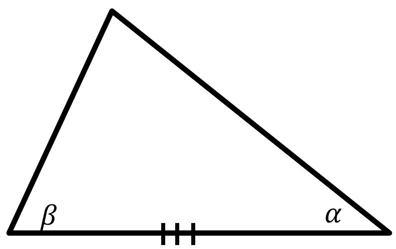
Section 7.4 – Oblique Triangle (Non-Right Angle)

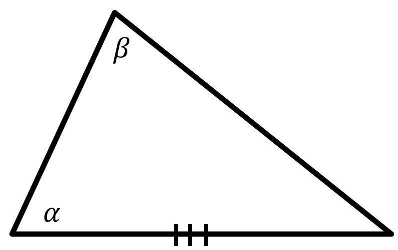
This booklet belongs to:______________________Block:______

- An **OBLIQUE TRIANGLE** is a triangle that does not have a right angle.
- There are four ways to look at the possible information
- In every case, at least one piece of information need to be a side of the triangle.

1. **Two angles** and **one side** of a triangle



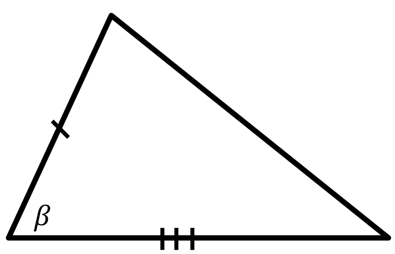
$\boldsymbol{ASA}\ (angle - side - angle)$



$\boldsymbol{AAS}\ (angle - angle - side)$

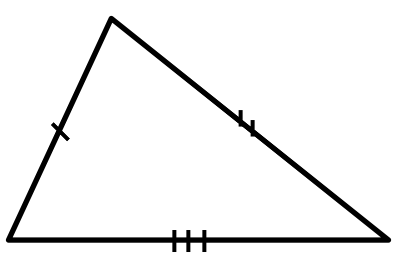
> If you **know two angles** of a triangle you can **solve for the third**, because they add up to $180°$

2. Two sides and **their included angle**



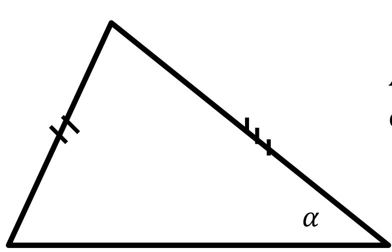
$\boldsymbol{SAS}\ (side - angle - side)$

3. Three sides



$\boldsymbol{SSS}\ (side - side - side)$

4. **Two sides** and the **angle opposite** one of the sides



$\boldsymbol{ASS}\ (angle - side - side)$ – **Ambiguous Case or Donkey Theorem**

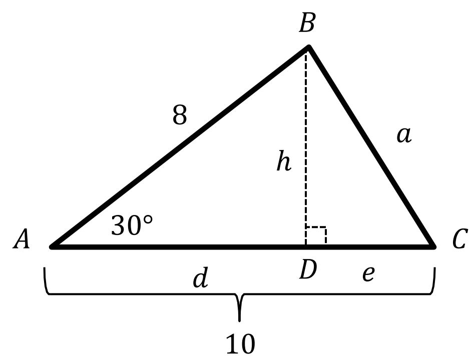
Example 1: Given ΔABC, with $\angle A = 30°$, $b = 10$, $c = 8$, find:

a) a
b) $\angle C$
c) $\angle B$

Remember we label side lengths the same letter as the angle that creates it

Solution 1: Start by constructing the triangle. It's good form to start with side $\angle A$ in the bottom left corner.

