Section 7.2 – Standard Form

This booklet belongs to:_____Block: _____

- This next equation in called: **STANDARD FORM**
- There is no obvious $\boldsymbol{y - intercept}$ **or SLOPE**
- It looks like this:

$$\boldsymbol{Ax + By = C}$$

  - $\boldsymbol{A > 0}$ and can't be a $\boldsymbol{fraction}$
  - It is still the equation of a **STRAIGHT LINE**

Solutions of the Line (Are the points in the line)

- ✓ We have a point with an $(\boldsymbol{x}, \boldsymbol{y})$ **coordinate**
- ✓ We have an **equation** with an $\boldsymbol{x\ and\ y}$
- ✓ If we plug in the **coordinates** and the equation **stays equal**, the line goes through the point!

Example 1: Does the line $3x - 2y = -6$ go through the point $(2,6)$?

Solution 1:

- In other words:

  The is the point $(2, 6)$ a point on the line $3x - 2y = -6$?

  - So, sub in $\boldsymbol{2\ for\ x}$ and $\boldsymbol{6\ for\ y}$

$$\begin{aligned} 3(2) - 2(6) &= -6 \\ 6 - 12 &= -6 \\ -6 &= -6 \end{aligned}$$

Yes, it is a solution; the **line goes through the point**!

Example 2: Is $(1, 3)$ a solution to $x - 3y = 9$?

Solution 2: So, sub in $\boldsymbol{1\ for\ x}$ and $\boldsymbol{3\ for\ y}$

$$\begin{aligned} (1) - 3(3) &= 9 \\ 1 - 9 &= 9 \\ -8 &= 9 \end{aligned}$$

No, it is not a solution; the **line does not go through the point**!

Example 3: Is $(4, 0)$ a solution to $x = 4$?

Solution 3: So, sub in $\boldsymbol{4\ for\ x}$ and since there is $\boldsymbol{no\ y}$, **sub in nothing**

$$4 = 4$$

Yes, it is a solution; the **line goes through the point**!

Example 4: Is $(-2,\ 5)$ a solution to $3x + 2y = 4$?

Solution 4: So, sub in $\boldsymbol{-2\ for\ x}$ and $\boldsymbol{5\ for\ y}$

$$3(-2) + 2(5) = 4$$
$$-6 + 10 = 4$$
$$4 = 4$$

Yes, it is a solution; the **line goes through the point**!

Graphing this Equation

- What do we know about (x, y)?
- What about when $x\ is\ 0$?
  - When $\boldsymbol{x\ is\ 0}$, we have the $\boldsymbol{y - intercept}$

    $$(\boldsymbol{0}, \boldsymbol{b})$$

- What about when $y\ is\ 0$?
  - When $\boldsymbol{y\ is\ 0}$, we have the $\boldsymbol{x - intercept}$

    $$(\boldsymbol{a}, \boldsymbol{0})$$

- It's easiest to start by finding these two points!
- We can set-up a **table of values**
  - It tells us **what $\boldsymbol{x}$ is** when **we have a particular $\boldsymbol{y}$**
  - Or **what $\boldsymbol{y}$ is** when **we have a particular $\boldsymbol{x}$**
- You can **literally pick any value for $\boldsymbol{x\ or\ y}$** and solve for the other!!!

Example 5: Graph $x + y = 4$

Solution 5: Set yourself up a **Table of Values**

x	y
0	$\mathbf{4}$
$\mathbf{4}$	0
$\mathbf{-4}$	$\mathbf{8}$

- It reads, when $x\ is$ … $y\ is$ …

i) $When\ x\ is\ 0$ (sub in 0 for x)

$0 + y = 4$
$\boldsymbol{y = 4}$

ii) $When\ y\ is\ 0$ (sub in 0 for y)

$x + 0 = 4$
$\boldsymbol{x = 4}$

iii) When $\boldsymbol{x\ is\ -4}$ (sub in x solve for y)

