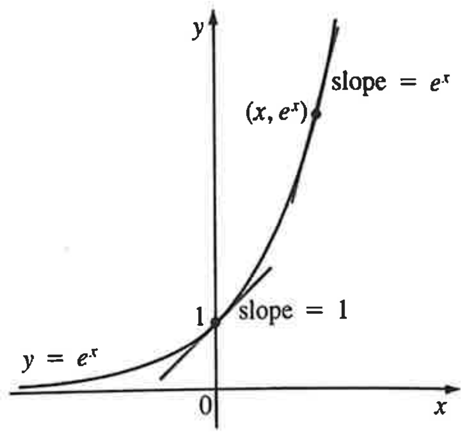
8.2 Derivatives of Exponential Functions in a Nutshell

In this section we will briefly discuss the mathematical value denoted by the letter e. Like π, e is a mathematical constant. The significance will be analysed at a later time, but it is important to understand the importance of e. With respect to exponential functions and their derivatives.

The exponential function e^x is such that is crosses the y-axis with a tangent slope of 1and that the slope of the tangent line at $(x, e^x) = e^x$.

$$\boxed{\frac{d}{dx}e^x = e^x}$$



We do need to consider Exponential Functions where the Chain Rule will need to be applied.

$$\boxed{\frac{d}{dx}e^{g(x)} = e^{g(x)} \cdot g'(x)}$$

Ex. 1 Differentiate

$y = x^2e^x$

$y' = x^2e^x + 2xe^x$

$= e^x(x^2+2x)$

$= xe^x(x+2)$

$y = e^{\sin x}$

$y' = e^{\sin x} \cdot \cos x$

Ex. 2 Find y' if $y = e^{-3x}\cos 2x$

$$y' = e^{-3x}(-\sin 2x)\cdot 2 + e^{-3x}\cdot(-3)\cos 2x$$

$$= -2e^{-3x}\sin 2x - 3e^{-3x}\cos 2x$$

$$= -e^{-3x}(2\sin 2x + 3\cos 2x)$$

Ex. 3 Find the absolute maximum value of the function $f(x) = xe^{-x}$

$$f'(x) = xe^{-x}(-1) + e^{-x}$$

$$= e^{-x}(1-x)$$

ct pt: $x = 1$

Interval	e^{-x}	$(1-x)$	$f'(x)$	$f(x)$
$(-\infty, 1)$	+	+	+	inc
$(1, \infty)$	+	−	−	dec

← local max

$$y = (1)(e^{-1})$$

$$\doteq 0.37$$

Ex. 4 Sketch the graph of $f(x) = e^{-x^2}$

Asymptote: no VA
no SA

$\lim_{x \to \pm\infty} \frac{1}{e^{x^2}} \to \frac{1}{\infty} = 0$

HA: $y = 0$

Domain: All Real #'s

Intercepts: $(0,1)$ y-int

no x-int

Symmetry: $f(-x) = e^{-(-x)^2}$
$= e^{-x}$

symmetric about the y-axis

$f'(x) = e^{-x^2} \cdot -2x$
$= -2xe^{-x^2}$ ← e^{-x^2} is always positive

when $x < 0$ $f'(x) > 0$ inc $(-\infty, 0)$
$x > 0$ $f'(x) < 0$ dec $(0, \infty)$

$f''(x) = -2xe^{-x^2} \cdot -2x + -2e^{-x^2}$
$= 4x^2e^{-x^2} - 2e^{-x^2}$
$= -2e^{-x^2}(-2x^2 + 1)$

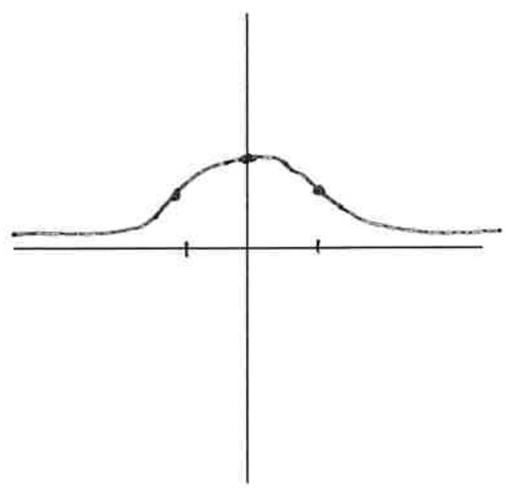
$f'(0) = 0$ so max at $(0,1)$

$f(-\frac{1}{\sqrt{2}}) = e^{-(-\frac{1}{\sqrt{2}})^2} \to e^{\frac{1}{2}}$
$f(\frac{1}{\sqrt{2}}) = e^{-(\frac{1}{\sqrt{2}})^2} \to e^{\frac{1}{2}}$

c pt: $\pm \frac{1}{\sqrt{2}}$

Interval	$-2e^{-x^2}$	$(-2x^2+1)$	f''	
$(-\infty, -\frac{1}{\sqrt{2}})$	−	−	+	cu
$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$	−	+	−	cd
$(\frac{1}{\sqrt{2}}, \infty)$	−	−	+	cu

← inflection
← inflection



Homework Assignment

- Practice Problems: 1ace, 2odd, 3-5