

Section 7.3 – Practice Problems1. Solve for: i) All solutions over $0 \leq x < 2\pi$ ii) The General Form

a) $\sin x = \frac{\sqrt{3}}{2}$ Ref angle: $\frac{\pi}{3}$

i) Q1: $\frac{\pi}{3}$ Q2: $\frac{2\pi}{3}$

ii) $x = \frac{\pi}{3} + 2\pi n$
 $x = \frac{2\pi}{3} + 2\pi n$ } n is an integer

b) $\cos x = \frac{\sqrt{2}}{2}$ Ref angle: $\frac{\pi}{4}$

i) Q1: $\frac{\pi}{4}$ Q4: $\frac{7\pi}{4}$

ii) $x = \frac{\pi}{4} + 2\pi n$
 $x = \frac{7\pi}{4} + 2\pi n$ } n is an integer

c) $\tan x = \frac{1}{\sqrt{3}}$ Ref angle: $\frac{\pi}{6}$

i) Q1: $\frac{\pi}{6}$ Q3: $\frac{7\pi}{6}$

ii) $x = \frac{\pi}{6} + \pi n$ } Period for Tan is π , n is an integer

d) $\cot x = \frac{1}{\sqrt{3}}$ Ref angle: $\frac{\pi}{3}$

i) Q1: $\frac{\pi}{3}$ Q3: $\frac{4\pi}{3}$

ii) $x = \frac{\pi}{3} + \pi n$

e) $\sec x = \frac{2}{\sqrt{3}}$ Ref angle: $\frac{\pi}{6}$

i) Q1: $\frac{\pi}{6}$ Q4: $\frac{11\pi}{6}$ also $-\frac{\pi}{6}$

ii) $\pm \frac{\pi}{6} + 2\pi n$

f) $\csc x = 2$ Ref angle: $\frac{\pi}{6}$

i) Q1: $\frac{\pi}{6}$ Q2: $\frac{5\pi}{6}$

ii) $\frac{\pi}{6} + 2\pi n$

$\frac{5\pi}{6} + 2\pi n$

g) $\sin x = -\frac{1}{2}$ Ref angle: $\frac{\pi}{6}$

but negative
so Q3 and Q4

i) Q3: $\frac{7\pi}{6}$

Q4: $\frac{11\pi}{6}$

ii) $\frac{7\pi}{6} + 2\pi n$

$\frac{11\pi}{6} + 2\pi n$

h) $\cos x = -1$

Ref angle: π

i) π

ii) $\pi + 2\pi n$

i) $\tan x = -\sqrt{3}$ Ref angle: $\frac{\pi}{3}$

i) Q2: $\frac{2\pi}{3}$

Q4: $\frac{5\pi}{3}$

ii) $\frac{2\pi}{3} + \pi n$ n is an integer

j) $\cot x = 0$

$\frac{x}{y} \rightarrow \frac{\pi}{2}$
add
 $\frac{3\pi}{2}$

i) $\frac{\pi}{2}, \frac{3\pi}{2}$

ii) $\frac{\pi}{2} + \pi n$ n is an integer

k) $\sec x = -\sqrt{2}$ Ref angle: $\frac{\pi}{4}$

i) Q2: $\frac{3\pi}{4}$

Q3: $\frac{5\pi}{4}$

ii) $\left. \begin{array}{l} \frac{3\pi}{4} + 2\pi n \\ \frac{5\pi}{4} + 2\pi n \end{array} \right\} n \text{ is an integer}$

l) $\csc x = -\frac{2}{\sqrt{3}}$ Ref angle: $\frac{\pi}{3}$

Q3: $\frac{4\pi}{3}$

i) Q4: $\frac{5\pi}{3}$

ii) $\left. \begin{array}{l} \frac{4\pi}{3} + 2\pi n \\ \frac{5\pi}{3} + 2\pi n \end{array} \right\} n \text{ is an integer}$

