Section 7: Linear Equations

This book belongs to: ____________________ Block: ________

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<tr>
<th>Section</th>
<th>Due Date</th>
<th>Date Handed In</th>
<th>Level of Completion</th>
<th>Corrections Made and Understood</th>
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<td>7.1</td>
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<td>7.2</td>
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<td>7.3</td>
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Self-Assessment Rubric

<table>
<thead>
<tr>
<th>Category</th>
<th>Sub-Category</th>
<th>Description</th>
<th>Mark</th>
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<tbody>
<tr>
<td>Expert (Extending)</td>
<td>4</td>
<td>Work meets the objectives; is clear, error free, and demonstrates a mastery of the Learning Targets</td>
<td>“You could teach this!”</td>
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<td></td>
<td>3.5</td>
<td>Work meets the objectives; is clear, with some minor errors, and demonstrates a clear understanding of the Learning Targets</td>
<td>“Almost Perfect, one little error.”</td>
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<tr>
<td>Apprentice (Proficient)</td>
<td>3</td>
<td>Work almost meets the objectives; contains errors, and demonstrates sound reasoning and thought concerning the Learning Targets</td>
<td>“Good understanding with a few errors.”</td>
</tr>
<tr>
<td>Apprentice (Developing)</td>
<td>2</td>
<td>Work is in progress; contains errors, and demonstrates a partial understanding of the Learning Targets</td>
<td>“You are on the right track, but key concepts are missing.”</td>
</tr>
<tr>
<td>Novice (Emerging)</td>
<td>1.5</td>
<td>Work does not meet the objectives; frequent errors, and minimal understanding of the Learning Targets is demonstrated</td>
<td>“You have achieved the bare minimum to meet the learning outcome.”</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Work does not meet the objectives; there is no or minimal effort, and no understanding of the Learning Targets</td>
<td>“Learning Outcomes not met at this time.”</td>
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Learning Targets and Self-Evaluation

<table>
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<tr>
<th>L – T</th>
<th>Description</th>
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<tbody>
<tr>
<td>7 – 1</td>
<td>• Visual representation of ordered pairs on a Cartesian Plane</td>
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<td></td>
<td>• Drawing and discussing how a slope can be created by two or more points</td>
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<td></td>
<td>• Using Slope-Intercept form to identify the slope, y-intercept of a line</td>
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<td></td>
<td>• Graphing and interpreting graphs of straight lines</td>
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<tr>
<td>7 – 2</td>
<td>• Using the Standard Form of a linear equation to find intercepts</td>
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<td>• Identifying Graphs to match to a given equation and vice-versa</td>
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<td></td>
<td>• Converting from slope-intercept to standard form using algebraic principles</td>
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Comments:

________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
Competency Evaluation

A valuable aspect to the learning process involves self-reflection and efficacy. Research has shown that authentic self-reflection helps improve performance and effort, and can have a direct impact on the growth mindset of the individual. In order to grow and be a life-long learner we need to develop the capacity to monitor, evaluate, and know what and where we need to focus on improvement. Read the following list of Core Competency Outcomes and reflect on your behaviour, attitude, effort, and actions throughout this unit.

- Rank yourself on the left of each column: 4 (Excellent), 3 (Good), 2 (Satisfactory), 1 (Needs Improvement)
- I will rank your Competency Evaluation on the right half of each column

<table>
<thead>
<tr>
<th>Competency Outcomes</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
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<tbody>
<tr>
<td><strong>Personal Responsibility</strong></td>
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<td>- I listen during instruction and come ready to ask questions</td>
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<tr>
<td>- I am on time for class</td>
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<td>- I am fully prepared for the class, with all the required supplies</td>
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<td>- I am fully prepared for Tests</td>
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<tr>
<td>- I follow instructions keep my Workbook organized and tidy</td>
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<td>- I am on task during work blocks</td>
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<td>- I complete assignments on time</td>
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<td><strong>Self-Regulation</strong></td>
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<td>- I keep track of my Learning Targets</td>
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<td>- I take ownership over my goals, learning, and behaviour</td>
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<td>- I can solve problems myself and know when to ask for help</td>
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<td>- I can persevere in challenging tasks</td>
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<td>- I am actively engaged in lessons and discussions</td>
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<td>- I only use my phone for school tasks</td>
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<td><strong>Classroom Responsibility and Communication</strong></td>
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<td>- I am focused on the discussion and lessons</td>
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<td>- I ask questions during the lesson and class</td>
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<td>- I give my best effort and encourage others to work well</td>
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<td>- I am polite and communicate questions and concerns with my peers and teacher in a timely manner</td>
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<td>- I clean up after myself and leave the classroom tidy when I leave</td>
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<td><strong>Collaborative Actions</strong></td>
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<td>- I can work with others to achieve a common goal</td>
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<td>- I make contributions to my group</td>
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<td>- I am kind to others, can work collaboratively and build relationships with my peers</td>
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<td>- I can identify when others need support and provide it</td>
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<td><strong>Communication Skills</strong></td>
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<td>- I present informative clearly, in an organized way</td>
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<td>- I ask and respond to simple direct questions</td>
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<td>- I am an active listener, I support and encourage the speaker</td>
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<td>- I recognize that there are different points of view and can disagree respectfully</td>
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<td>- I do not interrupt or speak over others</td>
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<td><strong>Overall</strong></td>
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Goal for next Unit – refer to the above criteria. Please select (underline/highlight) two areas you want to focus on