$-4 + y = 4$
$y = 4 + 4$
$\boldsymbol{y = 8}$

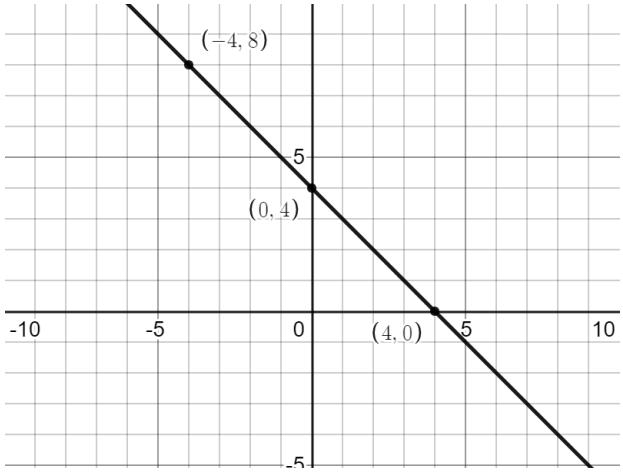
- For the **third point** you can **pick anything**
- I highly suggest you take the **value that you got** when you solved **for x**, and **flip the sign**
- If it was positive use the negative of it, and vice versa
- In this case we will take -4
- But remember you can pick any value, strategy helps though!

- So with the completed table of values we can graph it now
- We have three points:

$$(0, 4), (4, 0), and\ (-4, 8)$$

- Three points is enough to graph your line and it gives you a chance to check to see if there is an error, all your points should line up

x	y
0	$\mathbf{4}$
$\mathbf{4}$	0
$\mathbf{-4}$	$\mathbf{8}$



Example 6: Graph $4x - 3y = 12$

Solution 6: Start by identifying your intercepts

If $\boldsymbol{x = 0}$:

$$4(0) - 3y = 12$$

$$-3y = 12 \quad \rightarrow \quad \boldsymbol{y = -4}$$

If $\boldsymbol{y = 0}$:

$$4x - 3(0) = 12$$

$$4x = 12 \quad \rightarrow \quad \boldsymbol{x = 3}$$

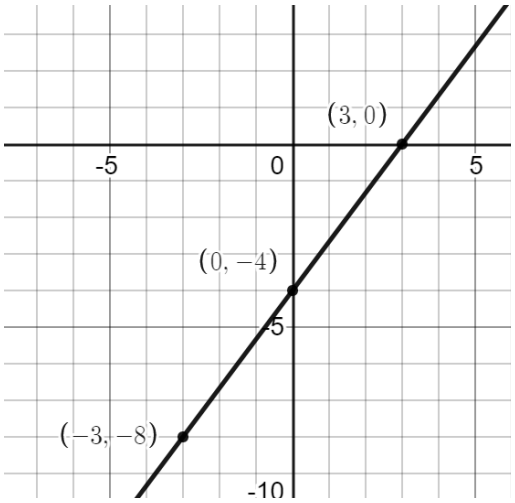
Pick any point, I'll use $\boldsymbol{x = -3}$

$$4(-3) - 3y = 12$$

$$-12 - 3y = 12 \quad \rightarrow \quad -3y = 24$$

$$\boldsymbol{y = -8}$$

x	y
0	-4
3	0
-3	-8

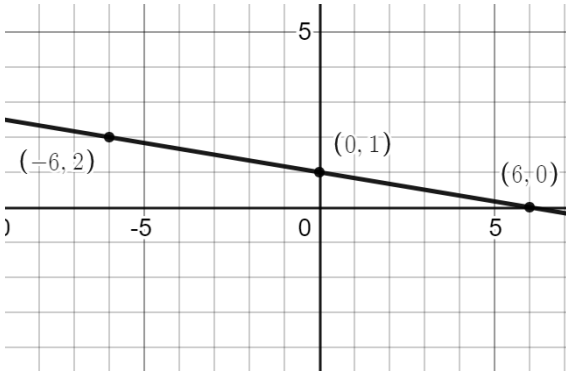


Example 7: Graph the following. $\frac{2}{3}x + 4y = 4$ ← Technically not Standard Form yet, the $A\ term$ is a fraction

Solution 7: This will be a little tricky.

