Section 7.5 – The Law of Sines

This booklet belongs to:________________________Block:______

- We are about to see a number of possible cases of designing Triangles
- Given the information we can either construct 0, 1 or 2 Triangles
- The Law of Sines allows you to solve oblique triangles

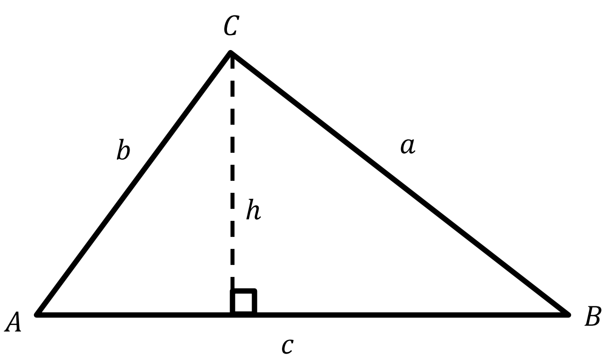
The Law of Sines

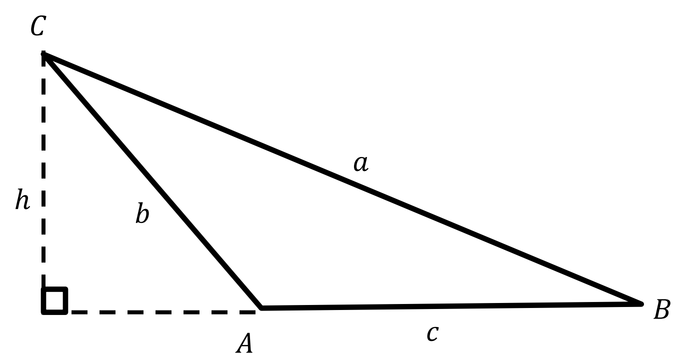
If ΔABC is a triangle with sides $a, b, and\ c, then$:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad or \quad \frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$$

The Law of Sines is derived from the following logic:

- $\Delta ABC\ can\ be\ an\ acute\ or\ obtuse\ oblique\ triangle$





- Let h be the altitude of either triangle

$\sin A = \frac{h}{b} \rightarrow \boldsymbol{b\sin A = h}$ and $\sin B = \frac{h}{a} \rightarrow \boldsymbol{a\sin B = h}$

Set them equal to one another we get: $bsinA = a\sin B$ or $\frac{\sin A}{a} = \frac{\sin B}{b}$

Similarly, in you move the vertex from vertex B to side AC, you end up with,

$$\frac{\sin A}{a} = \frac{sin\ C}{c}$$

Using the Sine Law

Example 1: Solve ΔABC, given $\angle B = 29°, \angle C = 105°, b = 30$

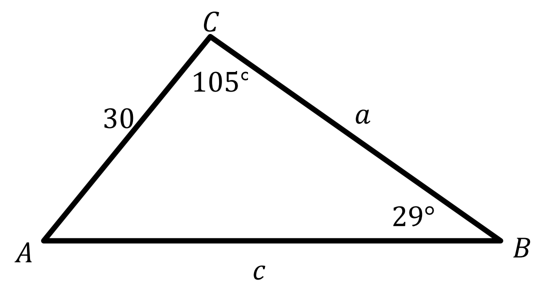
Solution 1: Draw the triangle to help visualize. **Set your Sine Law ratio up so the unknown is in the numerator, it makes the algebra more straight forward.**

Since we don't know angle A, let's find that first.

