Section 8.2 – Practice Problems

1. Simplify

a) $\frac{2}{e^{-x}}$ $= 2e^{x}$

b) $(e^x)^4$

$= e^{4x}$

c) $e^{1-x}e^{3x}$

$= e^1 \cdot e^{-x} \cdot e^{3x} \rightarrow e^{2x+1}$

d) $e^x e^{-x}$

$e^0 = 1$

e) $e^{2x}(1 - 5e^{3x})$

$e^{2x} - 5e^{5x}$

f) $\frac{6e^{8x}}{e^{3x}}$ $6e^{5x}$

2. Differentiate

a) $y = 2e^{-x}$

$\boxed{-2e^{-x}}$

b) $y = x^4e^x$

$y' = 4x^3e^x + x^4e^x$

$= \boxed{x^3e^x(4+x)}$

c) $y = e^{2x}\sin 3x$

$e^{2x} \cdot 2\sin 3x + e^{2x}\cos 3x \cdot 3$

$2e^{2x}\sin 3x + 3e^{2x}\cos 3x$

$\boxed{y' = e^{2x}(2\sin 3x + 3\cos 3x)}$

d) $y = e^{\sqrt{x}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$= \boxed{\frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

e) $y = e^{\tan x}$

$y' = e^{\tan x} \cdot \sec^2 x$

$\boxed{y' = \sec^2 x \, e^{\tan x}}$

f) $y = \tan(e^x)$

$y' = \sec^2(e^x) \cdot e^x$

$= \boxed{e^x \sec^2(e^x)}$

g) $y = \dfrac{e^x}{x}$

$y' = \dfrac{xe^x - e^x}{x^2} = \boxed{\dfrac{e^x(x-1)}{x^2}}$

h) $y = \dfrac{e^x}{1 - e^{2x}}$

$y' = \dfrac{(1-e^{2x})e^x - e^x(-e^{2x})(2)}{(1-e^{2x})^2}$

$y' = \dfrac{(1-e^{2x})e^x + 2e^{2x}e^x}{(1-e^{2x})^2} = \dfrac{e^x[(1-e^{2x})+2e^{2x}]}{(1-e^{2x})^2}$

$\boxed{y' = \dfrac{e^x(1+e^{2x})}{(1-e^{2x})^2}}$

i) $y = e^{\sin(x^2)}$

$y' = e^{\sin x^2} \cdot \cos x^2 \cdot 2x$

$= \boxed{2x\cos(x^2)e^{\sin(x^2)}}$

j) $y = xe^{\cot 4x}$

$y' = e^{\cot 4x} + xe^{\cot 4x} \cdot (-\csc^2 4x) \cdot 4$

$= e^{\cot 4x} - 4x\csc^2 4x \, e^{\cot 4x}$

$\Rightarrow \boxed{e^{\cot 4x}[1 - 4x\csc^2 4x]}$

k) $y = (1 + 5e^{-10x})^4$

$y' = 4(1+5e^{-10x})^3 \cdot 5e^{-10x} \cdot -10$

$= \boxed{-200e^{-10x}(1+5e^{-10x})^3}$

l) $y = \sqrt{x + e^{1-x^2}}$

$y' = \dfrac{1}{2\sqrt{x+e^{1-x^2}}} \cdot (1 + e^{1-x^2} \cdot -2x)$

$\boxed{y' = \dfrac{1 - 2xe^{1-x^2}}{2\sqrt{x+e^{1-x^2}}}}$

3. Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$.

$y' = xe^{2x} \cdot 2 + e^{2x}$

then $y = 1$

$= e^{2x}(2x+1)$

$y - 1 = 1(x - 0)$

at $x=0$

$\boxed{y = x+1}$

$e^0(1)$

$e^0 = 1$

4. Find y' if $e^{xy} = 2x + y$

$e^{xy} \cdot \left(x\frac{dy}{dx} + y\right) = 2 + \frac{dy}{dx}$

$xe^{xy}\frac{dy}{dx} + ye^{xy} = 2 + \frac{dy}{dx}$

$\frac{dy}{dx}xe^{xy} - \frac{dy}{dx} = 2 - ye^{xy}$

$\frac{dy}{dx}(xe^{xy} - 1) = 2 - ye^{xy}$

$\boxed{\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}}$

5. Find the intervals of increase and decrease for the function $f(x) = x^2e^{-x}$

$f'(x) = x^2e^{-x}(-1) + 2xe^{-x}$

$= xe^{-x}(2 - x)$

ct pt: $x = 0$

$x = 2$

Interval	xe^{-x}	$(2-x)$	$f'(x)$	$f(x)$
$(-\infty, 0)$	$-$	$+$	$-$	dec
$(0, 2)$	$+$	$+$	$+$	inc
$(2, \infty)$	$+$	$-$	$-$	dec

inc on $(0,2)$

dec on $(-\infty,0)$

$(2,\infty)$