- Go back to algebra, how do we **remove multiple fractions**?
- Multiply everything by the **LCM,** in this case: 3
- Then make your table of values
- And graph the results

$$3 \cdot \frac{2}{3}x + 4y \cdot 3 = 4 \cdot 3 \quad \rightarrow \quad 2x + 12y = 12$$



x	y
0	1
6	0
-6	2

If $\boldsymbol{x = 0}$:

$$2(0) + 12y = 12$$

$$12y = 12 \quad \rightarrow \quad \boldsymbol{y = 1}$$

If $\boldsymbol{y = 0}$:

$$2x + 12(0) = 12$$

$$2x = 12 \quad \rightarrow \quad \boldsymbol{x = 6}$$

Pick any point, I'll use $\boldsymbol{x = -6}$

$$2(-6) + 12y = 12$$

$$-12 + 12y = 12 \quad \rightarrow \quad 12y = 24$$

$$\boldsymbol{y = 2}$$

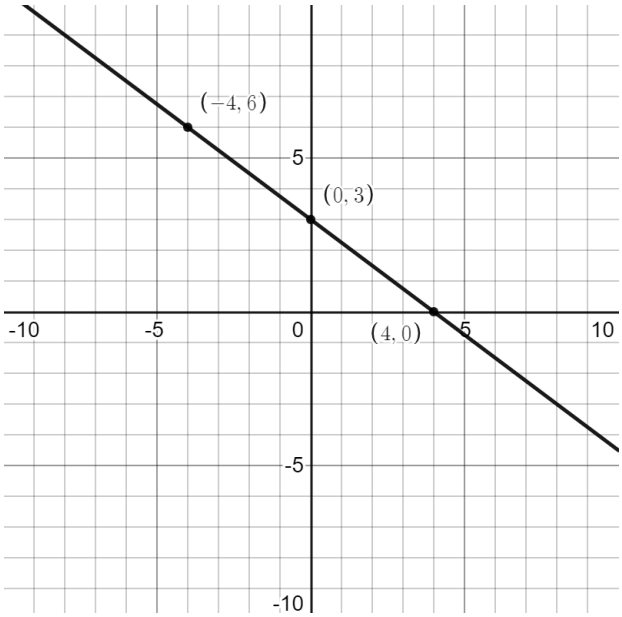
Example 8: Graph the following. $\frac{1}{2}x + \frac{2}{3}y = 2$

Solution 8: This will be similar to the previous example.

- Go back to algebra, how do we **remove multiple fractions**?
- Multiply everything by the **LCM, i**n this case: 6
- Then make your table of values
- And graph the results

$$6 \cdot \frac{1}{2}x + \frac{2}{3}y \cdot 6 = 2 \cdot 6 \quad \rightarrow \quad 3x + 4y = 12$$

x	y
0	4
4	0
-4	6



If $\boldsymbol{x = 0}$:

$$3(0) + 4y = 12$$

$$4y = 12 \quad \rightarrow \quad \boldsymbol{y = 3}$$

If $\boldsymbol{y = 0}$:

$$3x + 4(0) = 12$$

$$3x = 12 \quad \rightarrow \quad \boldsymbol{x = 4}$$

Pick any point, I'll use $\boldsymbol{x = -4}$

$$3(-4) + 4y = 12$$

$$-12 + 4y = 12 \quad \rightarrow \quad 4y = 24$$

$$\boldsymbol{y = 6}$$

Using **Standard Form** is really only **helpful sometimes**.

It **gives us the intercepts** to be sure, but what if they are not perfect whole number?

It can be cumbersome to use for graphing, so being able to **algebraically manipulate it to Slope-Intercept Form** has some significant benefits.

That is where we head next.

Converting from Standard to Slope-Intercept

- Now, there will come a time where you would like to have **SLOPE-INTERCEPT FORM** or **STANDARD FORM** but you have the opposite.
- The good news is that you can always use **ALGEBRA** to **manipulate the equation** you have into the **equation you want**.

Example: Change the equation from **Standard to Slope-Intercept Form**

$3x - 4y = 6$	• We need $y = mx + b$
$3x - 3x - 4y = 6 - 3x$	• Subtract $3x$ from both sides
$-4y = -3x + 6$	• Rearrange the equation
$\frac{-4y}{-4} = \frac{-3x}{-4} + \frac{6}{-4}$	• Divide everything by -4
$\boldsymbol{y = \frac{3}{4}x - \frac{3}{2}}$	• Simplify all the fractions

Example: Change the equation from **Standard to Slope-Intercept Form**

$-\frac{2}{3}x + \frac{1}{4}\mathrm{y} = 7$	• We need $y = mx + b$
$12 \cdot -\frac{2}{3}\mathrm{x} + \frac{1}{4}\mathrm{y} \cdot 12 = 7 \cdot 12$	• Multiply by the LCM to remove fractions
$-8x + 3y = 84$	• Simplify the equation
$-8x + 8x + 3y = 84 + 8x$	• Add $8x$ to both sides
$3y = 8x + 84$	• Rearrange the equation
$\frac{3y}{3} = \frac{8x}{3} + \frac{84}{3}$	• Divided everything by 3
$\boldsymbol{y = \frac{8}{3}\mathrm{x} + 28}$	• Simplify all the fractions

