Section 7.4 – Practice Problems

1. Find the exact value of each expression.

a) $\sin 15°$

$\sin 15 \rightarrow (\sin 45° - \sin 30°)$

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \rightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}$

b) $\cos(-75°)$

Q4 so cos still positive

$\cos 75 \rightarrow \cos 45 + \cos 30$

$\cos 45 \cos 30 - \sin 45 \sin 30$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \rightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}$

c) $\tan\frac{5\pi}{12} \rightarrow \tan\frac{3\pi}{12} + \tan\frac{2\pi}{12}$

$\tan\frac{\pi}{4} + \tan\frac{\pi}{6}$

$\frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}} \rightarrow \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\left(\frac{1}{\sqrt{3}}\right)} \rightarrow \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$\frac{\sqrt{3}+1}{\sqrt{3}-1}$ radicalize denominator $\rightarrow \frac{4+2\sqrt{3}}{3-1} \rightarrow \boxed{2+\sqrt{3}}$

d) $\cot\frac{11\pi}{12} \rightarrow \frac{3\pi}{12} + \frac{8\pi}{12} \rightarrow \frac{\pi}{4} + \frac{2\pi}{3}$

$\frac{1}{\tan\left(\frac{\pi}{4}+\frac{2\pi}{3}\right)}$

$\frac{1}{\frac{\tan\frac{\pi}{4} + \tan\frac{2\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{2\pi}{3}}} \rightarrow \frac{1}{\frac{1+(-\sqrt{3})}{1-1(-\sqrt{3})}} \rightarrow \frac{1+\sqrt{3}}{1-\sqrt{3}}$

$\frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{(1+\sqrt{3})} \rightarrow \frac{1+2\sqrt{3}+3}{1-3}$

$\frac{4+2\sqrt{3}}{-2} \rightarrow \boxed{-2-\sqrt{3}}$

e) $\sec\frac{19\pi}{12} \rightarrow \frac{1}{\cos\frac{19\pi}{12}} \rightarrow \frac{1}{\cos\frac{9\pi}{12} + \frac{10\pi}{12}}$

Q2

$\frac{1}{\cos\frac{3\pi}{4} + \frac{5\pi}{6}} \rightarrow \frac{1}{\cos\frac{3\pi}{4}\cos\frac{5\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{5\pi}{6}}$

$\frac{1}{-\frac{1}{\sqrt{2}} \cdot -\frac{\sqrt{3}}{2} - \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)} \rightarrow \frac{1}{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}}$

$\frac{1}{\frac{\sqrt{3}-1}{2\sqrt{2}}} \rightarrow \frac{2\sqrt{2}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{(\sqrt{3}+1)} \rightarrow \frac{2\sqrt{2}(\sqrt{3}+1)}{3-1}$

$\frac{2\sqrt{6}+2\sqrt{2}}{2} \rightarrow \sqrt{6}+\sqrt{2}$

f) $\csc(-105°) \rightarrow \frac{1}{\sin(-105)} \rightarrow \sin(-\theta) = -\sin\theta$

$\frac{1}{-\sin 105} = \frac{-1}{\sin 105} \rightarrow \frac{-1}{\sin(45+60)}$

$\frac{-1}{\sin 45 \cos 60 + \cos 45 \sin 60} \rightarrow \frac{-1}{\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}}$

$\frac{-1}{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}} = \frac{-1}{\frac{1+\sqrt{3}}{2\sqrt{2}}} \rightarrow \boxed{-\frac{2\sqrt{2}}{1+\sqrt{3}}}$

can rationalize but we'll leave it

2. Simplify each expression

a) $\sin 24° \cos 36° + \cos 24° \sin 36°$

$\sin(24+36)$

$\sin(60) = \frac{\sqrt{3}}{2}$

b) $\cos 55° \cos 10° + \sin 55° \sin 10°$

$\cos(55-10)$

$\cos 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

c) $\dfrac{\tan\frac{\pi}{5} - \tan\frac{\pi}{30}}{1 + \tan\frac{\pi}{5}\tan\frac{\pi}{30}}$