2. Solve for: i) All solutions over $0 \leq x < 2\pi$ ii) The General Form

a) $\sin x = 0.6234$

Sin positive
Q1 and Q2

i) $x = \sin^{-1}(0.6234)$

$x = 0.6731$ ← Q1

Q2 → $\pi - 0.6731 = 2.469$

ii) $\left. \begin{matrix} 0.6731 + 2\pi n \\ 2.469 + 2\pi n \end{matrix} \right\} n \text{ is an integer}$

b) $\cos x = 0.4821$

cos positive in Q1
Q4

i) $x = \cos^{-1}(0.4821)$

$x = 1.068$ Q1

$2\pi - 1.068$ Q4

5.215

ii) $\left. \begin{matrix} 1.068 + 2\pi n \\ 5.215 + 2\pi n \end{matrix} \right\} n \text{ is an integer}$

c) $\tan x = 1.7258$

tan positive
in Q1 Q3

$x = \tan^{-1}(1.7258)$

i) Q1: 1.046

Q3: $\pi + 1.046 = 4.1872$

ii) $1.046 + \pi n$ n is an integer

d) $\cot x = 0.7238$

Positive Q1 Q3

$\cot x = 0.7238$

$\tan x = \frac{1}{0.7238} = 1.3816$

$\tan^{-1}(1.3816) = x$

$x = 0.9443$

Q1: 0.9443

Q3: 4.086

ii) $0.9443 + \pi n$
n is an integer

e) $\sec x = 3.1743$ Pos Q1 Q4

$$\cos x = \frac{1}{3.1743} = 0.3150$$

$$x = \cos^{-1}(0.3150) = 1.25$$

i) Q1: 1.25

Q4: $2\pi - 1.25 = 5.03$

ii) $\left. \begin{array}{l} 1.25 + 2\pi n \\ 5.03 + 2\pi n \end{array} \right\} n \text{ is an integer}$

f) $\csc x = 1.5243$ Pos Q1 Q2

$$\sin x = \frac{1}{1.5243} = 0.6560$$

$$x = \sin^{-1}(0.6560) = 0.7156$$

i)

Q1: 0.7156

Q2: $\pi - 0.7156 = 2.426$

ii) $\left. \begin{array}{l} 0.7156 + 2\pi n \\ 2.426 + 2\pi n \end{array} \right\} n \text{ is an integer}$

g) $\sin x = -0.4173$ neg Q3 Q4

$$x = \sin^{-1}(+0.4173) \text{ ref angle pos.}$$

$$x = 0.4305$$

i)

Q3: $\pi + 0.4305 = 3.572$

Q4: $2\pi - 0.4305 = 5.8527$

ii) $\left. \begin{array}{l} 3.572 + 2\pi n \\ 5.853 + 2\pi n \end{array} \right\} n \text{ is an integer}$

h) $\cos x = -0.4821$ Neg Q2 Q3

Ref angle 0.4821

$$\cos^{-1}(0.4821) = 1.068$$

i)

Q2: $\pi - 1.068 = 2.074$

Q3: $\pi + 1.068 = 4.209$

ii) $\left. \begin{array}{l} 2.074 + 2\pi n \\ 4.209 + 2\pi n \end{array} \right\} n \text{ is an integer}$

i) $\tan x = -0.3124$

Neg Q2
Q4

Ref angle: 0.3124

$x = \tan^{-1}(0.3124)$

$x = 0.3028$

i) $\pi - 0.3028 = 2.839$

$2\pi - 0.3028 = 5.980$

ii) $2.839 + \pi n$ n is an integer

j) $\cot x = -1.1482$

Q2 Q4

$x = \tan^{-1}\left(\frac{1}{1.1482}\right)$

$x = 0.7165$

i) Q2: $\pi - 0.7165 = 2.425$

Q4: $2\pi - 0.7165 = 5.567$

ii) $2.425 + \pi n$ n is an integer

k) $\sec x = -1.9105$

Q2 Q3

$x = \cos^{-1}\left(\frac{1}{1.9105}\right)$

$x = 1.02$

i) Q2: $\pi - 1.02 = 2.12$

Q3: $\pi + 1.02 = 4.16$

ii) $2.12 + 2\pi n$
 $4.16 + 2\pi n$ n is an integer

l) $\csc x = -2.3124$

Q3 Q4

$x = \sin^{-1}\left(\frac{1}{2.3124}\right)$

$x = 0.447$

i)

