 1 and itself**.
- 1 doesn't count because it is only divided itself, not two different numbers
- Every number that is **not Prime** is called **a Composite number**
- **Every Composite number** is made up of **factors that are also PRIME**
- So, **every number** can be broken down into a product of **<u>Prime Factors</u>**

<u>Example 3:</u> Break the following down into Prime Factors

i) 6
ii) 13
iii) 122
iv) 57
v) 18
vi) 24

Solution 3:

i) $6 = 2 \cdot 3$

ii) Already Prime

iii) $122 = 2 \cdot 61$

iv) $57 = 3 \cdot 19$

$\textit{Isn't Prime yet, keep breaking it down}$

v) $18 = 2 * 9$
$= \mathbf{2 * 3 * 3}$

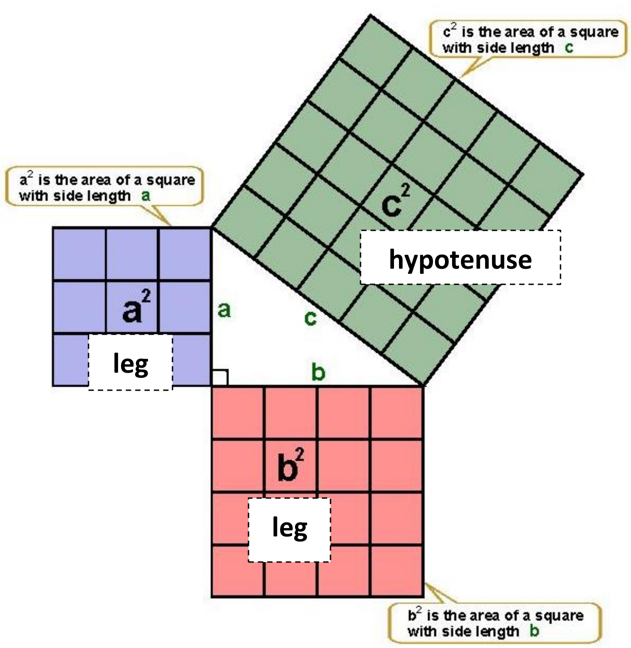
$\textit{Isn't Prime yet, keep breaking it down}$

vi) $24 = 3 * 8$
$= 3 * 2 * 4$
$= \mathbf{3 * 2 * 2 * 2}$

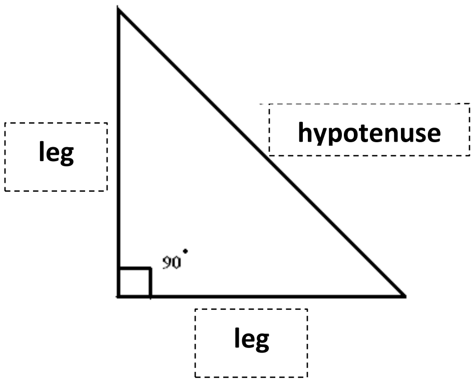
We can **determine if a number is PRIME** by trying to **divide it by all the Prime factors below it's Square Root**.

If it doesn't divide by any of them, it is a PRIME!!!!

Pythagorean Theorem



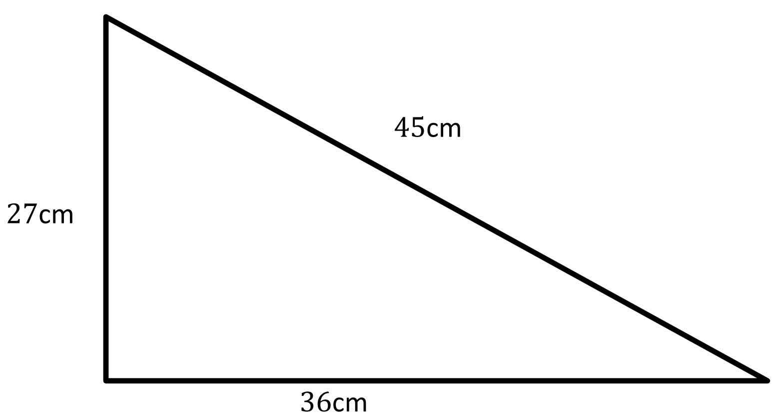
What is it exactly? $leg^2 + leg^2 = hypotenuse^2$



- It can **solve missing information** of right-angle triangles and to test to **see if triangles are right angle**.

<u>Example 1:</u> Is the following triangle a Right-Angle Triangle?