$\tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right)$

$\tan\left(\frac{6\pi}{30} - \frac{\pi}{30}\right) \rightarrow \tan\frac{5\pi}{30}$

$\tan\frac{\pi}{6} \rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

d) $\sin\frac{23\pi}{18}\cos\frac{\pi}{9} - \cos\frac{23\pi}{18}\sin\frac{\pi}{9}$

$\sin\left(\frac{23\pi}{18} - \frac{\pi}{9}\right) \rightarrow \sin\left(\frac{23\pi}{18} - \frac{2\pi}{18}\right)$

$\sin\left(\frac{21\pi}{18}\right) \rightarrow \sin\frac{7\pi}{6} \rightarrow \sin\frac{\pi}{6}$

Ref angle in Q3

$\downarrow$

$-\frac{1}{2}$

e) $\cos\frac{\pi}{8}\cos\frac{7\pi}{8} + \sin\frac{\pi}{8}\sin\frac{7\pi}{8}$

$\cos\left(\frac{\pi}{8} - \frac{7\pi}{8}\right)$

$\cos\left(-\frac{6\pi}{8}\right) = \cos\frac{6\pi}{8}$

$\downarrow$

$\boxed{-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}} \leftarrow \cos\frac{3\pi}{4}$ ✓ Q2 Ref angle $\frac{\pi}{4}$

f) $\dfrac{\tan\frac{2\pi}{9} + \tan\frac{\pi}{9}}{1 - \tan\frac{2\pi}{9}\tan\frac{\pi}{9}}$

$\tan\left(\frac{2\pi}{9} + \frac{\pi}{9}\right)$

$\tan\frac{3\pi}{9} \rightarrow \tan\frac{\pi}{3}$

$\downarrow$

$\sqrt{3}$

g) $\dfrac{\sin 3x}{\csc x} - \dfrac{\cos 3x}{\sec x}$

$\dfrac{\sin 3x}{\frac{1}{\sin x}} - \dfrac{\cos 3x}{\frac{1}{\cos x}} \rightarrow \sin 3x \sin x - \cos 3x \cos x$

$-\cos 3x \cos x + \sin 3x \sin x$

$-(\cos 3x \cos x - \sin 3x \sin x) \Rightarrow -\cos(3x + x)$

$\boxed{-\cos 4x}$

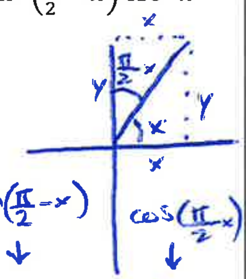
h) $\cos(A+B)\cos B + \sin(A+B)\sin B$

$\cos(A + B - B)$

$\boxed{\cos A}$

i) $\underbrace{\tan^2\left(\frac{\pi}{2} - x\right)\sec^2 x}$ $- \sin^2\left(\frac{\pi}{2} - x\right)\csc^2 x$

$\dfrac{\sin^2\left(\frac{\pi}{2} - x\right)}{\cos^2\left(\frac{\pi}{2} - x\right)} \cdot \dfrac{1}{\cos^2 x}$



$\sin\left(\frac{\pi}{2} - x\right) \downarrow \cos x$

$\cos\left(\frac{\pi}{2} - x\right) \downarrow \sin x$

$\dfrac{\cos^2 x}{\sin^2 x} \cdot \dfrac{1}{\cos^2 x} - \cos^2 x \cdot \dfrac{1}{\sin^2 x}$

$\dfrac{1}{\sin^2 x} - \dfrac{1}{\sin^2 x} = \boxed{0}$

j) $\sin\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right)\sin\left(\frac{\pi}{3} + x\right)$