Adrian Herlaar, School District 61
www.mrherlaar.weebly.com
Section 7.1 – Basics and Slope-Intercept Form

Mapping Points on a 2-D Grid

- Every equation of a straight-line (except 2 special ones) has specific criteria.
- They have 2 variables (unknowns), generally denoted \( x \), \( y \) and they have an \( = \) sign.
- All lines can be mapped on a 2-D grid, called it a Cartesian plane.

- The Grid is made up of 2 axes
- An \( x – axis \) and a \( y – axis \)
- The axes are both number lines
- The \( x – axis \) move left and right
- The \( y – axis \) move up and down

- In order to be a point found on the grid you need both an \( x – value \) and \( y – value \) denoted \( (x, y) \)
- Together they give the 2-D coordinates of points on the grid

Example: See above grid for placement

\((0, 0)\) Known and the ORIGIN

\((1, 2)\)

\((-4, 5)\)

\((8, -3)\)

\((-6, -9)\)
Slope-Intercept Equation

- Now think about this, a LINE is made up of an INFINITE number of individual points
- There are two equations we are going to talk about this year
- Here is the first one:

\[ y = mx + b \]

**SLOPE-INTERCEPT FORM**

- The variables in this equation are very important
  - The \( m \): Is the SLOPE of the line, represented \( \frac{\text{RISE}}{\text{RUN}} \)
  - \( \frac{\text{RISE}}{\text{RUN}} \) also \( \frac{\text{CHANGE IN HEIGHT}}{\text{CHANGE IN LENGTH}} \)
  - The Slope is the same from any point on the line to another
  - The SLOPE stays constant
  - The \( b \): Is the \( y \) - intercept, where the line crosses the \( y \) - axis
    - Why \( b \) and not \( y \) then?
    - You will soon find out...
    - Lastly, the \( x \) and \( y \).
    - They represent the \((x, y)\) coordinates of every possible point on the line

So, why \( b \) for the \( y \) - int?

- Have a look at the grid
- No matter where you cross the \( y \) - axis, what is the \( x \) - value?
- It is always, 0
- So every \( y \) - intercept, has the coordinates: \((0, b)\)
- The \( b \), is wherever it crosses the \( y \) - axis.
Every line (except 1 type) has a \( y - \text{intercept} \) and has a \( \text{slope} \). As an example:

\[
y = mx + b \quad \rightarrow \quad y = \frac{2}{3}x + 4
\]

- The \( \text{slope} (m) \) is: \( \frac{2}{3} \) For every 2 you go up, you go right 3, both positive

- Remember that the \( x \) and \( y \) represent every set of coordinates \((x, y)\)

- So the \( y - \text{intercept} \) is when the \( x - \text{value} \) is 0

- We can plug 0 in to the equation for \( x \) and then solve for \( y \)

\[
y = \frac{2}{3}x + 4
\]

\[
y = \frac{2}{3} \cdot 0 + 4 \quad \rightarrow \quad y = 0 + 4
\]

\[
y = 0 + 4
\]

\[
y = 4
\]

\[
\text{That's why } b \text{ is the } y - \text{intercept}
\]

Now let's go back to the \((x, y)\), remember that they represent the \textbf{coordinates of every possible point on the line}, they also are called \textbf{the solution} to the \( y = mx + b \) equation.

- What I mean by that is that when I plug the \( x \) and \( y \) values into the equation of a line, it \textbf{stays equal}.

**Example:**

\[
y = \frac{2}{3}x + 4
\]

i) When \( x = 0 \)

\[
y = \frac{2}{3} \cdot 0 + 4 \quad \rightarrow \quad y = 4
\]

Coordinates are: \((0, 4)\)

ii) When \( x = 3 \)

\[
y = \frac{2}{3} \cdot 3 + 4 \quad \rightarrow \quad y = 2 + 4 = 6
\]

Coordinates are: \((3, 6)\)

iii) When \( x = 6 \)

\[
y = \frac{2}{3} \cdot 6 + 4 \quad \rightarrow \quad y = 4 + 4 = 8
\]

Coordinates are: \((6, 8)\)
- We can continue this infinitely!
- Pick any \( x \) and solve for \( y \)
- Pick any \( y \) and solve for \( x \)

So, what do we know so far?