$\angle A + 29° + 105° = 180° \quad \rightarrow \quad \angle A = 46°$

$$\frac{30}{\sin 29°} = \frac{c}{\sin 105°} \quad \rightarrow \quad \mathbf{c} = \frac{30(\sin 105°)}{\sin 29°} \quad \rightarrow \quad \mathbf{59.8}$$

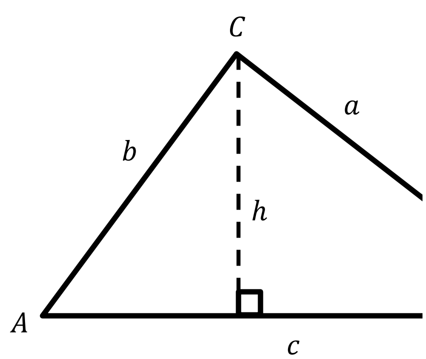
$$\frac{30}{\sin 29°} = \frac{a}{\sin 46°} \quad \rightarrow \quad \mathbf{a} = \frac{30(\sin 46°)}{\sin 29°} \quad \rightarrow \quad \mathbf{44.5}$$



The Ambiguous Case (ASS) Given $a, b, and\ \angle A$ in ΔABC

- Before looking at an example of an ASS triangle problem, the ambiguous case of ASS must be viewed.

Case 1: $a < h\ with\ \boldsymbol{h = b\ sinA}$ (No triangle Solution)



Possible triangles: 0

Example 2: Given ΔABC, with $\angle A = 30°, a = 4, b = 10, find\ \angle B$ in

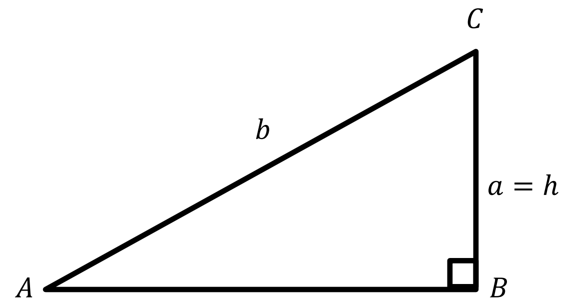
Solution 2:

$$\frac{\sin 30°}{4} = \frac{\sin B}{10} \rightarrow sinB = \frac{10(sin30°)}{4} = 1.25$$

- Since $sinB$ cannot be greater than 1, there is **no such $\boldsymbol{angle\ B}$**.
- Therefore, **no triangle can be made** with the given information

Case 2: $\boldsymbol{a = h}$ $(\boldsymbol{Right\ Triangle\ Solution})$

Possible Triangles: 1

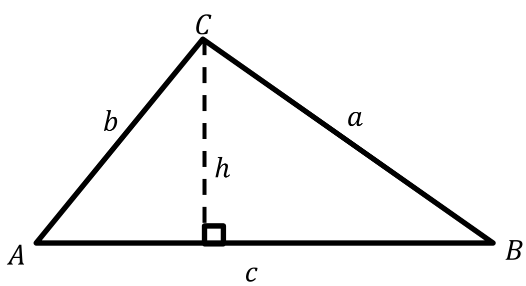


Example 3: Given ΔABC, with $\angle A = 30°$, $a = 5, b = 10, find\ \angle B$ in

Solution 3:

$$\frac{\sin 30°}{5} = \frac{\sin B}{10} \rightarrow\ sinB = \frac{10(sin30°)}{5} = 1 \rightarrow \angle B = 90°$$

Case 3: $\boldsymbol{a \geq b}$ $(\boldsymbol{Isosceles\ or\ Oblique\ Triangle\ Solution})$



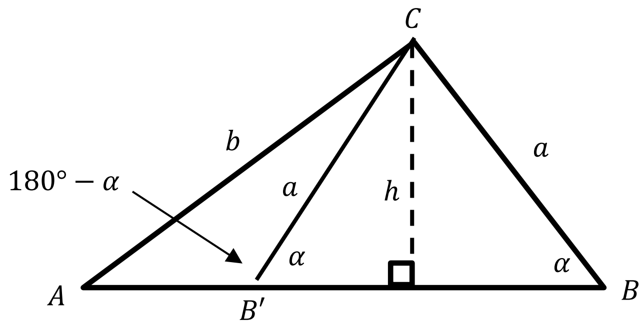
Possible Triangles: 1

Example 4: Given ΔABC, with $\angle A = 30°$, $a = 12, b = 10, find\ \angle B$ in