When calculating trigonometric ratios, it is prudent to use 3 decimals places during calculations



This is an SAS oblique triangle

a) First solve for h

$$\sin 30° = \frac{h}{8} \quad \rightarrow \quad h = 8(\sin 30°) \quad \rightarrow \quad \boldsymbol{h = 4}$$

Now **solve for $\boldsymbol{d}$** using the **Pythagorean Theorem** or **Right Angle Trigonometry with Cosine**

$$h^2 + d^2 = 8^2$$
$$4^2 + d^2 = 8^2$$
$$d^2 = 64 - 16 = 48$$
$$d = \sqrt{48} = \mathbf{6.928}$$

or

$$\cos 30° = \frac{d}{8}$$
$$d = \cos 30°(8)$$
$$\boldsymbol{d = 6.928}$$

Since you **have $\boldsymbol{d}$** you can now **solve for $\boldsymbol{e}$**

$$\mathbf{10 - 6.928 = 3.072}$$

Then you can use **Pythagorean Theorem to solve for $\boldsymbol{a}$**

$$a^2 = h^2 + e^2$$
$$a^2 = 4^2 + 3.072^2$$
$$a = \mathbf{5.04}$$

b) $\tan C = \frac{h}{e} = \frac{4}{3.072}$

$$\tan C = 1.3021$$
$$\angle \boldsymbol{C} = tan^{-1}(1.3021) = \mathbf{52.5°}$$

c) $\angle A = 30°$, $\angle C = 52.5°$

$$\angle A + \angle B + \angle C = 180°$$

So $\angle \boldsymbol{B} = 180° - 30 - 52.5° = \mathbf{97.5°}$

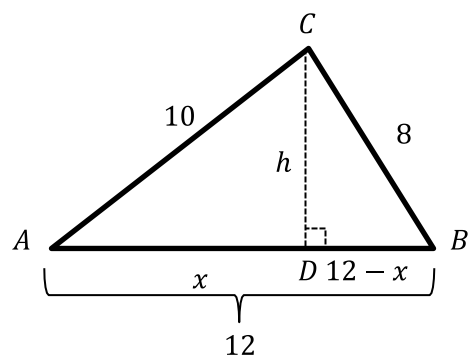
Example 2: Given ΔABC, with $a = 8$, $b = 10$, $c = 12$, find:

a) $\angle A$
b) $\angle B$
c) $\angle C$

Remember when drawing a triangle the largest side should reflect the largest angle

Solution 2: Start by constructing the triangle. It's good form to start with side $\angle A$ in the bottom left corner.

When calculating trigonometric ratios, it is prudent to use 3 decimals places during calculations



This is an SSS oblique triangle

a) First solve for $\angle A$, but you need x first

$$x^2 + h^2 = 10^2$$
$$h^2 = 10^2 - x^2$$

Then

$$h^2 + (12 - x)^2 = 8^2$$
$$h^2 = 8^2 - (12 - x)^2$$

Both expressions are equal to h^2. So you can set them equal to each other.

$$10^2 - x^2 = 8^2 - (12 - x)^2$$

$$10^2 - x^2 = 8^2 - (144 - 24x + x^2)$$

$$100 - x^2 = 64 - 144 + 24x - x^2$$

$$100 = -80 + 24x$$

$$24x = 180$$

$$\boldsymbol{x = 7.5}$$

$$\cos A = \frac{x}{10} = \frac{7.5}{10}$$

$$\angle \mathbf{A} = \mathbf{cos^{-1}(0.75) = 41.4°}$$

b) Solve for $\angle B$

$$\cos B = \frac{12 - x}{8} = \frac{12 - 7.5}{8}$$

$$\cos \mathrm{B} = 0.5625$$

$$\angle \boldsymbol{B} = cos^{-1}(0.5625) = \mathbf{55.8°}$$

c) $\angle A = 41.4°$, $\angle B = 55.8°$

$$\angle A + \angle B + \angle C = 180°$$

So $\angle \boldsymbol{C} = 180° - 41.4 - 55.8° = \mathbf{82.8°}$

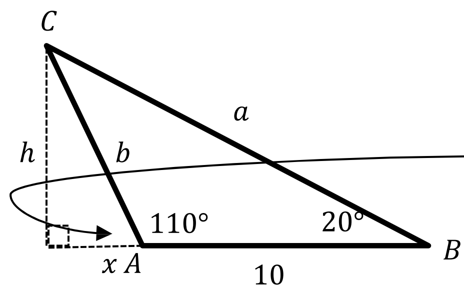
<u>Example 3:</u> Given ΔABC, with $\angle A = 110°$, $\angle B = 20°$, $c = 10$, find:

a) $\angle C$
b) b
c) a

Remember when drawing a triangle the largest side should reflect the largest angle

<u>Solution 3:</u> Start by constructing the triangle. It's good form to start with side $\angle A$ in the bottom left corner.

