8.4 Derivatives of Logarithmic Functions

In this section we will see the derivatives of the logarithmic functions $y = \log_b x$. But first we will look at the Natural Logarithm $y = \ln x$.

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Proof

Let $y = \ln x$

Then $e^y = x$

By differentiating implicitly with respect to x we get the following.

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Ex. 1   Differentiate the following

a) $y = x \ln x$

Product Rule

$x \cdot \frac{1}{x} + \ln x$

$\boxed{1 + \ln x}$

b) $y = \ln(x^2 + 2x - 5)$ chain rule

$y' = \frac{1}{x^2 + 2x - 5} \cdot 2x + 2$

$\Rightarrow \boxed{\dfrac{2x + 2}{x^2 + 2x - 5}}$

Ex. 2   Find the derivative of $f(x) = \ln(\cos x)$

$$f'(x) = \frac{1}{\cos x} \cdot -\sin x$$

$$= \frac{-\sin x}{\cos x}$$

$$= \boxed{-\tan x}$$

Ex. 3   Differentiate $y = (\ln x)^4$

$$y' = 4(\ln x)^3 \cdot \frac{1}{x}$$

$$\frac{4(\ln x)^3}{x}$$

Ex. 4   Find:

$$\frac{d}{dx}\ln\frac{x}{\sqrt{x+1}}$$

log derivative
and quotient rule
and chain rule

$$\frac{dy}{dx} = \frac{1}{\frac{x}{\sqrt{x+1}}} \cdot \frac{(x+1)^{\frac{1}{2}}(1) - \frac{1}{2(x+1)^{\frac{1}{2}}}\cdot x}{\sqrt{x+1}^2}$$

$$= \frac{\sqrt{x+1}}{x} \cdot \frac{2(x+1) - x}{\frac{2(x+1)^{1/2}}{\sqrt{x+1}^2}} \rightarrow \frac{\sqrt{x+1}}{x} \cdot \frac{2x+2-x}{2(x+1)^{3/2}}$$

$$= \frac{(x+2)(x+1)^{\frac{1}{2}}}{2x(x+1)^{3/2}} = \boxed{\frac{x+2}{2x(x+1)}}$$

Other Exponential and Logarithmic Derivatives

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}b^x = b^x \ln b$$

$$\frac{d}{dx}\log_b x = \frac{1}{x\ln b}$$

Ex. 6 Find y' if $y = 2^{x^2}$

$$y' = 2^{x^2}\ln 2\frac{d}{dx}x^2$$

but x^2 is
already
so chain
rule

$$2^{x^2}(\ln 2)\cdot 2x$$

$$\boxed{2x(\ln 2)2^{x^2}}$$

Ex. 7   Find $f'(x)$ if $f(x) = \log_{10}(3x+1)^4$

chain
rule again

log rules

$$f(x) = 4\log_{10}(3x+1)$$

$$f'(x) = 4\cdot\frac{1}{(3x+1)\ln 10}\frac{d}{dx}(3x+1)$$

$$= \frac{4}{(3x+1)\ln 10}\cdot 3 \rightarrow \boxed{\frac{12}{(3x+1)\ln 10}}$$

Homework Assignment
- Practice Problems: All