$\sin\left(\frac{\pi}{3} - x + \frac{\pi}{3} + x\right)$

$\sin\left(\frac{2\pi}{3}\right) \leftarrow$ Q2 ref angle: $\frac{\pi}{3}$

$\dfrac{\sqrt{3}}{2}$

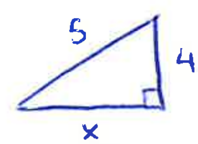
3. Find the exact value of each expression

a) $\tan x = 3$, *find* $\tan\left(x + \frac{\pi}{4}\right)$

$\dfrac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan\frac{\pi}{4}} \rightarrow \dfrac{3 + 1}{1 - 3(1)} = \dfrac{4}{-2}$

$\boxed{-2}$

b) $\sin x = \frac{4}{5}$, x is in $Q1$, find $\sin(x + \frac{\pi}{6})$

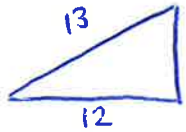


$x = 3$

$\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}$

$\frac{4}{5}\left(\frac{\sqrt{3}}{2}\right) + \frac{3}{5}\left(\frac{1}{2}\right)$

$\dfrac{2\sqrt{3}}{5} + \dfrac{3}{10} \rightarrow \dfrac{4\sqrt{3}}{10} + \dfrac{3}{10} = \boxed{\dfrac{4\sqrt{3} + 3}{10}}$



Pythagorean Theorem

Ref angle $\frac{\pi}{3}$ Q2 so negative

c) $\cos x = \frac{12}{13}$, x is in $Q1$, find $\cos(x + \frac{2\pi}{3})$

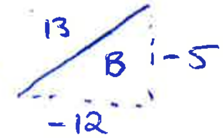
$\cos x \cos \frac{2\pi}{3} - \sin x \sin \frac{2\pi}{3}$

ref angle $\frac{\pi}{3}$ Q2 positive

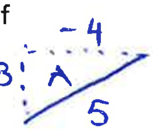
$\frac{12}{13} \cdot \left(\frac{-1}{2}\right) - \frac{5}{13}\left(\frac{\sqrt{3}}{2}\right)$

$\frac{-6}{13} - \frac{3\sqrt{3}}{26} \rightarrow \frac{-12}{26} - \frac{5\sqrt{3}}{26}$

$\boxed{\frac{-12-5\sqrt{3}}{26}}$



d) If both A and B are third quadrant angles, and $\cos B = -\frac{12}{13}$, what is the value of $\sin(A - B)$ if $\sin A = -\frac{3}{5}$



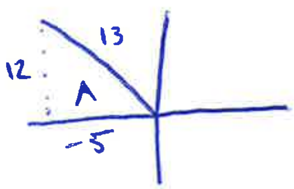
$\sin A \cos B - \cos A \sin B$

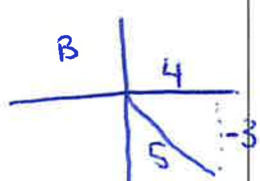
$\frac{-3}{5} \cdot \left(\frac{-12}{13}\right) - \left(\frac{-4}{5}\right)\left(-\frac{5}{13}\right)$

$\frac{36}{65} - \frac{20}{65}$

$\boxed{\frac{16}{65}}$

e) Given that $\sin A = \frac{12}{13}$ is in $Q2$, and $\sec B = \frac{5}{4}$, is in $Q4$, what is the value of $\cos(A + B)$





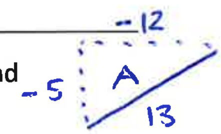
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

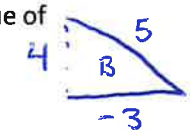
$\frac{-5}{13} \cdot \frac{4}{5} - \frac{12}{13} \cdot \left(\frac{-3}{5}\right)$

$-\frac{20}{65} - \left(\frac{-36}{65}\right)$

$-\frac{20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$

f) Given that $\tan A = \frac{5}{12}$ is in $Q3$, and $\cos B = -\frac{3}{5}$, is in $Q2$, what is the value of $\sin(A - B)$





$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\frac{-5}{13}\left(\frac{-3}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{4}{5}\right)$