With the information given, start by constructing a Right Angle in the left corner. Label the height h and added length x



This is an ASA or AAS oblique triangle

This angle is 70°, because angles on a straight line are 180°, and the other half is 110°.

a) Solve angle C first, using the sum of angles in a triangle

$$\angle A + \angle B + \angle C = 180°$$

$$110° + 20° + \angle C = 180° \quad \rightarrow \quad \angle \mathbf{C} = \mathbf{50°}$$

b) To solve for $\angle B$, we need x and h first, then we can use SOH CAH TOA (Next couple sections we'll find a new way to do this)

$$\tan 70° = \frac{h}{x}$$

$$h = x(tan\ 70°)$$

And

$$\tan 20° = \frac{h}{x + 10}$$

$$h = (x + 10)(tan\ 20°)$$

Both expressions are equal to h. So you can set them equal to each other.

$$x(tan\ 70°) = (x + 10)(tan\ 20°)$$

$$x(tan\ 70°) = x(\tan 20°) + 10(tan\ 20°)$$

$$x(tan\ 70°) - x(\tan 20°) = 10(tan\ 20°)$$

$$x(\tan 70° - \tan 20°) = 10tan\ 20°$$

$$x = \frac{10 \tan 20°}{\tan 70° - \tan 20°} = \mathbf{1.53}$$

Now:

$$h = xtan\ 70° = (1.53)tan\ 70° = \mathbf{4.2}$$

So for b:

$$b^2 = x^2 + h^2$$

$$b^2 = 1.53^2 + 4.2^2$$

$$b = \sqrt{19.98} \rightarrow \quad b = \mathbf{4.47}$$

c) $a^2 = (10 + x)^2 + h^2$

$$a^2 = 11.53^2 + 4.2^2$$

$$a = \sqrt{150.58} \quad \rightarrow \quad a = \mathbf{12.3}$$

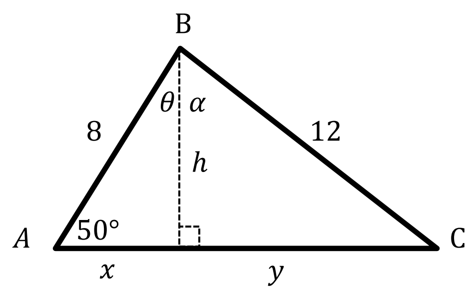
Example 4: Given ΔABC, with $\angle A = 50°$, $a = 12$, $c = 8$, find:

a) $\angle B$
b) $\angle C$
c) b

Remember when drawing a triangle the largest side should reflect the largest angle

Solution 4: Start by constructing the triangle. It's good form to start with side $\angle A$ in the bottom left corner.

When calculating trigonometric ratios, it is prudent to use 3 decimals places during calculations



This is an ASS oblique triangle, it can have several possible outcomes depending on the lengths of the sides. We will look at this further In the next section!

a) First solve for $\angle B$, but you need $\theta, \alpha, and\ h$ first

$$\sin 50° = \frac{h}{8} \quad \rightarrow \quad 8 \sin 50° = h \quad \rightarrow \quad \boldsymbol{h = 6.13}$$

$$\cos \alpha = \frac{h}{12} = \frac{6.13}{12} \qquad \cos \theta = \frac{h}{8} = \frac{6.13}{8}$$

$$\boldsymbol{\angle\alpha = \cos^{-1}(0.5108) = 59.3°} \qquad \boldsymbol{\angle\theta = \cos^{-1}(0.7663) = 40.0°}$$