Example: Change the equation from **Standard to Slope-Intercept Form**

$$\frac{5}{3}x - \frac{1}{2}y = -2$$

- We need $y = mx + b$

$$6 \cdot \frac{5}{3}x - \frac{1}{2}y \cdot 6 = -2 \cdot 6$$

- Multiply by the LCM to remove fractions

$$10x - 3y = -12$$

- Simplify the equation

$$10x - 10x - 3y = -12 - 10x$$

- Subtract $10x$ from both sides

$$-3y = -10x - 12$$

- Rearrange the equation

$$\frac{-3y}{-3} = \frac{-10x}{-3} - \frac{12}{-3}$$

- Divided everything by 3

$$\boldsymbol{y = \frac{10}{3}x + 4}$$

- Simplify all the fractions

Example: Change the equation from **Standard to Slope-Intercept Form**

$$\frac{x + y}{4} = -7$$

- We need $y = mx + b$

$$4 \cdot \frac{x + y}{4} = -7 \cdot 4$$

- Multiply by the LCM to remove fractions

$$x + y = -28$$

- Simplify the equation

$$x - x + y = -28 - x$$

- Subtract x from both sides

$$\boldsymbol{y = -x - 28}$$

- Simplify the equation

We can also change from Slope-Intercept Form to Standard Form.

It doesn't have the same benefits for Graphing, but will come in handy in the years ahead.

Example: Change the equation from **Slope-Intercept to Standard Form**

$y = \frac{2}{3}x - 4$	• We need $Ax + By = C$
$3 \cdot y = 3 \cdot \frac{2}{3}x - 4 \cdot 3$	• Multiply by the LCM to remove fractions
$3y = 2x - 12$	• Simplify the equation
$3y - 3y = 2x - 3y - 12$	• Subtract $3y$ from both sides
$0 = 2x - 3y - 12$	• Rearrange the equation
$12 + 0 = 2x - 3y - 12 + 12$	• Add 12 to both sides
$12 = 2x - 3y$	• Rearrange the equation
$2x - 3y = 12$	• Make sure A is a natural number

Example: Change the equation from **Slope-Intercept to Standard Form**

$y = 5x + \frac{2}{3}$	• We need $Ax + By = C$
$3 \cdot y = 3 \cdot 5x + \frac{2}{3} \cdot 3$	• Multiply by the LCM to remove fractions
$3y = 15x + 2$	• Simplify the equation
$3y - 3y = 15x - 3y + 2$	• Subtract $3y$ from both sides
$0 = 15x - 3y + 2$	• Rearrange the equation
$0 - 2 = 15x - 3y + 2 - 2$	• Subtract 2 from both sides
$-2 = 15x - 3y$	• Rearrange the equation
$15x - 3y = -2$	• Make sure A is a natural number

Example: Change the equation from **Slope-Intercept to Standard Form**

$y = -\frac{3}{4}x + \frac{2}{3}$	• We need $Ax + By = C$
$12 \cdot y = -\frac{3}{4}x \cdot 12 + \frac{2}{3} \cdot 12$	• Multiply by the LCM to remove fractions
$12y = -9x + 8$	• Simplify the equation
$12y - 12y = -9x - 12y + 8$	• Subtract $12y$ from both sides
$0 = -9x - 12y + 8$	• Rearrange the equation
$0 - 8 = -9x - 12y + 8 - 8$	• Subtract 8 from both sides
$-8 = -9x - 12y$	• Rearrange the equation
$-9x - 12y = -8$	• Rewrite as $Ax + By = C$
$\boldsymbol{9x + 12y = 8}$	• Multiply everything by -1 to make sure A is a natural number

Example: Change the equation from **Slope-Intercept to Standard Form**

$y = \frac{2}{3}x + 6$	• We need $Ax + By = C$
$3 \cdot y = 3 \cdot \frac{2}{3}x + 6 \cdot 3$	• Multiply by the LCM to remove fractions
$3y = 2x + 18$	• Simplify the equation
$3y - 3y = 2x - 3y + 18$	• Subtract $3y$ from both sides
$0 = 2x - 3y + 18$	• Rearrange the equation
$0 - 18 = 2x - 3y + 18 - 18$	• Subtract 18 from both sides
$-18 = 2x - 3y$	• Rearrange the equation
$\boldsymbol{2x - 3y = -18}$	• Make sure A is a natural number