✓ Every \( y \) – intercept has an \( x \) – value of 0
✓ Every \( x \) – intercept has a \( y \) – value of 0
✓ We can find an infinite number of coordinates (solutions) for a line

Now for Slope we know a few things too.

✓ The Slope of a straight line is consistent
✓ The Slope can go up or down
✓ The Slope is \( \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change of height}}{\text{Change in length}} = \frac{\text{Change in } y}{\text{Change in } x} \)

We will come across 4 different types of lines. Their characteristics will results in 4 types of Slope. Look at them from Left to Right.

✓ The Rise is Positive
✓ The Run is Positive
✓ So that means the Slope will be: \( \frac{\text{Positive}}{\text{Positive}} = \text{Positive} \)

✓ The Rise is 0
✓ The Run is Finite
✓ So that means the Slope will be: \( \frac{0}{\text{Anything}} = 0 \)
✓ Anything divided by 0 is 0

✓ The Rise is Finite
✓ The Run is 0
✓ So that means the Slope will be: \( \frac{\text{Anything}}{0} = \text{Undefined} \)
✓ Can’t divide by zero
Solutions to a Line

- Next is figuring out if a point is on a line. That is the same as saying: Is the following point a solution to the equation of the line.

  ✓ If the point is a solution, then when you plug the \((x, y)\) into the given equation, it will stay equal, and the point is on the line

  ✓ If the point is not a solution, then when you plug the \((x, y)\) into the given equation, it will not stay equal, and the point is not on the line

Example: Does the line \(y = 2x + 5\) go through the point \((1,8)\)?

Solution:

- Since \(x\) is 1, we plug 1 in for \(x\) and since \(y\) is 8, we plug 8 in for \(y\).
- Work through the equation and see if it stays equal.
- If it does, it’s a solution (A point on the line)
- If it doesn’t, it’s not a solution (Not a point on the line)

\[
y = 2x + 5 \\
8 = 2(1) + 5 \\
8 = 2 + 5 \\
8 = 7
\]

- 8 DOES NOT EQUAL 7
- So that means that \((1,8)\) is NOT a solution to \(y = 2x + 5\)
- In other words, the point at \((1,8)\) is not on the line with the equation \(y = 2x + 5\)

Example:

- Does the line \(y = -\frac{2}{5}x + 6\) go through the point \((10,2)\)?

\[
y = -\frac{2}{5}(10) + 6 \\
2 = -\frac{2}{5}(10) + 6 \\
2 = -4 + 6 \\
2 = 2
\]

So that means, the point \((10,2)\) is a point on the line.
Writing the Equation of a Line

- We can also identify information in a graph that will allow us to write the equation of a line.
- This technique is limited to SLOPE-INTERCEPT FORM and graphs where the \( y - \text{intercept} \) is easily discernible.

What is the equation of the given line?

- Identify the Slope and the \( y - \text{intercept} \) and you’re done

1. The \( y - \text{intercept} \) is easy to see: \( (0, 5) \)

2. Now from left to right, count the SLOPE
   - Our RUN: We move 7 places in the positive direction
   - Our RISE: We move 5 places up in the positive direction

So the SLOPE is: \( \frac{5}{7} \)

The Equation of the line then is:

\[ y = \frac{5}{7}x + 5 \]

Try another one:

1. The \( y - \text{intercept} \) is easy to see: \( (0, 4) \)

2. Now from left to right, count the SLOPE
   - Our RUN: We move 8 places in the positive direction
   - Our RISE: We move 4 places up in the negative direction

So the SLOPE is: \( \frac{-4}{8} = -\frac{1}{2} = -\frac{1}{2} \)

The Equation of the line then is:

\[ y = -\frac{1}{2}x + 4 \]
Graphing Lines

- With the SLOPE-INTERCEPT equation it is pretty easy to graph lines too.
- We are given the SLOPE and the Y-INTERCEPT, so it is really quite simple.

- **Identify** the $y$– intercept from the equation and plot it
- Then from that point, **count out your SLOPE**
- Up and left, up and right, down and left, or down and right

Graph this: $y = \frac{5}{3}x - 2$

$y$– intercept: $(0, -2)$

Slope: \[
\frac{\text{Rise}}{\text{Run}} = \frac{5}{3}
\]

Let’s try a couple more:

Graph: $y = -2x + 1$

$y$– intercept: $(0, 1)$

Slope: \[
\frac{\text{Rise}}{\text{Run}} = \frac{-2}{1}
\]

Graph: $y = \frac{3}{4}x - 5$

$y$– intercept: $(0, -5)$

Slope: \[
\frac{\text{Rise}}{\text{Run}} = \frac{3}{4}
\]
**Vertical and Horizontal Lines**

**Horizontal Lines**

Let’s look at an example:

- What is the Slope?
- What is the $y$ - intercept?