Solution 4:

$$\frac{\sin 30°}{12} = \frac{\sin B}{10} \rightarrow\ sinB = \frac{10(sin30°)}{12} = \frac{5}{12} \rightarrow \angle B = 24.6°$$

Case 4: $\boldsymbol{h < a < b\ (The\ Ambiguous\ Case)}$



Possible Triangles: 2

- Both triangles we can make $\Delta ABC\ and\ \Delta AB'C$ are two different triangles with the same ASS information

Example 5: Given ΔABC, with $\angle A = 30°, a = 7, b = 10, find\ \angle B$ in

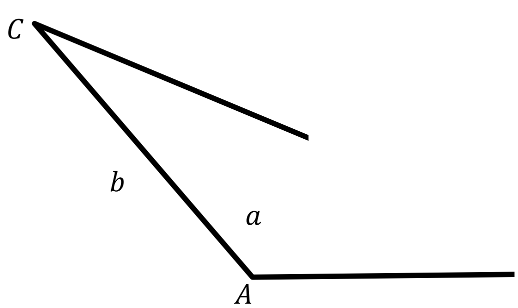
Solution 5:

$$\frac{\sin 30°}{7} = \frac{\sin B}{10} \rightarrow\ sinB = \frac{10(sin30°)}{7} = \frac{5}{7} \rightarrow \angle B = 45.6°$$

But since Sine is positive in quadrants I and II, another answer is:

$$\angle B' = 180 - 45.6° = 134.4°$$

Case 5: $\boldsymbol{\angle A\ Obtuse}$, $\boldsymbol{a \le b\ (No\ Triangle\ Solution)}$



Possible triangles: 0

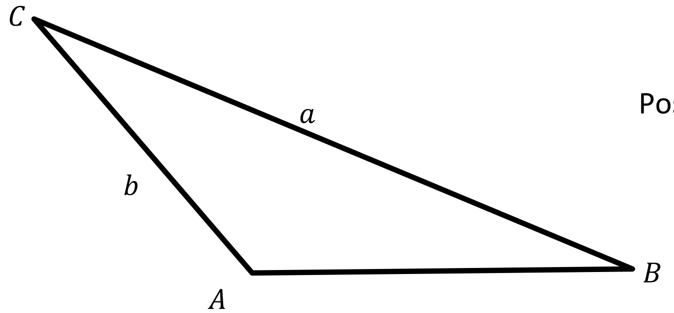
Example 6: Given ΔABC, with $\angle A = 120°, a = 8, b = 10, find\ \angle B$ in

Solution 6:

$$\frac{\sin 120°}{8} = \frac{\sin B}{10} \rightarrow\ sinB = \frac{10(sin120°)}{8} = 1.08$$

- Since $sinB$ cannot be greater than 1, there is no such $angle\ B$.
- Therefore, no triangle can be made with the given information

Case 6: $\angle A$ ***Obtuse and*** $a > b$ (***Obtuse Triangle Solution***)



Possible triangles: 1

Example 7: Given ΔABC, with $\angle A = 120°, a = 12, b = 10, find\ \angle B$ in

Solution 7:

$$\frac{\sin 120°}{12} = \frac{\sin B}{10} \rightarrow sinB = \frac{10(sin120°)}{12} = 0.72 \rightarrow \angle B = 46.2°$$

Example 8: The distance from the Sun (S) to Earth (E) and Venus (V) was $1.5\ x\ 10^8 km$ and $1.1\ x\ 10^8 km$ respectively when $\angle VES$ measured 28°. Find the distance between Venus and Earth.