Matching Equations to Graphs

When the equations are in **Standard Form** it's a little different

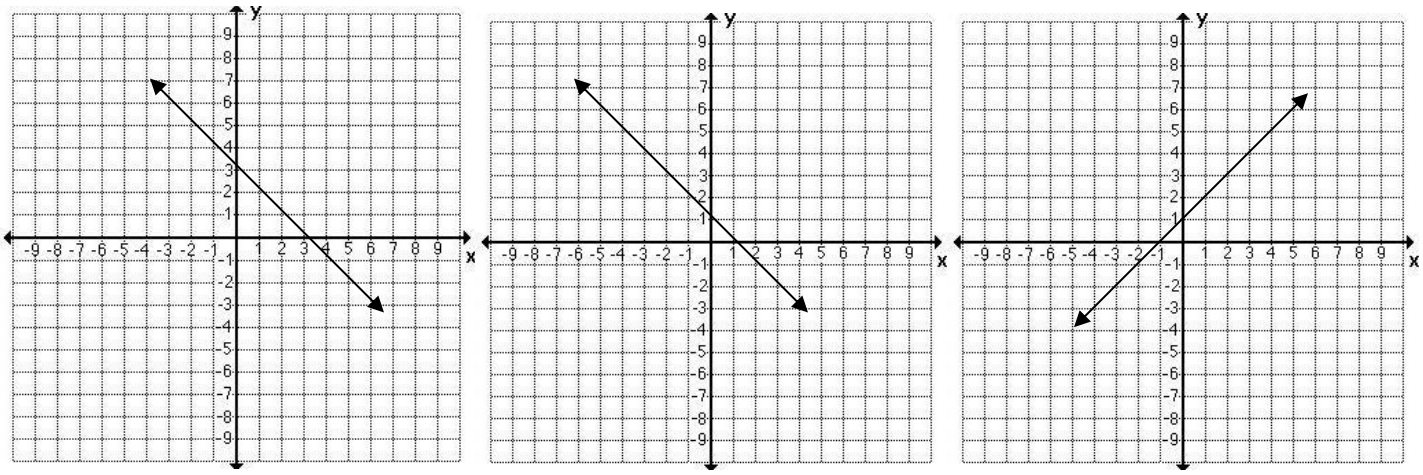
- You can't look at the **SLOPE**, **Standard Form** doesn't have it
- You need to look at the points you can get
    - Namely: **the x and y intercepts**
- Remember:
    - that to find the $x-intercept$ we set y to 0
    - That to find the $y-intercept$ we set x to 0