$\frac{15}{65} + \frac{48}{65}$

$\boxed{\frac{63}{65}}$

4. Find the solution, $0° \le \theta < 360°$, or $0 \le x < 2\pi$, for each equation.

a) $\cos\theta\cos 10° - \sin\theta\sin 10° = \frac{1}{2}$

$\cos\theta\cos 10 - \sin\theta\sin 10 = \frac{1}{2}$

$\cos(\theta + 10) = \frac{1}{2}$ ← positive Q1 Q4

Ref angle: $60°$ or $\frac{\pi}{3}$

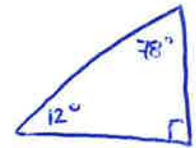
Q1: $\theta + 10 = 60$

$\boxed{\theta = 50°}$

Q4: $\theta + 10 = 300$

$\boxed{\theta = 290°}$

b) $\sin\theta\cos 12° + \cos\theta\cos 78° = \frac{\sqrt{3}}{2}$



$\cos 12° = \sin 78°$

$\sin 12° = \cos 78°$

↓

$\sin\theta\cos 12° + \cos\theta\sin 12° = \frac{\sqrt{3}}{2}$

$\sin(\theta + 12) = \frac{\sqrt{3}}{2}$ ← Q1 Q2

ref angle $\frac{\pi}{3}$ or $60°$

Q1

$\theta + 12 = 60°$

$\boxed{\theta = 48°}$

Q2

$\theta + 12 = 120°$

$\boxed{\theta = 108°}$

c) $\cos 3x\cos x + \sin 3x\sin x = 0$

A B A B

consider $P = \pi$ so 4 solutions

$\cos(3x - x) = 0$

$\cos 2x = 0$ ← $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

$2x = \frac{\pi}{2}$

$2x = \frac{\pi}{2} + 2\pi$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$2x = \frac{3\pi}{2}$

$2x = \frac{3\pi}{2} + 2\pi$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

d) $2\tan x + \tan(\pi - x) = \sqrt{3}$

$2\tan x + \frac{\tan\pi - \tan x}{1 - \tan\pi\tan x} = \sqrt{3}$

$\tan\pi = 0$

$2\tan x + \frac{0 - \tan x}{1 - 0} = \sqrt{3}$

$2\tan x - \tan x = \sqrt{3}$

$\tan x = \sqrt{3}$ ← Q1 Q3

Ref angle $\frac{\pi}{3}$

Q1: $\frac{\pi}{3}$

Q3: $\frac{4\pi}{3}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

e) $\sqrt{2}\sin 3x\cos 2x = 1 + \sqrt{2}\cos 3x\sin 2x$

$\sqrt{2}\sin 3x\cos 2x - \sqrt{2}\cos 3x\sin 2x = 1$

$\sqrt{2}(\sin 3x\cos 2x - \cos 3x\sin 2x) = 1$

$\sin(3x-2x) = \frac{1}{\sqrt{2}}$

$\sin x = \frac{1}{\sqrt{2}}$ Ref angle: $\frac{\pi}{4}$ Q1 Q2

Q1 $x = \frac{\pi}{4}$

Q2 $x = \frac{3\pi}{4}$

f) $\cos\left(x+\frac{\pi}{4}\right) + \cos\left(x-\frac{\pi}{4}\right) = 1$

$\left(\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4}\right) + \left(\cos x\cos\frac{\pi}{4} + \sin x\sin\frac{\pi}{4}\right) = 1$

$\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right) + \left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right) = 1$

$\frac{1}{\sqrt{2}}(\cos x - \sin x) + \frac{1}{\sqrt{2}}(\cos x + \sin x) = 1$

$\cos x - \sin x + \cos x + \sin x = \sqrt{2}$

$2\cos x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$ ← Ref angle: $\frac{\pi}{4}$ Q1 Q4