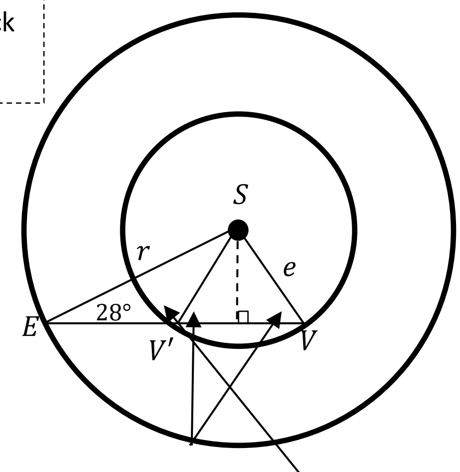
Solution 8:

Since this creates an ASS triangle, we need to check thee height to see if we have 1 or 2 possibilities

$$\sin 28° = \frac{h}{1.5\ x\ 10^8 km} \rightarrow h = 1.5\ x\ 10^8 km(\sin 28°)$$

$$h = 7.04\ x\ 10^7 km$$

Since $h < e < r$ $\quad$ *threre are 2 solutions*



$$\frac{\sin 28°}{1.1\ x\ 10^8} = \frac{\sin V}{1.5\ x\ 10^8} \rightarrow \sin V = \frac{1.5\ x\ 10^8(\sin 28°)}{1.1\ x\ 10^8} \rightarrow \boldsymbol{\angle V = 39.8°}$$

$$\angle SV'E = 180° - 39.8° = \boldsymbol{140.2°}$$

$$\angle ESV = 180° - 28° - 39.8° = \boldsymbol{112.2°}$$

$$\angle ESV = 180° - 28° - 140.2° = \boldsymbol{11.8°}$$

$$\frac{EV}{\sin 112.2°} = \frac{1.1\ x\ 10^8}{\sin 28°} \rightarrow \boldsymbol{EV = 2.2\ x\ 10^8 km}$$

$$\frac{EV'}{\sin 11.8°} = \frac{1.1\ x\ 10^8}{\sin 28°} \rightarrow \boldsymbol{EV' = 4.8\ x\ 10^7 km}$$

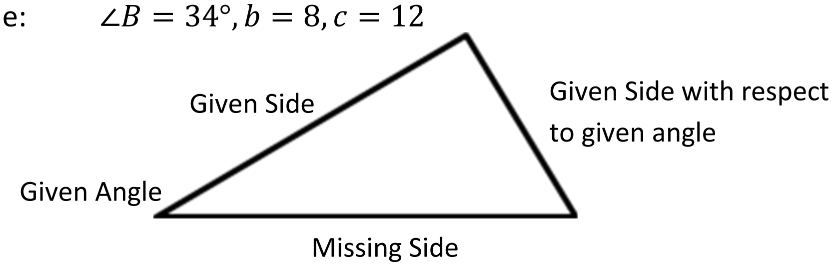
Summary

$$\frac{Sin\ A}{a} = \frac{SinB}{b} = \frac{Sin\ C}{c} \quad or \quad \frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$$

- $h = b * SinA \quad and \quad h = a * SinB$
- If $\angle A\ is\ acute\ (less\ than\ 90°)$
  - $a < h$ Possible triangles are: 0
  - $a = h$ Possible triangles are: 1
  - $a \geq b$ Possible triangles are: 1
  - $h < a < b$ Possible triangles are: 2
- If $\angle A\ is\ obtuse\ (more\ than\ 90°)$
  - $a \leq b$ Possible triangles are: 0
  - $a > b$ Possible triangles are: 1

- At both $0°\ and\ 180°$ $\sin t = 0$
- Anywhere between $0°\ and\ 180°$ $\sin t = positive$
- At $90°$ $\sin t = 1$
- $Sin\theta$ and $Sin(180 - \theta)$ are the same
- $Sin\theta$ is never greater than 1 or less than -1
- For the Ambiguous Case: The given sides are always the top two.

