Section 9.1 – Practice Problems

1. Find the most general antiderivative of the given function on the interval $(-\infty, \infty)$

a) $f(x) = 2x + 1$

$\boxed{F(x) = x^2 + x + C}$

b) $f(x) = 4x^3 - 11$

$\boxed{F(x) = x^4 - 11x + C}$

c) $f(x) = 16x^9 - 9x^4 + 3x$

$F(x) = \frac{16}{10}x^{10} - \frac{9}{5}x^5 + \frac{3}{2}x^2$

$= \boxed{\frac{8}{5}x^{10} - \frac{9}{5}x^5 + \frac{3}{2}x^2 + C}$

d) $f(x) = x^7 + x^5 + x^3 + x$

$\boxed{F(x) = \frac{1}{8}x^8 + \frac{1}{6}x^6 + \frac{1}{4}x^4 + \frac{1}{2}x^2 + C}$

2. Find the antiderivative of f on $(0, \infty)$

a)

$$f(x) = \frac{2}{x^7} + \frac{x^5}{2}$$

$f(x) = 2x^{-7} + \frac{1}{2}x^5$

$F(x) = 2\left(\frac{1}{-7+1}x^{-7+1}\right) + \frac{1}{2} \cdot \frac{1}{6}x^{5+1}$

$= -\frac{2}{6}x^{-6} + \frac{1}{12}x^6$

$= \boxed{-\frac{1}{3}x^{-6} + \frac{1}{12}x^6 + C}$

b)

$f(x) = \sqrt{x} + \sqrt[3]{x}$

$f(x) = x^{\frac{1}{2}} + x^{\frac{1}{3}}$

$F(x) = \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + \frac{1}{\frac{1}{3}+1}x^{\frac{1}{3}+1}$

$= \frac{1}{\frac{3}{2}}x^{3/2} + \frac{1}{\frac{4}{3}}x^{4/3}$

$= \boxed{\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C}$

c)

$$f(x) = -\frac{3}{x} + \frac{5}{x^2}$$

$f(x) = -3 \cdot \frac{1}{x} + 5x^{-2}$

$F(x) = -3\ln x + \frac{5}{-2+1} x^{-2+1}$

$= \boxed{-3\ln x - 5x^{-1} + C}$

d)

$$f(x) = \frac{1}{x^7} + \frac{1}{x^5} + \frac{1}{x^3} + \frac{1}{x}$$

$f(x) = x^{-7} + x^{-5} + x^{-3} + \frac{1}{x}$

$F(x) = \frac{1}{-7+1} x^{-6} + \frac{1}{-5+1} x^{-4} + \frac{1}{-3+1} x^{-2} + \ln x$

$= \boxed{-\frac{1}{6}x^{-6} - \frac{1}{4}x^{-4} - \frac{1}{2}x^{-2} + \ln x + C}$

3. Find the antiderivative of f on $(-\infty, 0)$

a)

$$f(x) = \frac{1}{x}$$

$F(x) = \ln(-x)$

b)

$$f(x) = \frac{2}{x^3} - \frac{3}{x^2}$$

$F(x) = -\frac{2}{2}x^{-2} - \left(-\frac{3}{1}x^{-1}\right) + C$

$= \boxed{-x^{-2} + 3x^{-1} + C}$

c)

$$f(x) = \sqrt{-x}$$

$f(x) = (-x)^{\frac{1}{2}}$

$F(x) = \frac{1}{\frac{1}{2}+1}(-x)^{3/2}(-1)$

$\boxed{= -\frac{2}{3}(-x)^{3/2} + C}$

d)