Q3: $\pi + 0.447 = 3.59$

Q4: $2\pi - 0.447 = 5.84$

ii) $3.59 + 2\pi n$
 $5.84 + 2\pi n$ n is an integer

3. How many solutions do the following equations have for $0 \leq x < 2\pi$?

a) $\sin 3x = -\frac{1}{4}$

sin has 2
negative
solutions
per 2π

$P = \frac{2\pi}{3}$

$\frac{2\pi}{\frac{2\pi}{3}} = 3$ full
revolutions

so 6 solutions

b) $\sin 3x = -1$

sin has 1 solution
for -1 in 2π

$P = \frac{2\pi}{3}$

$\frac{2\pi}{\frac{2\pi}{3}} = 3$ revolutions

so 3 solutions.

c) $\sin \frac{1}{2}x = \frac{1}{3}$

2 solⁿ in 2π

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$\frac{2\pi}{4\pi} = \frac{1}{2}$ rev.

sin occurs in the first half
of the revolution positive

so 2 solutions

d) $\cos \frac{1}{2}x = \frac{1}{3}$

solⁿ occurs in
Q1 and Q4
of 1 2π rev

$P = 4\pi$

$\frac{2\pi}{4\pi} = \frac{1}{2}$ rev.

only the Q1 solution occurs.

1 solution

e) $\tan^2 2x = 1$

squareroot both side

$\tan 2x = \pm 1$ ← this happens in each quadrant
 $\frac{\pi}{4}$ ref angle

$P = \frac{\pi}{2}$

$\frac{\frac{\pi}{2}}{\frac{\pi}{2}} = 2$ revs.

4 solⁿ · 2 revs

8 solutions.

f) $\sin bx = \frac{1}{2}$

This occurs Q1 Q2
 Twice over 2π

$P = \frac{2\pi}{b}$

b revolutions

2 · b solutions

4. Solve for: i) All solutions over $0 \leq x < 2\pi$ ii) The General Form

a) $\sin 2x = \frac{\sqrt{3}}{2}$

Q1 Q2

$P = \frac{2\pi}{2} = \pi$ 2 revs over 2π 4 solutions

$\frac{\sqrt{3}}{2}$ is special angle.

Q1
 $2x = \frac{\pi}{3}$

Q2
 $2x = \frac{2\pi}{3}$

$\frac{\pi}{3} + 2\pi =$

$\frac{7\pi}{3}$

$\frac{2\pi}{3} + 2\pi =$

$\frac{8\pi}{3}$

$2x = \frac{\pi}{3}$

$x = \frac{\pi}{6}$

$2x = \frac{2\pi}{3}$

$x = \frac{2\pi}{6} = \frac{\pi}{3}$

$2x = \frac{7\pi}{3}$

$x = \frac{7\pi}{6}$

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i) $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

ii) $2x = \frac{\pi}{3} + 2\pi n \rightarrow x = \frac{\pi}{6} + \pi n$
 $2x = \frac{2\pi}{3} + 2\pi n \rightarrow x = \frac{\pi}{3} + \pi n$

b) $\tan 3x = -1$

ref angle: $\frac{\pi}{4}$ Q2 Q4

← occurs 2 times

$P = \frac{\pi}{3}$ so 3 revs.

so 6 solutions

i)

Q2: $3x = \frac{3\pi}{4}$

$x = \frac{\pi}{4}$

$3x = \frac{3\pi}{4} + 2\pi$

$3x = \frac{11\pi}{4}$

$x = \frac{11\pi}{12}$

$3x = \frac{3\pi}{4} + 4\pi$

$3x = \frac{19\pi}{4}$

$x = \frac{19\pi}{12}$

Q4: $3x = \frac{7\pi}{4}$

$x = \frac{7\pi}{12}$

$3x = \frac{7\pi}{4} + 2\pi$

$3x = \frac{15\pi}{4}$

$x = \frac{15\pi}{12} = \frac{5\pi}{4}$

$3x = \frac{7\pi}{4} + 4\pi$

$3x = \frac{23\pi}{4}$

$x = \frac{23\pi}{12}$

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ii) $3x = \frac{3\pi}{4} + \pi n$

