9.1 Antiderivatives

A function F is an **antiderivative** of f on an interval if $F'(x) = f(x)$ for all x in that interval. To illustrate how to find an antiderivative consider the problem of finding all the antiderivatives of $f(x) = 2$. The obvious answer (thinking of differentiation in reverse) is $F(x) = x^2$ since $\frac{d}{dx}(x^2) = 2x$ by the Power Rule. However, $G(x) = x^2 + 1$and $H(x) = x^2 - 5$ are also possible antiderivatives because the 1 and -5 differentiate to zero. In general, the antiderivative of $f(x) = 2x$ is $F(x) = x^2 + C$.

> If F is an antiderivative of f on an interval, then the most general antiderivative of f on that interval is
> $$F(x) + C$$
> Where C is an arbitrary constant and $F(x)$ is the particular antiderivative of $f(x)$.

Ex. 1
Find the most general antiderivative of $f(x) = 4x^3 - 6x^2 + 11$ on the interval $(-\infty, \infty)$.

Consider the derivative of each term individually

Derivative of x^4 is $4x^3$!
Derivative of $-2x^3$ is $-6x^2$!
Derivative of $11x$ is 11!

So general antiderivative is:

$x^4 - 2x^3 + 11x + C$

Some arbitrary constant.

Function	Particular Antiderivative
0	1
1	x
$x^n \ (n \neq -1)$	$\frac{1}{n+1}x^{n+1}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
$e^x \ (k \neq 0)$	$\frac{1}{k}e^{kx}$
$\cos kx \ (k \neq 0)$	$\frac{1}{k}\sin kx$
$\sin kx \ (k \neq 0)$	$-\frac{1}{k}\cos kx$

Ex. 2
Find the antiderivative of f.

(a) $f(x) = 2x^2 - x + 7$ (b) $f(x) = -3x^4 + x^2 - 5$

(c) $f(x) = -3e^{-x} + 6e^{2x}$ (d) $\cos 3x - 5\sin 2x$

a) $2\cdot\frac{1}{3}x^3 - \frac{1}{2}x^2 + 7x + C = \boxed{\frac{2x^3}{3} - \frac{1}{2}x^2 + 7x + C}$

b) $-3\cdot\frac{1}{5}x^5 + \frac{1}{3}x^3 - 5x + C = \boxed{-\frac{3}{5}x^5 + \frac{1}{3}x^3 - 5x + C}$

c) $-3\left(\frac{1}{-1}e^{-x}\right) + 6\left(\frac{1}{2}e^{2x}\right) + C = \boxed{3e^{-x} + 3e^{2x} + C}$

d) $\frac{1}{3}(\sin 3x) - 5\left(-\frac{1}{2}\cos 2x\right) + C = \boxed{\frac{1}{3}\sin 3x + \frac{5}{2}\cos 2x + C}$

Ex. 3
Find the antiderivative of f on the interval $(0, \infty)$.

(a) $f(x) = \dfrac{2}{x^2} - \dfrac{5}{x} + x$ (b) $f(x) = \sin x + \dfrac{1}{x^3}$

strategy

$2x^{-2} - 5\left(\frac{1}{x}\right) + x$

$F(x) = 2\left(\frac{1}{-1}\right)x^{-1} - 5\ln|x| + \frac{1}{2}x^2 + C$

$\boxed{-\frac{2}{x} - 5\ln x + \frac{1}{2}x^2 + C}$

$F(x) = -\cos x + \frac{1}{-3+1}x^{-3+1} + C$

$= -\cos x - \frac{1}{2}x^{-2} + C$

$= \boxed{-\cos x - \frac{1}{2x^2} + C}$

Ex. 4
Find the most general antiderivative of $f(x) = \sin x \cos x$.

Solution 1: let $\sin x = g(x)$

$\cos x = \frac{d}{dx}\sin$ $\frac{d}{dx}\sin x = g'(x)$

$f(x) = g(x)g'(x)$ *chain rule

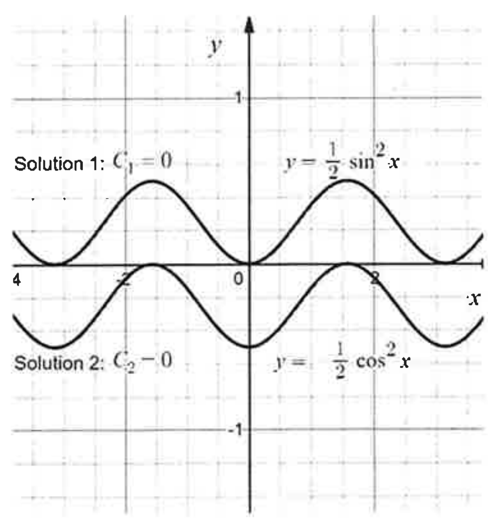
$F(x) = \frac{1}{2}g(x)^2 = \frac{1}{2}\sin^2 x + C_1$

Solution 2: let $\cos x = g(x)$

$\sin x =$ $-\frac{d}{dx}\cos x = g'(x)$

$f(x) = g(x)g'(x)$

$F(x) = -\frac{1}{2}\cos^2 x + C_2$



- We differ purely by our constants which is a vertical displacement.
- Shape does not change and location of our tangents stays constant.

Homework Assignment

- Practice Problems: #1 – 6ace