<u>Solution 1:</u>



I need to see if: $leg^2 + leg^2 = hyp^2$

- $36^2 + 27^2 = 45^2$ Fill in Values
- $1296 + 729 = 2025$ Check to see if they balance
- $2025 = 2025$

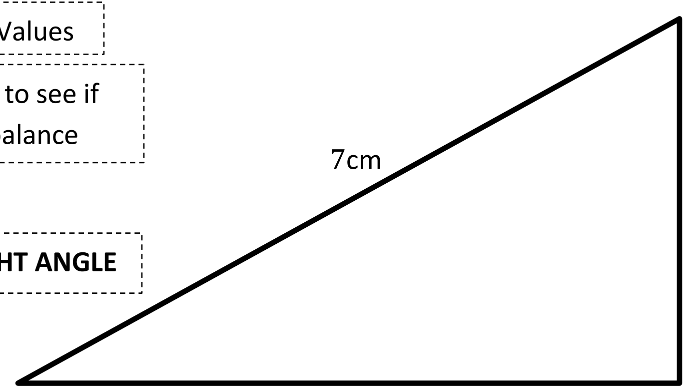
They **Balance** so it is a **RIGHT ANGLE TRAINGLE**

<u>Example 2:</u> Is the following triangle a Right-Angle Triangle?

<u>Solution 2:</u> I need to see if: $leg^2 + leg^2 = hyp^2$

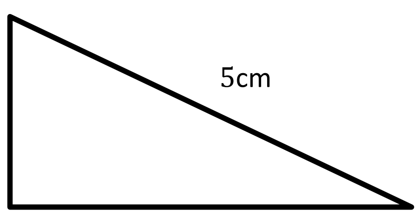
- $4^2 + 2^2 = 7^2$ Fill in Values
- $16 + 4 = 49$ Check to see if they balance
- $20 = 49$

They **Don't Balance** so it is **a NOT A RIGHT ANGLE**



<u>Example 3:</u> Is the following triangle a Right-Angle Triangle?

<u>Solution 3:</u> I need to see if: $leg^2 + leg^2 = hyp^2$

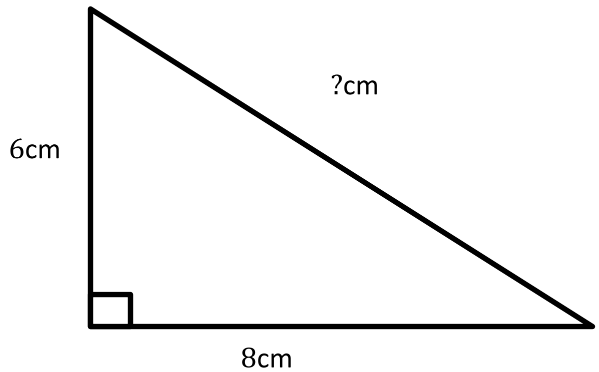


- $4^2 + 3^2 = 5^2$ Fill in Values
- $16 + 9 = 25$ Check to see if they balance
- $25 = 25$

They **Balance** so it is a **RIGHT ANGLE TRAINGLE**

Example 4: Solve the following Right-Angle Triangles

Solution 4:



- We know the triangle is a **Right-Angle Triangle** so we can use **Pythagorean Theorem**.
- The **missing side** is the **BIGGEST side** so the equation is used as is.

$$leg^2 + leg^2 = hyp^2$$

$$6^2 + 8^2 = hyp^2$$

$$36 + 64 = hyp^2$$

$$100 = hyp^2$$

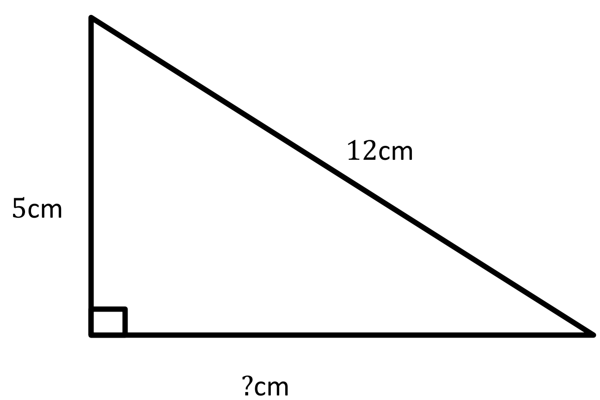
- Now $100 = hyp^2$ and we want hyp
- So, remember to get hyp from hyp^2 we **SQUARE ROOT**

$$hyp = \sqrt{100}$$

$$hyp = 10$$

Example 5: Solve the following Right-Angle Triangle

Solution 5:



