Section 7.6 – The Law of Cosines

This booklet belongs to:______________________Block:_____

- There five cases in which it is possible to solve a general triangle ABC
  - ASA, AAS, ASS (Ambiguous), SAS, and SSS
  - ASA and AAS were solved using the LAW of SINES
  - SAS and SSS are solved using the LAW of COSINES

The Law of Cosines

For any triangle $ABC$with correspomding sides $a, b, and\ c$:

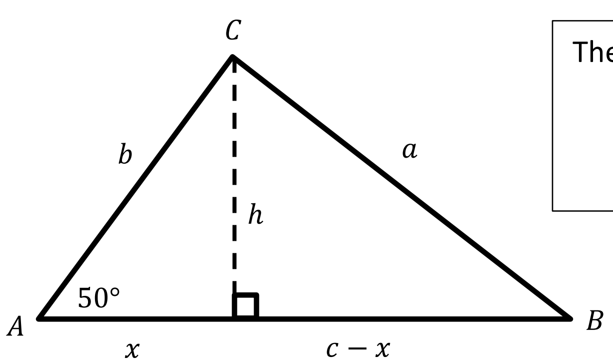
$$a^2 = b^2 + c^2 - 2bcCosA \qquad Cos\ A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2acCosB \qquad Cos\ B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2abCosC \qquad Cos\ C = \frac{a^2 + b^2 - c^2}{2ab}$$

Where did this come from?

- **Consider the Oblique triangle ABC**



The length c, is divided into two parts: $x\ and\ c - x$

$$Cos\ A = \frac{x}{b} \quad \rightarrow \quad x = bCosA$$

By Pythagorean Theorem:

$$b^2 = h^2 + x^2 \quad \rightarrow \quad h^2 = b^2 - x^2$$

$$a^2 = h^2 + (c - x)^2 \quad \rightarrow \quad h^2 = a^2 - (c - x)^2$$

By setting $h^2\ equal\ to\ one\ another$:

$$a^2 - (c - x)^2 = b^2 - x^2$$

$$\rightarrow \quad a^2 = b^2 - x^2 + (c - x)^2$$

$$\rightarrow \quad a^2 = b^2 - x^2 + (c - x)(c - x)$$

$$\rightarrow \quad a^2 = b^2 - x^2 + c^2 - 2cx + x^2$$

$$\rightarrow \quad a^2 = b^2 + c^2 - 2cx$$

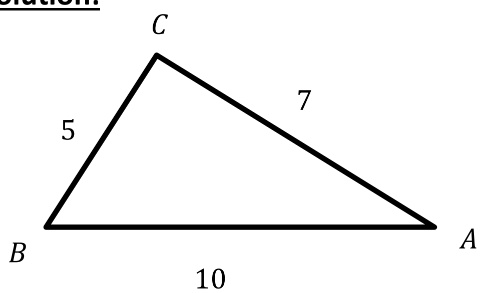
$$\rightarrow \quad a^2 = b^2 + c^2 - 2bcCosA$$

Using the Law of Cosines for SSS

- When you have a SSS triangle **ALWAYS** find the **largest angle first**.
- This will **guarantee** that the other two angles are **ACUTE**
- There is **NO AMBIGUOUS CASE** for the LAW of Cosines (THANK YOU!!)

Example: Solve $\Delta ABC, given\ a = 5, b = 7, and\ c = 10$

Solution:



$$c^2 = a^2 + b^2 - 2abCosC$$

$$10^2 = 5^2 + 7^2 - 2(5)(7)CosC$$

$$100 = 25 + 49 - 70CosC$$

$$100 = 74 - 70CosC$$

$$100 - 74 = -70CosC$$

$$26 = -70CosC$$

$$-\frac{26}{70} = CosC$$

$$Cos^{-1}\left(-\frac{26}{70}\right) = \angle C$$

$$\angle C = 111.8°$$

- Now we can use the LAW of SINES to find one of the other two angles.

$$\frac{Sin\ A}{a} = \frac{Sin\ C}{c} \rightarrow \frac{Sin\ A}{5} = \frac{Sin\ 111.8°}{10} \rightarrow Sin\ A = \frac{5(Sin\ 111.8)}{10} \rightarrow \angle A = 27.7°$$

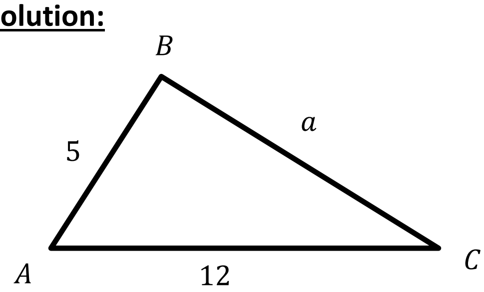
So, $180° - 27.7° - 111.8° = \angle B \quad \rightarrow \quad \angle B = 40.5°$

Note: If we had solved for another angle first we would have gotten the **WRONG** solution. **ALWAYS** solve the **LARGEST ANGLE FIRST** in a **SSS** problem!