Example: $\angle B = 34°, b = 8, c = 12$



Section 7.5 – Practice Questions

Explain why no triangle is possible given the following information of ΔABC.

1.

$A = 38°$	$B = 69°$	$C = 73°$
$a = 12$	$b = 14$	$c = 13$

2.

$A = 42°$	$B = 65°$	$C = 70°$
$a = 7$	$b = 11$	$c = 12$

3.

$A = 39°$	$B = 46°$	$C = 95°$
$a = 5$	$b = 6$	$c = 12$

4.

$A = 120°$	$B = 20°$	$C = 40°$
$a = 12$	$b = 6$	$c = 12$

Find the sine angle equivalent to the following. $0° \leq \theta \leq 180°$

5. $Sin\ 10°$

6. $Sin\ 30°$

7. $Sin\ 42°$

8. $Sin\ 71°$

9. $Sin\ 121°$

10. $Sin\ 137°$

Determine if the set of data leads to 0, 1, or 2 triangles. Drawings help!

11. $\angle A = 60°, a = 11, b = 12$

12. $\angle A = 60°, a = 10, b = 12$

13. $\angle A = 60°, a = 12, b = 12$

14. $\angle A = 110°, a = 16, b = 12$

Given $\angle A$and side b, determine the lengths of side a that result in 0, 1, or 2 triangles.

15. $\angle A = 30°, b = 12$

16. $\angle A = 60°, b = 6\sqrt{3}$

Solve the unknown. If possible, determine the second angle, $0° \leq \theta \leq 180°$, and see if it satisfies the proportion.

17. $\frac{Sin\ A}{10} = \frac{Sin\ 40°}{30}$

18. $\frac{Sin\ A}{200} = \frac{Sin\ 20°}{50}$

19. $\frac{Sin\ A}{10} = \frac{Sin\ 30°}{5}$

20. $\frac{Sin\ A}{40} = \frac{Sin\ 57°}{53}$

21. $\frac{Sin\ A}{3} = \frac{Sin\ 125°}{5}$

22. $\frac{Sin\ A}{7.3} = \frac{Sin\ 12°}{1.3}$

Solve each triangle using the Law of Sines. If two triangles exist, solve both. Drawings help. Think about where each angle is placed, the letters my be different than you are used too.

23. $\angle A = 140°, \angle C = 25°, a = 20$

24. $\angle B = 46°, \angle C = 27°, a = 120$

25. $\angle B = 60°, b = 4\sqrt{3}, a = 8$

26. $\angle A = 74°, b = 8.1, a = 7$

27. $\angle A = 10°, \angle B = 60°, a = 4.5$

28. $\angle B = 40°, b = 55, c = 80$

Answer Key – Section 7.5

1. Largest Angle DOES NOT have the largest side	15. $a < 6$ (0); $a = 6, a \geq 12$ (1); $6 < a < 12$ (2)
2. Angles DO NOT add to 180°	16. $a < 9$ (0); $a = 9, a \geq 6\sqrt{3}$ (1); $9 < a < 6\sqrt{3}$ (2)
3. Sum of the two smaller sides LESS than the other	17. $12.4°, 167.6°$
4. Two angles of different degrees have same sides	18. $None$
5. $170°$	19. $90°$
6. $150°$	20. $39.3°, 140.7°$
7. $138°$	21. $29.4°, 150.6°$
8. $109°$	22. $None°$
9. $59°$	23. $\angle B = 15°, b = 8.1, c = 13.1$
10. $43°$	24. $\angle A = 107°, b = 90.3, c = 57$
11. 2	25. $\angle A = 90°, \angle C = 30°, c = 4$
12. 0	26. $No\ Triangle\ can\ be\ Made$
13. 1	27. $\angle C = 110°, b = 22.4, c = 24.4$
14. 1	28. $\angle A = 70.8°\ or\ 29.2°, \angle C 69.2°\ or\ 110.8°,$ $a = 80.8\ or\ 41.7$

Extra Work Space