- So the **Slope is 0**, and the $y$ - intercept is 5.
- But when else is $y = 5$?
- Does it matter what the $x$ - value is?
- So do we even need $x$ in our equation?

It turn out that every horizontal line is simply:

$$y = b$$

So in this case, the equation of the horizontal line is:

$$y = 5$$

**Vertical Lines**

- Vertical lines don’t have the same $y$ - value all the time, they have the same $x$ - value.
- So does the $y$ - value matter?
- Do we need it in our equation?

It turn out that every vertical line is simply:

$$x = a$$

So in this case, the equation of the vertical line is:

$$x = -3$$
Summary

\[ y = mx + b \]  
Is the equation for a diagonal line (Slope-Intercept)

\[ y = ? \]  
Is the equation of a horizontal line

\[ x = ? \]  
Is the equation of a vertical line

\[ b \]  
Is the value of the \( y \) – intercept

\( (x, y) \)  
The coordinates of the point on a line (also the Solution to the Equation)

\[ m \]  
Is the Slope

Is written:  
\[
\frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in height}}{\text{change in length}} = \frac{\text{Change in } y}{\text{Change in } x}
\]

Remember when counting out the Slope

You have a fraction so you can count 4 possible ways:

The first two give you a consistent POSITIVE SLOPE regardless of the direction you count

i) Up and to the Right (POSITIVE RISE/POSITIVE RUN)  
\[ \frac{A}{B} \]  which equals \[ \frac{A}{B} \]

ii) Down and to the Left (NEGATIVE RISE/NEGATIVE RUN)  
\[ \frac{-A}{-B} \]  which equals \[ \frac{A}{B} \]

The second two give you a consistent NEGATIVE SLOPE regardless of the direction you count

iii) Down and to the Right (NEGATIVE RISE/POSITIVE RUN)  
\[ \frac{-A}{B} \]  which equals \[ -\frac{A}{B} \]

iv) Up and the Left (POSITIVE RISE/NEGATIVE RUN)  
\[ \frac{A}{-B} \]  which equals \[ -\frac{A}{B} \]
Section 7.1 – Practice Questions

1. Map the following Coordinate \((x, y)\) on the 2-D plane (GRID)

\[
\begin{align*}
A(1, 3) & \quad B(9, -1) \\
C(-4, 4) & \quad D(-7, -7) \\
E(-5, -3) & \quad F(1, 8) \\
G(8, -2) & \quad H(-5, 2)
\end{align*}
\]

2. Identify the Coordinates of the given points

3. What does it mean to be a solution to an equation with respect to coordinates of a point?
4. What is the $y$-intercept? What is the $x$-coordinate of every $y$-intercept point? Example?

5. What is the $x$-intercept? What is the $y$-coordinate of every $x$-intercept point? Example?

6. What do you think of when you see the word SLOPE?

7. For the sake of our Math Vocabulary then:

$$SLOPE = \quad = \quad$$

8. Are the following points solutions to the given equations? Are they POINTS on the given LINE?

A) $(1, -3) \quad y = 3x - 5$

B) $(0, 5) \quad y = \frac{2}{3}x + 5$

C) $(-2, 7) \quad y = \frac{3}{2}x + 4$

D) $(8, -1) \quad y = -\frac{1}{8}x$
9. What is the SLOPE and Y-INTERCEPT of the following lines?

\[ \text{Slope} = \quad \text{Slope} = \]
\[ y - int = \quad y - int = \]

10. Graph the following lines.

a) \[ y = x \]

b) \[ y = \frac{2}{5} x + 4 \]
c) \( y = -2x + 7 \)

d) \( y = -\frac{3}{5}x - 5 \)

e) \( y = 3 \)

f) \( x = -4 \)
Section 7.2 – Standard Form

- This next equation is called: STANDARD FORM
- There is no obvious \( y \) – intercept or SLOPE
- It looks like this:

\[ Ax + By = C \]

- \( A > 0 \) and can’t be a fraction
- It is still the equation of a STRAIGHT LINE

Solutions of the Line

- We have a point with an \((x, y)\) coordinate
- We have an equation with an \(x\) and \(y\)
- If we plug in the coordinates and the equation stays equal, the line goes through the point!