$x = \frac{\pi}{4} + \frac{\pi n}{3}$

c) $\sec \frac{x}{2} = -\frac{2}{\sqrt{3}}$

2π is $\frac{1}{2}$ rev

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Q2 Q3
↑
only this solution

$\sec \frac{x}{2} = -\frac{2}{\sqrt{3}} \rightarrow \cos \frac{x}{2} = -\frac{\sqrt{3}}{2}$ ref angle $\frac{\pi}{6}$

$\frac{x}{2} = \frac{\pi}{6} \rightarrow x = \frac{\pi}{3}$

i) Q2: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$; $\frac{x}{2} = \frac{5\pi}{6} \rightarrow x = \frac{5\pi}{3}$

ii) $\frac{x}{2} = \frac{5\pi}{6} + 2\pi n \rightarrow x = \frac{5\pi}{3} + 4\pi n$

e) $\tan 2x = 1.7258$ Q1 Q3

$P = \frac{2\pi}{2} = \pi$ 2 revolutions, 4 solⁿ

i) Q1: $2x = 1.046$ $2x = \tan^{-1}(1.7258)$
 $2x = 1.046$

$x = 0.523$

Q1: $2x = 1.046 + 2\pi = 7.329$

$x = 3.665$

Q3: $2x = \pi + 1.046 = 4.188$

$x = 2.094$

$2x = 4.188 + 2\pi = 10.471$

$x = 5.236$

ii) $2x = 1.046 + \pi n \rightarrow x = \frac{1.046 + \pi n}{2}$

d) $\sin 2x = -0.4173$

so 2π is 2 rev

$P = \frac{2\pi}{2} = \pi$

4 solutions Q3 Q4

$2x = \sin^{-1}(0.4173)$ (ref angle)

Q3: $\pi + 0.430 = 3.572$

Q4: $2\pi - 0.430 = 5.853$

i) $2x = 3.572$ $2x = 3.572 + 2\pi = 9.855$

$x = 1.786$

$x = 4.928$

$2x = 5.853$

$x = 2.927$

$2x = 5.853 + 2\pi$

$2x = 12.136$

$x = 6.068$

ii) $2x = 3.572 + 2\pi n \rightarrow x = 1.786 + \pi n$ $x = 2.927 + \pi n$
 $2x = 5.853 + 2\pi n$

f) $\tan bx = 1.7258$, b is an integer ← Q1 Q3

$\frac{2\pi}{b}$, b revolutions, $2b$ solutions

i) Q1: $bx = 1.046$

$bx = \tan^{-1}(1.7258)$

$bx = 1.046$

$x = \frac{1.046}{b}$

Next rev. $bx = 1.046 + 2\pi = 7.329$

$x = \frac{7.329}{b}$

and so on $< 2\pi$

Q3: $bx = \pi + 1.046 = 4.188$

$x = \frac{4.188}{b}$

next rev $4.188 + 2\pi = 10.471$ and so on

$x = \frac{10.471}{b}$ until $< 2\pi$

ii) $bx = 1.046 + \pi n$

$x = \frac{1.046 + \pi n}{b}$

5. Solve the given equations algebraically; provide exact answers when possible

i) All solutions over $0 \leq x < 2\pi$ ii) The General Form

a) $2 \cos x + 1 = 0$

$$2 \cos x = -1 \rightarrow \cos x = -\frac{1}{2}$$

Ref angle $\frac{\pi}{3}$ Q2 and Q3

Q3: $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Q2: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

i) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

ii) $\frac{2\pi}{3} + 2\pi n$ and $\frac{4\pi}{3} + 2\pi n$

b) $(2 \sin x - 1)(\cos x + 1) = 0$

case 1: $2 \sin x - 1 = 0 \rightarrow \sin x = \frac{1}{2}$

Ref angle $\frac{\pi}{6}$ Q1, Q2

Q1: $\frac{\pi}{6}$ Q2: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

case 2: $\cos x + 1 = 0 \rightarrow \cos x = -1$

occurs at π

i) $\frac{\pi}{6}, \frac{5\pi}{6}, \pi$

ii) $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \pi + 2\pi n$

c) $\sqrt{2} \cos^2 x - \cos x = 0$

$$\cos x (\sqrt{2} \cos x - 1) = 0$$

case 1: $\cos x = 0$ occurs at $\frac{\pi}{2}, \frac{3\pi}{2}$

case 2: $\sqrt{2} \cos x - 1 = 0 \rightarrow \cos x = \frac{1}{\sqrt{2}}$

Q1, Q4 Ref angle: $\frac{\pi}{4}$

Q1: $\frac{\pi}{4}$ Q4: $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

i) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{4}$

ii) $\frac{\pi}{4} + 2\pi n, \frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n$

$\frac{7\pi}{4} + 2\pi n$

d) $4 \sin^2 x = 3$

$$\sin^2 x = \frac{3}{4} \rightarrow \sin x = \pm \sqrt{\frac{3}{4}}$$

$\sin x = \pm \frac{\sqrt{3}}{2}$ Q1 Q2 Ref angle: $\frac{\pi}{3}$
Q3 Q4

Q1: $\frac{\pi}{3}$ Q2: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q3: $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Q4: $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

i) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

ii) $\frac{\pi}{3} + \pi n$ and $\frac{2\pi}{3} + \pi n$

e) $\sin^2 x = \sin x$

$\sin^2 x - \sin x = 0$

$\sin x (\sin x - 1) = 0$

↑
0 at 0 and π

$\sin x - 1 = 0 \rightarrow \sin x = 1$

↑
at $\frac{\pi}{2}$

i) $0, \frac{\pi}{2}, \pi$

ii) $0 + \pi n$
 $\frac{\pi}{2} + 2\pi n$

g) $5 \cos^2 x + 6 \cos x - 8 = 0$

$(5 \cos x - 4)(\cos x + 2) = 0$

✓
 $\cos x = \frac{4}{5}$ Q1
Q4

↑
 $\cos x = -2$
DNE

$x = \cos^{-1}(\frac{4}{5}) = 0.644$

Q1: 0.644

Q4: $2\pi - 0.644 = 5.6397$

i) 0.644 and 5.64

ii) $0.644 + 2\pi n$
 $5.64 + 2\pi n$

f) $6 \sin^2 x + 11 \sin x - 10 = 0$

$(3 \sin x - 2)(2 \sin x + 5) = 0$

↑
 $\sin x = \frac{2}{3}$

$x = \sin^{-1}(\frac{2}{3})$

$x = 0.730$

Q1 and Q2

Q1: 0.730

Q2: $\pi - 0.730 = 2.412$

i) 0.730 and 2.412

ii) $0.730 + 2\pi n$
 $2.412 + 2\pi n$

↑
 $\sin x = -\frac{5}{2}$

↑
not possible
reject as potential solution

h) $2 \cos^2 x - \cos x = 1$

$2 \cos^2 x - \cos x - 1 = 0$

$(2 \cos x + 1)(\cos x - 1) = 0$

↓
 $\cos x = -\frac{1}{2}$ Q2 Q3 Ref angle $\frac{\pi}{3}$

Q2: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q3: $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

i) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

ii) $\frac{2\pi}{3} + 2\pi n$
 $\frac{4\pi}{3} + 2\pi n$ } or $\frac{2\pi n}{3}$

i) $2 \cos^2 x - 3 \cos x - 2 = 0$

$(2 \cos x + 1)(\cos x - 2) = 0$

\downarrow $\hookrightarrow \cos x = 2$
DNE

$\cos x = -\frac{1}{2}$ Q2 Q3 Reference $\frac{\pi}{3}$

Q2: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q3: $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

i) $2\pi/3, 4\pi/3$

ii) $2\pi/3 + 2\pi n, 4\pi/3 + 2\pi n$

j) $2 \tan^2 x + 5 \tan x + 2 = 0$

$(2 \tan x + 1)(\tan x + 2) = 0$

$\hookrightarrow \tan x = -\frac{1}{2}$ $\hookrightarrow \tan x = -2$

$x = \tan^{-1}(-\frac{1}{2})$ $x = \tan^{-1}(-2)$
 $= 0.464$ (reference) 1.107 (reference)