Q1: $\frac{\pi}{4}$ Q4: $\frac{7\pi}{4}$

5. Prove the identities

a) $\sin(A+B) - \sin(A-B) = 2\sin B\cos A$

$\sin A\cos B + \cos A\sin B - [\sin A\cos B - \cos A\sin B] = 2\sin B\cos A$

$\cancel{\sin A\cos B} + \cos A\sin B - \cancel{\sin A\cos B} + \cos A\sin B = 2\sin B\cos A$

$2\cos A\sin B = 2\sin B\cos A$

↑

rearrange

$2\sin B\cos A$ = RHS

b) $\frac{\sin(A+B)}{\cos(A-B)} = \frac{\cot A + \cot B}{1 + \cot A\cot B}$

↓

$\frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B + \sin A\sin B}$

Multiply top/bottom by $\frac{1}{\sin A\sin B}$ ← will create cot scenarios

$$\frac{(\sin A\cos B + \cos A\sin B)\cdot\frac{1}{\sin A\sin B}}{(\cos A\cos B + \sin A\sin B)\cdot\frac{1}{\sin A\sin B}}$$

$$\frac{\frac{\cancel{\sin A}\cos B}{\cancel{\sin A}\sin B} + \frac{\cos A\cancel{\sin B}}{\sin A\cancel{\sin B}}}{\frac{\cos A\cos B}{\sin A\sin B} + \frac{\cancel{\sin A\sin B}}{\cancel{\sin A\sin B}}} \rightarrow \frac{\cot B + \cot A}{\cot A\cot B + 1} = \text{RHS}$$

$\tan\frac{\pi}{4} = 1$

c) $\dfrac{\sin(A-B)}{\sin B} + \dfrac{\cos(A-B)}{\cos B} = \dfrac{\sin A}{\sin B\cos B}$

$\dfrac{\sin A\cos B - \cos A\sin B}{\sin B} + \dfrac{\cos A\cos B + \sin A\sin B}{\cos B}$

$\dfrac{\cos B(\sin A\cos B - \cos A\sin B) + \sin B(\cos A\cos B + \sin A\sin B)}{\sin B\cos B}$

$\dfrac{\sin A\cos^2 B - \cancel{\cos A\cos B\sin B} + \cancel{\cos A\cos B\sin B} + \sin A\sin^2 B}{\sin B\cos B}$

$\dfrac{\sin A\cos^2 B + \sin A\sin^2 B}{\sin B\cos B} \Rightarrow \dfrac{\sin A\overbrace{(\cos^2 B + \sin^2 B)}^{=1}}{\sin B\cos B}$

$\dfrac{\sin A}{\sin B\cos B} = RHS$

d) $\dfrac{1+\tan A}{\tan\left(A+\frac{\pi}{4}\right)} = 1 - \tan A$

$\dfrac{1+\tan A}{\left[\dfrac{\tan A + \tan\frac{\pi}{4}}{1 - \tan A\tan\frac{\pi}{4}}\right]}$

$\downarrow$

$1+\tan A \cdot \dfrac{1 - \tan A\tan\frac{\pi}{4}}{\tan A + \tan\frac{\pi}{4}}$

$(1+\tan A) \cdot \dfrac{1-\tan A}{(\tan A + 1)} \rightarrow 1 - \tan A = RHS$

e) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

$(\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$\sin^2 A\cos^2 B - \cancel{\sin A\sin B\cos A\cos B} + \cancel{\sin A\sin B\cos A\cos B} - \cos^2 A\sin^2 B$

$\sin^2 A\cos^2 B - \cos^2 A\sin^2 B$ Pythagorean Identities

$\sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$\sin^2 A - \cancel{\sin^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}$

$\sin^2 A - \sin^2 B = RHS$

f) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$(\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$\cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$\cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$\cos^2 A - \cos^2 A\cos^2 B - \sin^2 B + \cos^2 A\sin^2 B$

$\cos^2 A - \sin^2 B = RHS$

g) $\sec(A+B) = \dfrac{\sec A \sec B}{1-\tan A \tan B}$

$\dfrac{1}{\cos(A+B)}$

$\dfrac{1}{\cos A\cos B - \sin A\sin B}$

$\dfrac{\frac{1}{\cos A}\cdot\frac{1}{\cos B}}{1-\frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}}$