Example:

- Does the line \( 3x - 2y = -6 \) go through the point \((2,6)\)?
- In other words:
- The \((2,6)\) a solution to the equation \(3x - 2y = -6\)?
  - So sub in 2 for \(x\) and 6 for \(y\)
    \[
    3(2) - 2(6) = -6 \\
    6 - 12 = -6 \\
    -6 = -6
    \]
  - Yes it is a solution; the line goes through the point!

Example:  Is \((1, 3)\) a solution to \(x - 3y = 9\)?
  - So sub in 1 for \(x\) and 3 for \(y\)
    \[
    (1) - 3(3) = 9 \\
    1 - 9 = 9 \\
    -8 = 9
    \]
  - No it is not a solution; the line does not go through the point!
Example: Is $(4, 0)$ a solution to $x = 4$?

- So sub in $4$ for $x$ and since there is no $y$, sub in nothing

$$4 = 4$$

Yes it is a solution; the line goes through the point!

Example: Is $(-2, 5)$ a solution to $3x + 2y = 4$?

- So sub in $-2$ for $x$ and $5$ for $y$

$$3(-2) + 2(5) = 4$$
$$-6 + 10 = 4$$
$$4 = 4$$

Yes it is a solution; the line goes through the point!

Graphing this Equation

- What do we know about $(x, y)$?
- What about when $x$ is $0$?
  - When $x$ is $0$, we have the $y$ – intercept $$(0, b)$$

- What about when $y$ is $0$?
  - When $y$ is $0$, we have the $x$ – intercept $$(a, 0)$$

- It’s easiest to start by finding these two points!
Example:
Let’s graph this: \( x + y = 4 \)

- Set yourself up a Table of Values

- It reads, when \( x \) is ... \( y \) is ...

i) \( \text{When } x \text{ is } 0 \)

\[
0 + y = 4 \\
y = 4
\]

ii) \( \text{When } y \text{ is } 0 \)

\[
x + 0 = 4 \\
x = 4
\]

iii) \( \text{When } x \text{ is } -4 \)

\[
-4 + y = 4 \\
y = 4 + 4 \\
y = 8
\]

- For the third point you can pick anything
- I highly suggest you take the value that you got when you solved for \( x \), and flip the sign
- If it was positive use the negative of it, and vice versa
- In this case we will take \(-4\)

So with the completed table of values we can graph it now

We have three points: \((0, 4), (4, 0), \text{ and } (-4, 8)\)
Example: Graph $4x - 3y = 12$

If $x = 0$:

$4(0) - 3y = 12$
$-3y = 12 \rightarrow y = -4$

If $y = 0$:

$4x - 3(0) = 12$
$4x = 12 \rightarrow x = 3$

Pick any point, I’ll use $x = -3$

$4(-3) - 3y = 12$
$-12 - 3y = 12 \rightarrow -3y = 24$
$y = -8$

Example: Graph the following. This will be a little tricky.

- Go back to algebra, how do we **remove multiple fractions**?
- Multiply everything by the **LCM**, in this case: 3
- Then make your table of values
- And graph the results

If $x = 0$:

$2(0) + 12y = 12$
$12y = 12 \rightarrow y = 1$

If $y = 0$:

$2x + 12(0) = 12$
$2x = 12 \rightarrow x = 6$

Pick any point, I’ll use $x = -6$

$2(-6) + 12y = 12$
$-12 + 12y = 12 \rightarrow 12y = 24$
$y = 2$
Example: Graph the following. This will be a little tricky.

\[ \frac{1}{2}x + \frac{2}{3}y = 2 \]

- Go back to algebra, how do we remove multiple fractions?
- Multiply everything by the LCM, in this case: 6
- Then make your table of values
- And graph the results

\[
6 \cdot \frac{1}{2}x + \frac{2}{3}y \cdot 6 = 2 \cdot 6 \rightarrow 3x + 4y = 12
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>-4</td>
<td>6</td>
</tr>
</tbody>
</table>

If \( x = 0 \):

\[ 3(0) + 4y = 12 \]

\[ 4y = 12 \rightarrow y = 3 \]

If \( y = 0 \):

\[ 3x + 4(0) = 12 \]

\[ 3x = 12 \rightarrow x = 4 \]

Pick any point, I'll use \( x = -4 \)

\[ 3(-4) + 4y = 12 \]

\[ -12 + 4y = 12 \rightarrow 4y = 24 \]

\[ y = 6 \]
Converting from Standard to Slope-Intercept

- Now, there will come a time where you would like to have SLOPE-INTERCEPT FORM or STANDARD FORM but you have the opposite.
- The good news is that you can always use ALGEBRA to manipulate the equation you have into the equation you want.