Q2: $\pi - 0.464 = 2.678$ and $\pi - 1.107 = 2.03$

Q4: $2\pi - 0.464 = 5.819$ $2\pi - 1.107 = 5.176$

i) $x = 2.03, 2.678, 5.176, 5.819$

ii) $2.03 + \pi n$ and $2.678 + \pi n$

k) $\tan^2 x - 2 \tan x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

\downarrow $\hookrightarrow \tan x = -1$
 $\tan x = 3$ Q2, Q4 Reference $\frac{\pi}{4}$
Q1 Q3

$\tan^{-1}(3) = +1.25$

i) $1.25, \pi + 1.25 = 4.39, \frac{3\pi}{4}, \frac{7\pi}{4}$

ii) $1.25 + \pi n$
 $\frac{3\pi}{4} + \pi n$

l) $\cot^2 x - \cot x - 6 = 0$

$(\cot x - 3)(\cot x + 2) = 0$

\downarrow \swarrow Q1 Q3 \nwarrow Q2 Q4
 $\tan x = \frac{1}{3}$ $\tan x = -\frac{1}{2}$

$x = 0.322$ $x = 0.464$ (reference)

Q1: 0.322 Q3: $\pi + 0.322 = 3.464$

Q2: $\pi - 0.464 = 2.678$ Q4: $2\pi - 0.464 = 5.82$

i) $0.322, 2.678, 3.464, 5.82$

ii) $0.322 + \pi n$
 $2.678 + \pi n$

m) $\tan x - 2 \tan x \cdot \sin x = 0$

$\tan x (1 - 2 \sin x) = 0$

\downarrow
 $\tan x = 0$ occurs at $0, \pi$
 $\hookrightarrow \sin x = \frac{1}{2}$ occurs at $\frac{\pi}{6}$ Q1 Q2

Q1: $\frac{\pi}{6}$ Q2: $\frac{5\pi}{6}$

i) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

ii) $x = \pi n$

$x = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$

o) $\sec^2 x - 3 \sec x + 2 = 0$

$(\sec x - 2)(\sec x - 1) = 0$

\downarrow
 $\sec x = 2$ Q1 Q4
 $\cos x = \frac{1}{2}$ occurs at 0
 $\hookrightarrow \sec x = 1$

Reference: $\frac{\pi}{3}$

Q1: $\frac{\pi}{3}$ Q4: $\frac{5\pi}{3}$

i) $0, \frac{\pi}{3}, \frac{5\pi}{3}$

ii) $0 + 2\pi n$
 $\frac{\pi}{3} + 2\pi n$
 $\frac{5\pi}{3} + 2\pi n$

n) $3 \sin^2 x + 4 \sin x - 4 = 0$

$(3 \sin x - 2)(\sin x + 2)$

\downarrow
 $\sin x = \frac{2}{3}$ Q1 Q2 $\hookrightarrow \sin x = -2$ DNE

$x = \sin^{-1}(\frac{2}{3}) = 0.730$

Q1: 0.730

Q2: $\pi - 0.730 = 2.412$

i) 0.730, 2.412

ii) $0.730 + 2\pi n$
 $2.412 + 2\pi n$

p) $2 \cos^2 x - 3 \sin x - 3 = 0$

$\cos^2 x = 1 - \sin^2 x$

$2(1 - \sin^2 x) - 3 \sin x - 3 = 0$

$2 - 2 \sin^2 x - 3 \sin x - 3 = 0$

$-2 \sin^2 x - 3 \sin x - 1 = 0$

$2 \sin^2 x + 3 \sin x + 1 = 0$

$(2 \sin x + 1)(\sin x + 1) = 0$

\downarrow
 $\sin x = -\frac{1}{2}$ Reference $\frac{\pi}{6}$ occurs at $\frac{3\pi}{2}$