- We know the triangle is a **Right-Angle Triangle** so we can use **Pythagorean Theorem**.
- The **missing side** is a **SMALLER side** so the equation has to be adjusted.

$$leg^2 + leg^2 = hyp^2 \quad \rightarrow \quad leg^2 = hyp^2 - leg^2$$

$$leg^2 = 12^2 - 5^2$$

$$leg^2 = 144 - 25$$

$$leg^2 = 119$$

- Now $119 = leg^2$ and we want leg
- So, remember to get leg from leg^2 we **SQUARE ROOT**

$$\boldsymbol{leg = \sqrt{119}}$$

We can't simplify that, so that is the answer.

Determining Lowest Common Multiple (LCM) – This is an Extending Concept

- The **words Lowest and Greatest** can throw people off
- What really matters is: **Multiple and Factor**

The Lowest Common Multiple must be a <u>Multiple</u> of all the numbers being considered

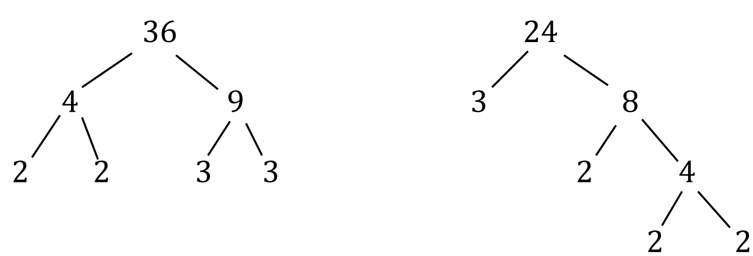
- This means it has to be at **least as big as the largest number** in the group
- In order **to be a multiple, you are divisible by the given numbers**
- The examples will help explain this
- Greatest Common Factor will become more intuitive, but requires factors!

➢ Using our Prime Factors allows us to see what factors make up a number and determine what factors are required in order to be a multiple of a given number

➢ As long as all the numbers in consideration are represented by the factor breakdown we have a Common Multiple

<u>Example 4:</u> Determine the LCM of $24\ and\ 36$

<u>Solution 4:</u> Use a factor tree to first determine Prime Factors of each



$36 = 2 \cdot 2 \cdot 3 \cdot 3$ — I need at least two 2's and two 3's

$24 = 2 \cdot 2 \cdot 2 \cdot 3$ — I need at least three 2's and one 3

In order to be a multiple of each number, each prime factor has to appear in the prime factor breakdown of the LCM. This means each number is represented, but with no redundancy or additional factors.

This covers 36

The LCM of $24\ and\ 36$ then is: $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \quad = 72$

This covers 24

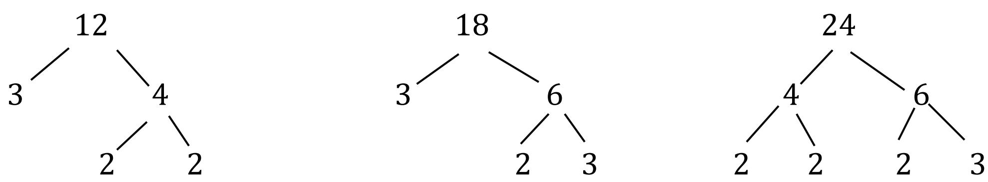
You see the extra factor is what by what amount the LCM is multiplied by:

$72 = 36 \cdot \mathbf{2}$

$72 = 24 \cdot \mathbf{3}$

Example 5: Determine the LCM of $12, 18, and\ 24$

Solution 5: Use a factor tree, then make sure the prime number factorization for each number is expressed in the prime number factorization of the LCM



$$12 = 2 \cdot 2 \cdot 3$$

$$18 = 2 \cdot 3 \cdot 3$$

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

In order to be a multiple of each number, each prime factor has to appear in the prime factor breakdown of the LCM. This means each number is represented, but with no redundancy or additional factors.

The LCM of $12, 18\ and\ 24$ then is:

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 3}_{\text{This covers } 24} \cdot 3 = 72$$

$$2 \cdot \underbrace{2 \cdot 2 \cdot 3}_{\text{This covers } 12} \cdot 3 = 72$$

$$2 \cdot 2 \cdot \underbrace{2 \cdot 3 \cdot 3}_{\text{This covers } 18} = 72$$

You see the extra factor is what by what amount the LCM is multiplied by:

$$72 = 12 \cdot \mathbf{6}$$

$$72 = 24 \cdot \mathbf{3}$$

$$72 = 18 \cdot \mathbf{4}$$

There is nothing extra, everything is covered the least amount of times making it the:

LCM

R.1 – Practice Problems

Write the following numbers out **numerically**.