Example: Change the equation from Standard to Slope-Intercept Form

\[
3x - 4y = 6
\]

- Subtract 3x from both sides

\[
3x - 3x - 4y = 6 - 3x
\]

- Rearrange the equation

\[
-4y = -3x + 6
\]

- Divide everything by \(-4\)

\[
\frac{-4y}{-4} = \frac{-3x}{-4} + \frac{6}{-4}
\]

- Simplify all the fractions

\[
y = \frac{3}{4}x - \frac{3}{2}
\]

Example: Change the equation from Standard to Slope-Intercept Form

\[
-\frac{2}{3}x + \frac{1}{4}y = 7
\]

- We need \(y = mx + b\)

\[
12 \times -\frac{2}{3}x + \frac{1}{4}y \times 12 = 7 \times 12
\]

- Multiply by the LCM to remove fractions

\[
-8x + 3y = 84
\]

- Simplify the equation

\[
-8x + 8x + 3y = 84 + 8x
\]

- Add 8x to both sides

\[
3y = 8x + 84
\]

- Rearrange the equation

\[
\frac{3y}{3} = \frac{8x}{3} + \frac{84}{3}
\]

- Divide everything by 3

\[
y = \frac{8}{3}x + 28
\]

- Simplify all the fractions
Example: Change the equation from Standard to Slope-Intercept Form

\[
\frac{5}{3}x - \frac{1}{2}y = -2
\]

- We need \( y = mx + b \)

\[
6 \cdot \frac{5}{3}x - \frac{1}{2}y \cdot 6 = -2 \cdot 6
\]

- Multiply by the LCM to remove fractions

\[10x - 3y = -12\]

- Simplify the equation

\[10x - 10x - 3y = -2 - 10x\]

- Subtract 10x from both sides

\[-3y = -10x - 2\]

- Rearrange the equation

\[-3y = -\frac{10x}{3} \cdot \frac{2}{3}\]

- Divided everything by 3

\[y = \frac{10}{3}x - \frac{2}{3}\]

- Simplify all the fractions

Example: Change the equation from Standard to Slope-Intercept Form

\[
\frac{x + y}{4} = -7
\]

- We need \( y = mx + b \)

\[
4 \cdot \frac{x + y}{4} = -7 \cdot 4
\]

- Multiply by the LCM to remove fractions

\[x + y = -28\]

- Simplify the equation

\[x - x + y = -28 - x\]

- Subtract \( x \) from both sides

\[y = -x - 28\]

- Simplify the equation
Example: Change the equation from Slope-Intercept to Standard Form

\[ y = \frac{2}{3}x - 4 \]

- We need \( Ax + By = C \)

\[ 3 \cdot y = 3 \cdot \frac{2}{3}x - 4 \cdot 3 \]

- Multiply by the LCM to remove fractions

\[ 3y = 2x - 12 \]

- Simplify the equation

\[ 3y - 3y = 2x - 3y - 12 \]

- Subtract \( 3y \) from both sides

\[ 0 = 2x - 3y - 12 \]

- Rearrange the equation

\[ 12 + 0 = 2x - 3y - 12 + 12 \]

- Add 12 to both sides

\[ 12 = 2x - 3y \]

- Rearrange the equation

\[ 2x - 3y = 12 \]

- Make sure \( A \) is a natural number

Example: Change the equation from Slope-Intercept to Standard Form

\[ y = 5x + \frac{2}{3} \]

- We need \( Ax + By = C \)

\[ 3 \cdot y = 3 \cdot 5x + \frac{2}{3} \cdot 3 \]

- Multiply by the LCM to remove fractions

\[ 3y = 15x + 2 \]

- Simplify the equation

\[ 3y - 3y = 15x - 3y + 2 \]

- Subtract \( 3y \) from both sides

\[ 0 = 15x - 3y + 2 \]

- Rearrange the equation

\[ 0 - 2 = 15x - 3y + 2 - 2 \]

- Subtract 2 from both sides

\[ -2 = 15x - 3y \]

- Rearrange the equation

\[ 15x - 3y = -2 \]

- Make sure \( A \) is a natural number
Example: Change the equation from **Slope-Intercept to Standard Form**

\[ y = -\frac{3}{4} x + \frac{2}{3} \]

- We need \( Ax + By = C \)

\[ 12 \cdot y = -\frac{3}{4} x \cdot 12 + \frac{2}{3} \cdot 12 \]

- Multiply by the LCM to remove fractions

\[ 12y = -9x + 8 \]

- Simplify the equation

\[ 12y - 12y = -9x - 12y + 8 \]

- Subtract 12\(y\) from both sides

\[ 0 = -9x - 12y + 8 \]

- Rearrange the equation

\[ 0 - 8 = -9x - 12y + 8 - 8 \]

- Subtract 8 from both sides

\[ -8 = -9x - 12y \]

- Rearrange the equation

\[ -9x - 12y = -8 \]