So $\angle \boldsymbol{B} = \mathbf{99.3°}$

b) $\angle A = 50.0°, \angle B = 99.3°$

$$\angle A + \angle B + \angle C = 180°$$

So $\angle \boldsymbol{C} = 180° - 50° - 99.3° = \mathbf{30.7°}$

c) Solve for side b

$$\cos 50° = \frac{x}{8} \qquad \cos 30.7° = \frac{y}{12}$$

$$8(\cos 50°) = x \qquad 12(\cos 30.7°) = y$$

$$x = 5.14 \qquad y = 10.32$$

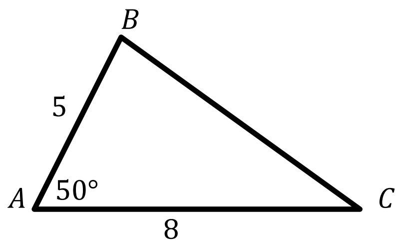
$$b = x + y$$

$$b = 5.14 + 10.32 = \mathbf{15.46}$$

Section 7.4 – Practice Problems

Find all the missing sides and angles. Round answer to the nearest tenth.

1.



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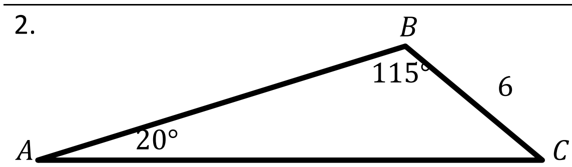

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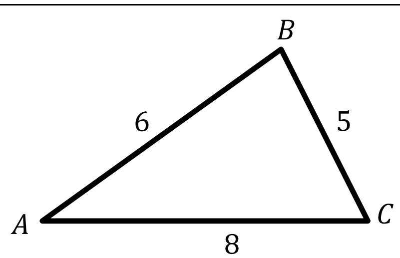

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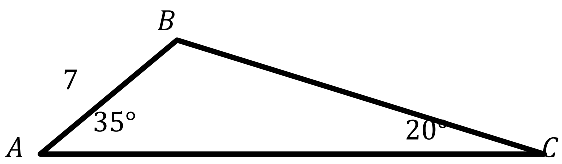

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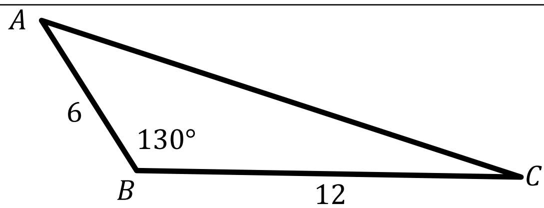

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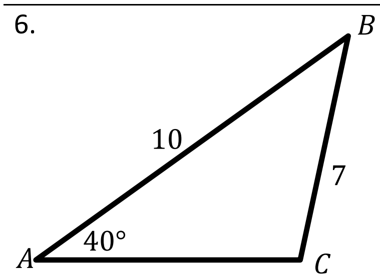

Using the information given find what is missing. Side letters are opposite of the same angle letter. You will have to draw an image of the triangle, it helps and is good practice in visualization.

7. $\angle A = 50°, a = 35, c = 27, find\ \angle B$

8. $a = 23, b = 31, c = 19, find\ \angle A$

9. $\angle B = 94°, a = 29, c = 21, find\ \angle C$

10. $\angle C = 21°, \angle A = 48°, b = 26, find\ c$

11. $\angle B = 108°, \angle C = 34°, b = 68, find\ a$

12. $\angle A = 80°, a = 57, c = 22, find\ \angle B$

13. $\angle C = 73°, a = 12, b = 14, find\ c$

14. $a = 10, b = 12, c = 17, find\ \angle C$

Answer Key – Section 7.4

1. $\angle B = 91.3°$, $\angle C = 38.7°$, $a = 6.1$
2. $\angle C = 45°$, $b = 15.9$, $c = 12.4$
3. $\angle A = 38.6°$, $\angle B = 92.9°$, $\angle C = 48.5°$
4. $\angle B = 125°$, $a = 11.7$, $b = 16.8$
5. $\angle A = 33.8°$, $\angle C = 16.2°$, $b = 16.5$
6. $\angle B = 26.7°$, $\angle C = 113.3°$, $b = 4.9$
7. $93.8°$
8. $47.7°$
9. $34.5°$
10. 10.0
11. 44.0
12. $77.7°$
13. 15.5
14. $100.8°$

Extra Work Space