$\downarrow$

$\dfrac{\frac{1}{\cos A\cos B}}{\frac{\cos A\cos B - \sin A\sin B}{\cos A\cos B}}$

$\dfrac{1}{\cancel{\cos A\cos B}}\cdot\dfrac{\cancel{\cos A\cos B}}{\cos A\cos B - \sin A\sin B}$

$\dfrac{1}{\cos A\cos B - \sin A\sin B}$

LHS = RHS

h) $\csc(A-B) = \dfrac{\csc A \csc B}{\cot B - \cot A}$

$\dfrac{1}{\sin(A-B)}$

$\downarrow$

$\dfrac{1}{\sin A\cos B - \cos A\sin B}$

$\dfrac{\frac{1}{\sin A}\cdot\frac{1}{\sin B}}{\frac{\cos B}{\sin B}-\frac{\cos A}{\sin A}} \rightarrow \dfrac{\frac{1}{\sin A\sin B}}{\frac{\cos B\sin A - \cos A\sin B}{\sin B\sin A}}$

$\downarrow$

$\dfrac{1}{\sin A\sin B}\cdot\dfrac{\sin B\sin A}{\cos B\sin A - \cos A\sin B}$

$\dfrac{1}{\cos B\sin A - \cos A\sin B}$

$\downarrow$

$\dfrac{1}{\sin A\cos B - \cos A\sin B}$

LHS = RHS

6. Simplify the following statements

a) $\cos(90° - A)\sin(180° - B) + \cos(360° - A)\sin(90° - B)$

$(\cos 90\cos A + \sin 90\sin A)(\sin 180\cos B - \cos 180\sin B) + (\cos 360\cos A + \sin 360\sin A)(\sin 90\cos B - \cos 90\sin B)$

$(0+\sin A)(0-(-1)(\sin B)) + (\cos A + 0)(\cos B - 0)$

$(\sin A)(\sin B) + (\cos A)(\cos B) \rightarrow \cos A\cos B + \sin A\sin B = \boxed{\cos(A-B)}$

b) $\cos(A - 90°)\sin(90° - B) - \sin(B - 270°)\cos(90° - A)$

$(\cos A\cos 90 + \sin A\sin 90)(\sin 90\cos B - \cos 90\sin B) - [(\sin B\cos 270 - \cos B\sin 270)(\cos 90\cos B + \sin 90\sin A)]$

$(0+\sin A)(\cos B - 0) - [(0-(-\cos B))(0+\sin A)]$

$(\sin A)(\cos B) - (\cos B)(\sin A)$

$\sin A\cos B - \sin A\cos B = \boxed{0}$

Be aware $\tan 90°$ DNE use identity $\tan(90-A) = \cot A$

c) $\tan(90° - A)\tan(180° - A)\sec A + \csc B \sin(90° - B)\csc(90° - B)$

cancel

$$\cot A\left(\frac{\tan 180 - \tan A}{1 + \tan 180 \tan A}\right) \cdot \sec A + \csc B(\sin 90 \cos B - \cos 90 \sin B)\left(\frac{1}{\sin 90 \cos B - \cos 90 \sin B}\right)$$

cancel

$$\cot A\left(\frac{0 - \tan A}{1+0}\right) \cdot \sec A + \csc B \rightarrow \cot A(-\tan A)\sec A + \csc B$$

$$-1 \sec A + \csc B$$

$$\boxed{-\sec A + \csc B}$$

d) $\sec(180° - A)\csc(270° - A) - \cot(630° + A)\tan(540° - A)$

check angles first: $630° \rightarrow 270°$ $\tan 270$ DNE so convert to sin and cos