1. Three hundred and forty-two __________
2. Twenty-three and forty-seven hundredths __________
3. Three tenths __________
4. Three thousand, two hundred twenty-nine and eight hundredths __________
5. Twelve thousand three hundred and five __________
6. Forty-two thousandths __________
7. Thirteen and fifty-seven hundredths __________

Write the following numbers out **verbally**.

8. 14.1
9. 23.76
10. 0.003
11. 34 540.3
12. 1 456 345.234

What are the factors of the following numbers. Write them in factor pairs.

13. 64	14. 144	15. 36
16. 76	17. 112	18. 98

What are the **Square Roots** of the following numbers? Demonstrate by getting two identical factors.

19. 64	20. 196
21. 1296	22. 900

23. 1764

24. 225

25. 256

26. 676

Between what **two whole numbers** do the following square roots fall?

27. $\sqrt{156}$

28. $\sqrt{402}$

29. $\sqrt{11}$

30. What is the **difference** between a **perfect square and a perfect cube**? Explain your answers verbally, numerically, and with a picture.

Break the following numbers down to **Prime Factors,** use your **divisibility rules** to help break them down

31. 42	32. 36
33. 72	34. 124
35. 153	36. 144

37. $\sqrt{99}$

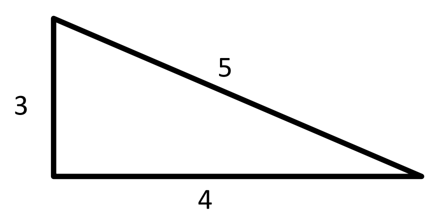
38. $\sqrt{23}$

39. $\sqrt{1170}$

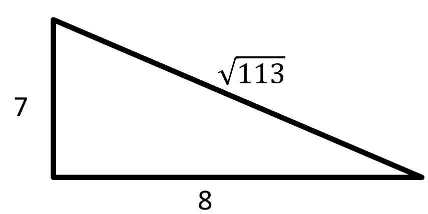
40. $\sqrt{100}$

Are the following triangles **Right Angle Triangles**? Prove your answer using the **Pythagorean Theorem**.

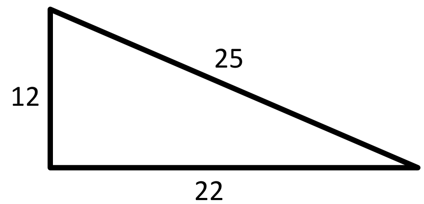
41.



42.



43.



44. If you worked for a construction company and your foreman tasked you with building a ramp exactly 10ft in length, but had to reach a height of 6 ft. How far back from the top of the ramp would you have to start building? Draw a picture to demonstrate your idea.

Find the LCM of the following sets of Numbers (Extending)

45. $18, 27$

46. $8, 12$

47. $5, 12, 15$

48. $8, 12, 18$

49. $12, 24, 36$

50. $3, 5, 18$

Answer Key – Section R.1

1.	342	2.	23.47
3.	0.3	4.	3229.08
5.	12305	6.	0.042
7.	13.57	8.	Fourteen and one tenth
9.	Twenty-three and seventy-six hundredths	10.	Three thousandths
11.	Thirty-four thousand, five hundred forty and three tenths	12.	One million, four hundred fifty-six thousand, three hundred forty-five and two hundred thirty-four thousandths
13.	See Website Key	14.	See Website Key
15.	See Website Key	16.	See Website Key
17.	See Website Key	18.	See Website Key
19.	8	20.	14
21.	36	22.	30
23.	42	24.	15
25.	16	26.	26
27.	12 and 13	28.	20 and 21
29.	3 and 4	30.	See Website Key
31.	$2 \cdot 3 \cdot 7$	32.	$2 \cdot 2 \cdot 3 \cdot 3$
33.	$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$	34.	$2 \cdot 2 \cdot 31$
35.	$3 \cdot 3 \cdot 17$	36.	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
37.	$3 \cdot 3 \cdot 11$	38.	All ready Prime
39.	$2 \cdot 3 \cdot 3 \cdot 5 \cdot 13$	40.	$2 \cdot 2 \cdot 5 \cdot 5$
41.	Yes	42.	Yes
43.	No	44.	8ft
45.	54	46.	24
47.	60	48.	72
49.	72	50.	90

Extra Work Space