Section 8.4 – Practice Problems

1. Differentiate

a) $f(x) = x^2 \ln x$

$f'(x) = 2x \ln x + x^2 \frac{1}{x}$

$= 2x \ln x + x$

$= \boxed{x(2\ln x + 1)}$

b) $f(x) = \sqrt{\ln x}$

$f(x) = (\ln x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$

$= \boxed{\frac{1}{2x\sqrt{\ln x}}}$

c) $g(x) = \ln(x^3 + 1)$

$g'(x) = \frac{1}{x^3+1} \cdot 3x^2$

$= \boxed{\frac{3x^2}{x^3+1}}$

d) $g(x) = \ln(5x)$

$g'(x) = \frac{1}{5x} \cdot 5$

$= \boxed{\frac{1}{x}}$

e) $y = \sin(\ln x)$

$y' = \cos(\ln x) \cdot \frac{1}{x}$

$\boxed{\frac{\cos(\ln x)}{x}}$

f) $y = \ln(\sin x)$

$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$

$\boxed{\cot x}$

g) $y = \dfrac{\ln x}{x^3}$

$$y' = \frac{x^3 \cdot \frac{1}{x} - 3x^2 \ln x}{x^6}$$

$$\Rightarrow \frac{x^2 - 3x^2 \ln x}{x^6} = \frac{x^2(1 - 3\ln x)}{x^6}$$

$$\Rightarrow \boxed{\frac{1 - 3\ln x}{x^4}}$$

h) $y = (x + \ln x)^3$

$$y' = 3(x + \ln x)^2 \cdot \left(1 + \frac{1}{x}\right)$$

$$\boxed{y' = 3(x + \ln x)^2\left(1 + \frac{1}{x}\right)}$$

2. If $f(x) = \ln(\ln x)$, find $f'(x)$

$$f'(x) = \frac{1}{\ln x} \cdot \frac{1}{x} = \boxed{\frac{1}{x \ln x}}$$

3. Find the derivative of each function

a) $f(x) = \log_2(x^2 + 1)$

$$f'(x) = \frac{1}{(x^2+1)\ln 2} \cdot 2x = \boxed{\frac{2x}{(x^2+1)\ln 2}}$$

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b) $g(x) = x \log_{10} x$

$$g'(x) = x \cdot \frac{1}{x \ln 10} + \log_{10} x \rightarrow \boxed{\log_{10} x + \frac{1}{\ln 10}}$$

c) $f(x) = \log_5(3x - 8)$

$f'(x) = \dfrac{1}{(3x-8)\ln 5} \cdot 3 = \boxed{\dfrac{3}{(3x-8)\ln 5}}$

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d)

$g(x) = \dfrac{1 + \log_3 x}{x}$

$g'(x) = (1+\log_3 x)\cdot(-1)x^{-2} + x^{-1}\left(\dfrac{1}{x\ln 3}\right)$

$= (1+\log_3 x)x^{-1}$

$= -\dfrac{(1+\log_3 x)}{x^2} + \dfrac{1}{x^2\ln 3} \rightarrow \dfrac{-\ln 3(1+\log_3 x)+1}{x^2\ln 3}$

$\rightarrow \dfrac{1 - \ln 3 - \ln 3 \log_3 x}{x^2\ln 3} = \dfrac{1 - \ln 3 - \ln 3 \cdot \frac{\ln x}{\ln 3}}{x^2\ln 3}$ * change of base $\log_3 x = \dfrac{\log_e x}{\log_e 3} = \dfrac{\ln x}{\ln 3}$

$= \boxed{\dfrac{1 - \ln x - \ln 3}{x^2\ln 3}}$

4. Differentiate

a) $y = x^3 + 3^x$

$\boxed{y' = 3x^2 + 3^x \ln 3}$

b) $y = 2^{x^4 - x}$

$y' = 2^{x^4-x}\ln 2\,(4x^3 - 1)$

$y' = (4x^3-1)(\ln 2)2^{x^4-x}$

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c) $y = x5^{\sqrt{x}}$

$y' = x5^{\sqrt{x}}\ln 5 \cdot \dfrac{1}{2\sqrt{x}} + 5^{\sqrt{x}}$

$= \dfrac{5^{\sqrt{x}}(\ln 5)\sqrt{x}}{2} + \dfrac{2\cdot 5^{\sqrt{x}}}{2}$

$= \boxed{\dfrac{5^{\sqrt{x}}}{2}\left[(\ln 5)\sqrt{x} + 2\right]}$

d) $y = 10^{\tan \pi x}$

$y' = 10^{\tan \pi x}(\ln 10)\cdot \sec^2 \pi x \cdot \pi$

$= \boxed{\pi(\ln 10)\sec^2 \pi x \cdot 10^{\tan \pi x}}$

5. Find the equation of the tangent line to each curve at the given point.

a) $y = \ln(x-1)$; $(2,0)$

$y' = \frac{1}{x-1} \cdot 1$

$= \frac{1}{x-1}$ at $(2,0)$

$\frac{dy}{dx} = 1$

$y - 0 = 1(x-2)$

$\boxed{y = x-2}$

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b) $y = x^2 \ln x$; $(1,0)$

$y' = x^2 \cdot \frac{1}{x} + 2x \ln x$

$= x + 2x \ln x$

$= x(1 + 2\ln x)$

at $x = 1$

$1 + 2\ln 1$

$= 1 + 0$

$= 1$

$\frac{dy}{dx} = 1$

$y - 0 = 1(x-1)$

$\boxed{y = x-1}$

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c) $y = 10^x$; $(1,10)$

$y' = 10^x \ln 10$

at $x = 1$

$y' = 10^1 \ln 10$

$\boxed{10 \ln 10}$

$\boxed{y - 10 = 10\ln 10 (x-1)}$

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d) $y = \log_{10} x$; $(100,2)$

$y' = \frac{1}{x \ln 10}$ at $x = 100$

$\frac{1}{100 \ln 10}$

$\boxed{y - 2 = \frac{1}{100 \ln 10}(x - 100)}$