- Rewrite as \( Ax + B y = C \)

\[ 9x + 12y = 8 \]

- Multiply everything by \(-1\) to make sure \(A\) is a natural number

---

Example: Change the equation from **Slope-Intercept to Standard Form**

\[ y = \frac{2}{3} x + 6 \]

- We need \( Ax + By = C \)

\[ 3 \cdot y = 3 \cdot \frac{2}{3} x + 6 \cdot 3 \]

- Multiply by the LCM to remove fractions

\[ 3y = 2x + 18 \]

- Simplify the equation

\[ 3y - 3y = 2x - 3y + 18 \]

- Subtract 3\(y\) from both sides

\[ 0 = 2x - 3y + 18 \]

- Rearrange the equation

\[ 0 - 18 = 2x - 3y + 18 - 18 \]

- Subtract 18 from both sides

\[ -18 = 2x - 3y \]

- Rearrange the equation

\[ 2x - 3y = -18 \]

- Make sure \(A\) is a natural number
Section 7.2 – Practice Questions

1. Are the following points solutions to the given equations? Are they POINTS on the given LINE?
   
a) \((2, 4)\) \hspace{1cm} 2x + 3y = 16
   
b) \((-6, 0)\) \hspace{1cm} \frac{1}{6}x + 13y = 1
   
c) \((8, -2)\) \hspace{1cm} x - 2y = 4
   
d) \((-3, -4)\) \hspace{1cm} 4x + 2y = -20

2. Graph the following equations, use the table of values to organize your coordinates.

i) \hspace{1cm} 2x - 3y = 12

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ ii) \quad -4x + 5y = 40 \]

\[ iii) \quad \frac{2}{3}x - y = 2 \]
3. How many points are there on a line? Explain your thinking.

4. Using your algebraic logic, manipulate the STANDARD FORM equations in to SLOPE-INTERCEPT equations and graph them.

a) $2x + 2y = -4$
b) \( \frac{3}{5} x - \frac{2}{3} y = \frac{2}{3} \)

c) \( 12x - 5y = 10 \)

d) \( -\frac{1}{6} x - \frac{2}{3} y = 2 \)
5. Using your algebraic logic, manipulate the SLOPE-INTERCEPT to STANDARD FORM, remember that \( Ax + By = C \) has NO FRACTIONS and \( A > 0 \)

a) \( y = \frac{2}{5}x + 6 \)

b) \( y = -7x - 4 \)

c) \( y = 5x - \frac{2}{3} \)

d) \(-4 + 3x = y\)
Section 7.3 – Identifying Graphs and Writing Equations

Writing Equations of Lines

Step 1: Identify the $y$ – intercept

Step 2: Identify two points on the grid where the line crosses an intersection of $x$ and $y$ gridlines perfectly

Step 3: Count your SLOPE Rise and Run

Step 4: Fill in the equation:

So the equation can be filled in to be:

$$y = \frac{5}{4}x + 2$$
Write the Equation of the Following Graphs

**Slope is:** \(-\frac{8}{6} = -\frac{4}{3}\)

**y - intercept is:** \((0, -3)\)

So: \(y = -\frac{4}{3}x - 3\)

**Slope is:** Undefined

**y - intercept is:** No y - int

Vertical Line, so: \(x = -6\)

**Slope is:** 0

**y - intercept is:** \((0, 4)\)

Horizontal Line, so: \(y = 4\)

**Slope is:** \(\frac{2}{12} = \frac{1}{6}\)

**y - intercept is:** \((0, 5)\)

So: \(y = \frac{1}{6}x + 5\)

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Matching Equations to Graphs

- The last concept we will look at in this section is matching graphs to equations and vice versa.
- The trick is to take the information we have and use it to our advantage.
- If the equation is in Slope-Intercept Form:
  - Just identify the $SLOPE$ and $y$ – intercept and use the process of elimination.
- If the equation is in Standard Form:
  - Use our skills to determine the $x$ and $y$ intercepts and match it to the graph.

Example: Identify the graph that matches the equation $y = 2x + 7$

- Right Away I know what?
  - My $y$ – intercept is $+7$
    - That immediately excludes the $2^{nd}$ graph
    - It goes through $-7$
  - My Slope is POSITIVE 2
    - That removes the $3^{rd}$ graph
    - It has a negative Slope

➢ Sometimes it’s even easier
Example:

Which graph matches the equation: \( y = -x + 1 \)

- Why is this so easy?

- Look at the Slope of the equation?
  - It’s NEGATIVE

- Which graph has a negative slope?
  - Only the third one!