$540° \rightarrow 180°$ $\tan 180 = 0$

$$\frac{1}{\cos(180-A)} \cdot \frac{1}{\sin(270-A)} - \left[\frac{\cos(630+A)}{\sin(630+A)} \cdot \frac{\tan 540 - \tan A}{1 + \tan 540 \tan A}\right]$$

$$\frac{1}{\cos 180 \cos A + \sin 180 \sin A} \cdot \frac{1}{\sin 270 \cos A + \cos 270 \sin A} - \left[\frac{\cos(630+A)}{\sin(630+A)} \cdot \frac{0 - \tan A}{1}\right]$$

$$\frac{1}{-\cos A} \cdot \frac{1}{-\cos A} - \left[\frac{\cos 630 \cos A - \sin 630 \sin A}{\sin 630 \cos A + \cos 630 \sin A} \cdot \left(-\frac{\sin A}{\cos A}\right)\right]$$

$$\frac{1}{\cos^2 A} - \left[\frac{0 - (-\sin A)}{-\cos A + 0} \cdot \frac{(-1)(\sin A)}{\cos A}\right]$$

$$\frac{1}{\cos^2 A} - \left[\frac{(-1)\sin A \cdot (-1)\sin A}{\cos^2 A}\right] \rightarrow \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A} \rightarrow \frac{1 - \sin^2 A}{\cos^2 A}$$

$$\frac{\cos^2 A}{\cos^2 A} = \boxed{1}$$

7. Find the Amplitude, Period, and Phase Shift of the following trigonometric functions.

a) $y = \cos 3x \cos x - \sin 3x \sin x$

$y = \cos(3x + x)$

$y = \cos 4x$

Amp: 1

PS: None

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

b) $y = -2\sin 2x \cos\frac{\pi}{3} + 2\cos 2x \sin\frac{\pi}{3}$

$y = -2(\sin 2x \cos\frac{\pi}{3} - \cos 2x \sin\frac{\pi}{3})$

$y = -2(\sin(2x - \frac{\pi}{3}))$

$y = -2\sin 2(x - \frac{\pi}{6})$

Amp: 2

Phase Shift: $\frac{\pi}{6}$

Period: $\frac{2\pi}{2} = \pi$

c) $y = 3\sin\frac{\pi}{6}x \cos\frac{\pi}{3} + 3\cos\frac{\pi}{6}x \sin\frac{\pi}{3}$

$y = 3[\sin\frac{\pi}{6}x \cos\frac{\pi}{3} + \cos\frac{\pi}{6}x \sin\frac{\pi}{3}]$

$y = 3\sin(\frac{\pi}{6}x + \frac{\pi}{3})$

$y = 3\sin\frac{\pi}{6}(x + 2)$

Amp: 3

PS: -2

Period: $\frac{2\pi}{\frac{\pi}{6}} = 12$

d) $y = -\sin\frac{\pi}{4}x \sin\frac{\pi}{2} - \cos\frac{\pi}{4}x \cos\frac{\pi}{2}$

$y = -[\sin\frac{\pi}{4}x \sin\frac{\pi}{2} + \cos\frac{\pi}{4}x \cos\frac{\pi}{2}]$

$y = -\cos(\frac{\pi}{4}x - \frac{\pi}{2})$

$y = -\cos\frac{\pi}{4}(x - 2)$

Amp: 1

PS: 2

Period: $\frac{2\pi}{\frac{\pi}{4}} = 8$

8. Consider the expression $\tan\left(\frac{\pi}{2} + x\right)$. Why can't we use the identity for $\tan(A + B)$ to express it as a function of x alone?

Because, as seen in number 6. $\tan\frac{\pi}{2}$ is undefined

9. Demonstrate that $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$

$\tan(\frac{\pi}{2} + x) \rightarrow \frac{\sin(\frac{\pi}{2} + x)}{\cos(\frac{\pi}{2} + x)} \rightarrow \frac{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x} \rightarrow \frac{\cos x + 0}{0 - \sin x} \rightarrow \frac{\cos x}{-\sin x} \rightarrow -\cot x$

See Website for Detailed Answer Key

Extra Work Space