$$f(x) = \frac{1}{x^4} + x^3 + \frac{1}{x^2}$$

$F(x) = \frac{1}{-4+1}x^{-3} + \frac{1}{4}x^4 + \frac{1}{-2+1}x^{-1}$

$= \boxed{-\frac{1}{3}x^{-3} + \frac{1}{4}x^4 - x^{-1} + C}$

4. Find the antiderivative of f on $(-\infty, \infty)$

a) $f(x) = \sin 2x + 2\cos x$

$F(x) = \boxed{-\frac{1}{2}\cos 2x + 2\sin x + C}$

b) $f(x) = -3\cos 5x + 8\sin x$

$\boxed{F(x) = -\frac{3}{5}\sin 5x - 8\cos x + C}$

c) $f(x) = 7\cos x - 11\sin 11x$

$F(x) = 7\sin x - \frac{11(-\cos 11x)}{11}$

$= \boxed{7\sin x + \cos 11x + C}$

d) $f(x) = -4\cos(x+2)$

$\boxed{F(x) = -4\sin(x+2) + C}$

5. Find the most general antiderivative of f on $(-\infty, \infty)$

a) $f(x) = e^x + e^{-x}$

$F(x) = e^x + e^{-x}(-1) + C$

$\boxed{F(x) = e^x - e^{-x} + C}$

b) $f(x) = e^x - e^{-x}$

$\boxed{F(x) = e^x + e^{-x} + C}$

c) $f(x) = 4e^{2x} - 6e^{-3x}$

$F(x) = \frac{4e^{2x}}{2} - \frac{6e^{-3x}}{-3}$

$= \boxed{2e^{2x} + 2e^{-3x} + C}$

d) $f(x) = e^x - e^{-2x} + e^{3x}$

$F(x) = e^x - \frac{e^{-2x}}{-2} + \frac{e^{3x}}{3}$

$= \boxed{e^x + \frac{1}{2}e^{-2x} + \frac{1}{3}e^{3x} + C}$

6. Find the antiderivative of f on $(0, 1)$

a)
$$f(x) = \sqrt{x} - \sqrt{1-x}$$

$f(x) = x^{\frac{1}{2}} - (1-x)^{\frac{1}{2}}$

$F(x) = \frac{1}{\frac{1}{2}+1} x^{3/2} - \frac{1}{\frac{1}{2}+1}(1-x)^{3/2}(-1) + C$

$= \boxed{\frac{2}{3}x^{3/2} + \frac{2}{3}(1-x)^{3/2} + C}$

b)
$$f(x) = \frac{1}{x} - \frac{1}{1-x}$$

$F(x) = \ln x - \ln(1-x)(-1) + C$

$= \ln x + \ln(1-x) + C$

$= \ln(x(1-x)) + C \Rightarrow \boxed{\ln(x-x^2) + C}$

c)
$$f(x) = \frac{1}{\sqrt{1-x}} + \frac{1}{\sqrt{x}}$$

$f(x) = (1-x)^{-\frac{1}{2}} + x^{-\frac{1}{2}}$

$F(x) = \frac{1}{-\frac{1}{2}+1}(1-x)^{\frac{1}{2}}(-1) + \frac{1}{-\frac{1}{2}+1} x^{\frac{1}{2}}$

$= \boxed{-2(1-x)^{\frac{1}{2}} + 2x^{\frac{1}{2}} + C}$

d)
$$f(x) = \frac{4}{x} + \frac{5}{1-x}$$

$f(x) = 4 \cdot \frac{1}{x} + 5(1-x)^{-1}$

$F(x) = 4\ln x + 5\ln(1-x)(-1)$

$= \boxed{4\ln x - 5\ln(1-x) + C}$

7. Find the antiderivative of f on $(-\infty, \infty)$ chain rule scenarios

a) $f(x) = xe^{x^2}$

$\boxed{F(x) = \frac{1}{2}e^{x^2} + C}$

b) $f(x) = \sin^2 x \cos x$

$\boxed{F(x) = \frac{1}{3}\sin^3 x + C}$

c)
$$f(x) = \frac{2x}{x^2+1}$$

$\boxed{F(x) = \ln(x^2+1) + C}$

d)
$$f(x) = \frac{x}{\sqrt{x^2+1}}$$

$\frac{d}{dx}(x^2+1)^{\frac{1}{2}}$

$= \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}} \cancel{2}x$

$\boxed{F(x) = \sqrt{x^2+1} + C}$