When the equations are in Standard Form it’s a little different

- You can’t look at the SLOPE, Standard Form doesn’t have it
- You need to look at the points you can get
  - Namely: the \( x \) and \( y \) intercepts
- Remember:
  - That to find the \( x \) - intercept we set \( y \) to 0
  - That to find the \( y \) - intercept we set \( x \) to 0

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\( y \) - intercept
\( x \) - intercept
Example: Which graph matches the equation? \( x + y = 1 \)

When: \( x = 0; \quad y = 1 \)
When: \( y = 0; \quad x = 1 \)
The \( y \)-int = (0, 1)
The \( x \)-int = (1, 0)
So the only graph that matches that is: Graph 2

Example: Which graph matches the equation? \( 2x + y = 4 \)

When: \( x = 0; \quad y = 4 \)
When: \( y = 0; \quad x = 2 \)
The \( y \)-int = (0, 4)
The \( x \)-int = (2, 0)
So the only graph that matches that is: Graph 2
What about 1 graph and multiple equations?

- Use elimination and the information you have to narrow down the equations that fit

**Example:** What equation matches the following graph?

- Example: What equation matches the following graph?

**Example:** What equation matches the following graph?

**Example:** What equation matches the following graph?
Unit 7.3 – Practice Problems

1. Which graph represents \( y = -2x + 3 \) How do you know?

2. Which graph represents \( x + y = 7 \) How do you know?
3. Which graph represents \( y = \frac{3}{4}x - 4 \) How do you know?

4. Which graph represents \( 3x - 2y = 12 \) How do you know?
5. Which equation matches the graph below:

\[ y = 3x + 2 \]
\[ y = -3x + 2 \]
\[ y = 3x - 2 \]

6. Which equation matches the graph below:

\[ 2x + 3y = 6 \]
\[ -2x - 3y = 6 \]
\[ 2x - 3y = -6 \]
7. Write the equation of the following graph in SLOPE-INTERCEPT form, then manipulate it to STANDARD FORM
**Answer Key**

**Section 7.1**

1. ![Graph 1](image1)

3. It means the \((x, y)\) of a point sub into \(y = mx + b\) for the \(x\) and \(y\) and the equation stays equal.

4. Where the line goes through the \(y\) - axis; \(x\) - coordinate always 0; \((0, 4)\)

5. Where the line goes through the \(x\) - axis; \(y\) - coordinate always 0; \((4, 0)\)

6. Answers Vary

7. \[\text{Slope} = \frac{\text{Change in Height}}{\text{Change in Length}} = \frac{\text{RISE}}{\text{RUN}}\]

8. A: NO     B: YES     C: NO     D: YES

9. i) Slope is \(\frac{3}{4}\), \(y\) - int is \((0, 3)\) ii) Slope is \(\frac{1}{7}\), \(y\) - int is \((0, 4)\)

10. a) \[y = x\]

    ![Graph 2](image2)

    b) \[y = \frac{2}{5}x + 4\]

    ![Graph 3](image3)
c) \[ y = -2x + 7 \]

\[ \begin{align*}
\text{Graph for } y &= -2x + 7
\end{align*} \]

d) \[ y = -\frac{3}{5}x - 5 \]

\[ \begin{align*}
\text{Graph for } y &= -\frac{3}{5}x - 5
\end{align*} \]

e) \[ y = 3 \]

\[ \begin{align*}
\text{Graph for } y &= 3
\end{align*} \]

f) \[ x = -4 \]

\[ \begin{align*}
\text{Graph for } x &= -4
\end{align*} \]
Section 7.2

1. a) Yes  b) No  c) No  d) Yes

2. i) \(2x - 3y = 12\)

   iii) \(\frac{2}{3}x - y = 2\)

3. Infinite Number of Points on a line
   Explanation will Vary
4.  
   a) $2x + 2y = -4 \quad \rightarrow \quad y = -x - 2$

   

   b) $\frac{3}{5}x - \frac{2}{3}y = \frac{2}{3} \quad \rightarrow \quad y = \frac{9}{10}x - 1$

   

   c) $12x - 5y = 10 \quad \rightarrow \quad y = \frac{12}{5}x - 2$

   

   d) $-\frac{1}{6}x - \frac{2}{3}y = 2 \quad \rightarrow \quad y = -\frac{1}{4}x - 3$

   

5.  
   a) $2x - 5y = -30$
   b) $7x + y = -4$
   c) $15x - 3y = 2$
   d) $3x - y = 4$

---

Section 7.3

1. $2^{\text{nd}}$ Graph, Reasoning will vary
2. $1^{\text{st}}$ Graph, Reasoning will vary
3. $2^{\text{nd}}$ Graph, Reasoning will vary
4. $2^{\text{nd}}$ Graph, Reasoning will vary
5. $y = 3x + 2$
6. $2x - 3y = -6$
7. $2x + 7y = 42$