Q3: $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Q4: $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

i) $\frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

ii) $\frac{3\pi}{2} + 2\pi n$
 $\frac{7\pi}{6} + 2\pi n$
 $\frac{11\pi}{6} + 2\pi n$

q) $3 \csc x - \sin x - 2 = 0$

$3\left(\frac{1}{\sin x}\right) - \sin x - 2 = 0$

$\frac{3}{\sin x} - \frac{\sin^2 x}{\sin x} - \frac{2\sin x}{\sin x} = 0$

$\frac{-\sin^2 x - 2\sin x + 3}{\sin x} = 0$ multiply both sides by $-\sin x$

$\sin^2 x + 2\sin x - 3 = 0$

$(\sin x + 3)(\sin x - 1) = 0$

\downarrow $\hookrightarrow \sin x = 1$ occurs at $\frac{\pi}{2}$
 $\sin x = -3$
 DNE

- i) $\frac{\pi}{2}$
- ii) $\frac{\pi}{2} + 2\pi n$

r) $3 \sin x = \sqrt{3} \cos x$

$\frac{3 \sin x}{\sqrt{3} \cos x} = 1 \rightarrow \sqrt{3} \tan x = 1 \rightarrow \tan x = \frac{1}{\sqrt{3}}$

Q1 Q3 \uparrow
 ref angle $\frac{\pi}{6}$

Q1: $x = \frac{\pi}{6}$

Q3: $x = \frac{7\pi}{6}$

i) $\frac{\pi}{6}, \frac{7\pi}{6}$

ii) $\frac{\pi}{6} + 2\pi n$

$\frac{7\pi}{6} + 2\pi n$

s) $\sin x \tan 2x = \sin x$

$\sin x \tan 2x - \sin x = 0$

$\sin x (\tan 2x - 1) = 0$ Q1 Q3

$\sin x = 0$
 $x = 0, \pi$

$\tan 2x = 1$ Ref angle $\frac{\pi}{4}$

$2x = \frac{\pi}{4} \rightarrow x = \frac{\pi}{8}$

$2x = \frac{5\pi}{4} \rightarrow x = \frac{5\pi}{8}$

$2x = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$

$x = \frac{9\pi}{8}$

$2x = \frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}$

$x = \frac{13\pi}{8}$

$P = \frac{\pi}{2}$
 in 2π we have
 4 solutions

- i) $0, \pi, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
- ii) $0 + \pi n, \frac{\pi}{8} + \pi n, \frac{5\pi}{8} + \pi n$

t) $3 \sin^2 x - 2 \sin 2x - 1 = 0$

$(3 \sin x + 1)(\sin x - 1) = 0$ P: π
 4 solutions

\downarrow Q3 Q4 \downarrow
 $\sin 2x = -\frac{1}{3}$ $\sin 2x = 1$ occurs at $\frac{\pi}{2}$

$2x = \sin^{-1}\left(-\frac{1}{3}\right)$ $2x = \frac{\pi}{2} \rightarrow x = \frac{\pi}{4}$

$2x = 0.34 + \pi$ $2x = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$

$2x = 2\pi - 0.34$ $x = \frac{5\pi}{4}$

$2x = 3.48 \rightarrow 3.48 + 2\pi$

$x = 1.74 \rightarrow 1.74 + \pi = 4.88$

$2x = 5.94 \rightarrow 5.94 + 2\pi$

$x = 2.97 \rightarrow 2.97 + \pi = 6.11$

- i) $1.74, 2.97, 4.88, 6.11, \frac{\pi}{4}, \frac{5\pi}{4}$
- ii) $\frac{\pi}{4} + \pi n, 1.74 + \pi n, 2.97 + \pi n$

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6. Solve using Desmos.

a) $\tan x - \sin 3x = 1, \quad 0 \leq x < 2\pi$

$$x = 0.931$$

$$x = 3.411$$

b) $\sin 3x - \cos 2x = -1, \quad 0 \leq x < 2\pi$

$$x = 0$$

$$x = \pi$$

$$x = 3.85$$

$$x = 5.574$$

c) $\cot 2x + \tan \frac{1}{2}x = 0, \quad 0 \leq x < 2\pi$

$$x = \frac{\pi}{3}$$

$$x = \frac{5\pi}{3}$$

See Website for Detailed Answer Key

Extra Work Space