Using the Law of Cosines for SAS

Example: Solve $\Delta ABC, given\ \angle A = 50°, b = 12, and\ c = 5$

Solution:



$$a^2 = b^2 + c^2 - 2bcCosA$$

$$a^2 = 12^2 + 5^2 - 2(5)(12)CosA$$

$$a^2 = 144 + 25 - 120Cos50°$$

$$a^2 = 169 - 120Cos50°$$

$$a^2 = 169 - 77.135$$

$$a^2 = 91.865$$

$$a = \sqrt{91.865}$$

$$a = 9.5846$$

- Now we can use the LAW of SINES to find one of the other two angles.

$$\frac{Sin\ A}{a} = \frac{Sin\ C}{C} \rightarrow \frac{Sin\ 50°}{9.58} = \frac{Sin\ C}{5} \rightarrow SinC = \frac{5(Sin\ 50°)}{9.58} \rightarrow \angle C = 23.6°$$

So, $180° - 23.6° - 50° = \angle B \rightarrow \angle B = 106.4°$

Note: If we had solved for another angle first we would have gotten the **WRONG** solution. **<u>ALWAYS</u>** solve the **SMALLEST ANGLE FIRST** in a **SAS** problem!

Summary of Law of Sines and Law of Cosines

Given	Method of Solving
ASA or AAS	1. Find the remaining angle using $\angle A + \angle B + \angle C = 180°$ 2. Find the remaining sides using the Law of Sines
ASS	Be aware of the ambiguous case, there may be 2 triangles possible 1. Find the angle using Law of Sines 2. Find the remaining angle using $\angle A + \angle B + \angle C = 180°$ 3. Find the remaining sides using the Law of Sines
SAS	1. Find the remaining side using the Law of Cosines 2. Find the **smaller** of the two remaining angles using Law of Sines 3. Find the remaining angle using $\angle A + \angle B + \angle C = 180°$
SSS	1. Find the **largest** angle using the Law of Cosines 2. Find one remaining angle using the Law of Sines 3. Find the remaining angle using $\angle A + \angle B + \angle C = 180°$

Section 7.6 – Practice Questions

Solve each Law of Cosines for the unknown part. Leave answer to 2 decimal places.

1. $a^2 = 5^2 + 3^2 - 2(5)(3)Cos43°$

2. $b^2 = 7^2 + 8^2 - 2(7)(8)Cos115°$

3. $c^2 = 4^2 + 6^2 - 2(4)(6)Cos90°$

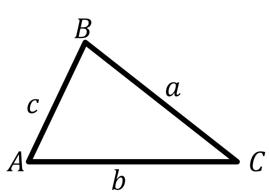
4. $7^2 = 3^2 + 6^2 - 2(3)(6)CosA°$

5. $5.3^2 = 2.7^2 + 4.6^2 - 2(2.7)(4.6)CosB°$

6. $9.3^2 = 6.2^2 + 4.5^2 - 2(6.2)(4.5)CosC°$

Given the following triangles, what angle should be solved for first, and which formula do you use?

7.



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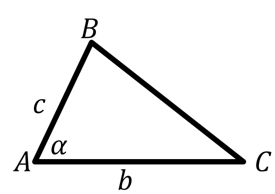

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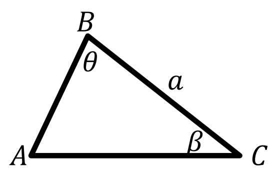

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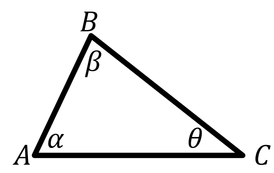

Solve ΔABC. Round answers to the 1 decimal place.

11. $\angle A = 50°, b = 10, c = 15$

12. $\angle B = 36°, a = 4, c = 10$

13. $\angle C = 60°, b = 4, a = 8$

14. $a = 7, b = 24, c = 25$

15. $a = 6, b = 7, c = 13$

16. $\angle A = 120°, b = 4, c = 1$

Solve ΔABC, using either the Law of Sines or Cosines to begin the answer.

17. $\angle A = 126°, b = 9, c = 12.2$

18. $\angle A = 28°, \angle B = 42°, c = 18.2$

19. $\angle C = 38°, b = 9, c = 7$

20. $\angle C = 100°, a = 10, c = 10$

21. $\angle A = 60°, a = 2\sqrt{3}, c = 4$

22. $a = 12.3, b = 9.6, c = 8.9$

Answer Key – Section 7.6

1. 3.47
2. 12.66
3. 7.21
4. $96.38°$
5. $89.17°$
6. $119.88°$
7. Find $\angle B$ by Law of Cosines
8. Find a by Law of Cosines
9. Find $\angle A$ by Sum of Angles Law
10. Nothing can be determined
11. $\angle B = 41.8°, \angle C = 88.2°, a = 11.5$
12. $\angle A = 19.2°, \angle C = 124.8°, b = 7.2$
13. $\angle A = 89.9°, \angle B = 30.1°, c = 6.9$
14. $\angle A = 16.3°, \angle B = 73.7°, \angle C = 90°$
15. $No\ Solution$
16. $\angle B = 49.1°, \angle C = 10.9°, a = 4.6$
17. $\angle B = 22.6°, \angle C = 31.4°, a = 18.9$
18. $\angle C = 110°, a = 9.1, b = 13.0$
19. $\angle A = 89.7°\ or\ 14.3°\ \angle B = 52.3°\ or\ 127.7°, a = 11.4\ or\ 2.8$
20. $No\ Solution$
21. $\angle B = 30°, \angle C = 90°, b = 2$
22. $\angle A = 83.3°, \angle B = 50.8°, \angle C = 45.9°$

Extra Work Space