x	y
0	
	0

$\leftarrow$ $y-intercept$

$\leftarrow$ $x-intercept$

Example: Which graph matches the equation? $x+y=1$



When: $x = 0$; $y = 1$

When: $y = 0$; $x = 1$

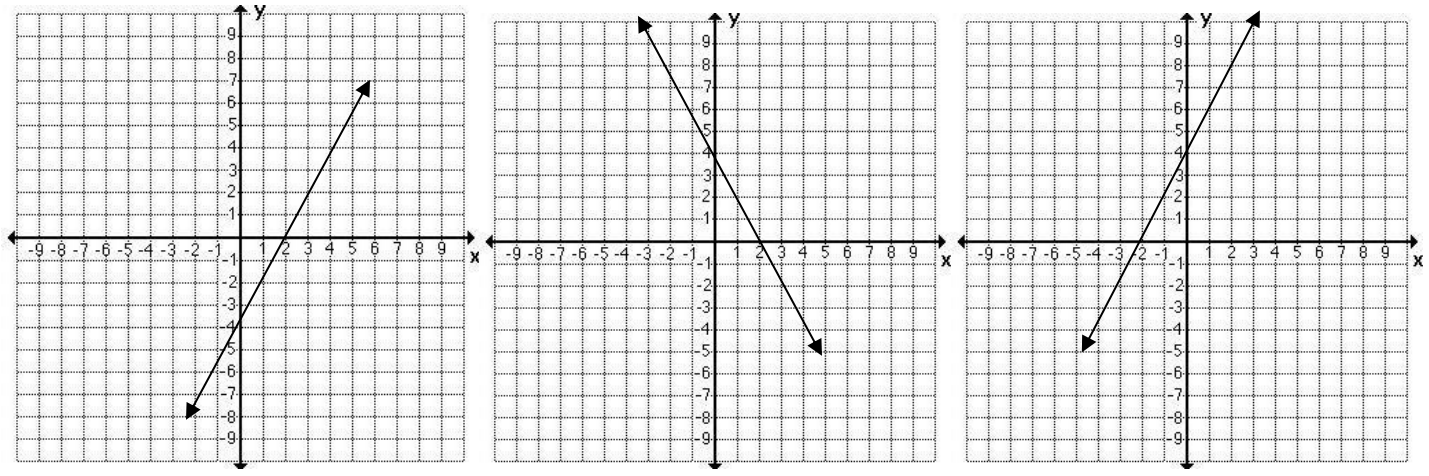
The $y-int = (0,1)$

The $x-int = (1,0)$

So, the only graph that matches that is:

Graph 2

Example: Which graph matches the equation? $2x + y = 4$



When: $x = 0$; $y = 4$

When: $y = 0$; $x = 2$

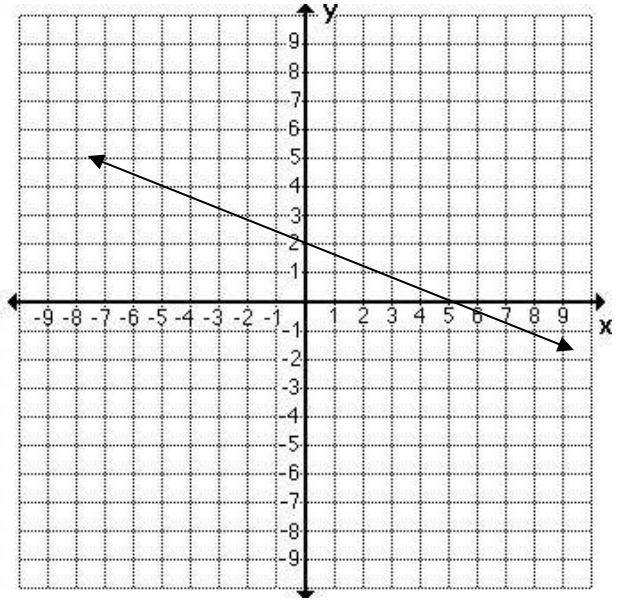
The $y - int = (0, 4)$

The $x - int = (2, 0)$

So, the only graph that matches that is:

Graph 2

Example: What equation matches the following graph?



i) $-2x + 5y = 10$ ← When x *is* 0; y *is* 2

ii) $\boldsymbol{2x + 5y = 10}$ ← When: y *is* 0; x *is* 5

iii) $-2x + 5y = -10$

Need to Satisfy Both:

When: x *is* 0; y *is* 2

y *is* 0; x *is* 5

Section 7.2 – Practice Questions

1. Are the following points solutions to the given equations? Are they POINTS on the given LINE?

   a) $(2, 4)$ $2x + 3y = 16$

   b) $(-6, 0)$ $\frac{1}{6}x + 13y = 1$

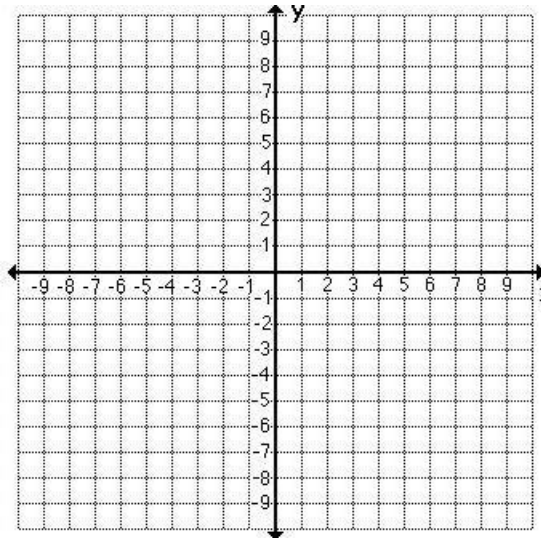
   c) $(8, -2)$ $x - 2y = 4$

   d) $(-3, -4)$ $4x + 2y = -20$

2. Graph the following equations, use the table of values to organize your coordinates.

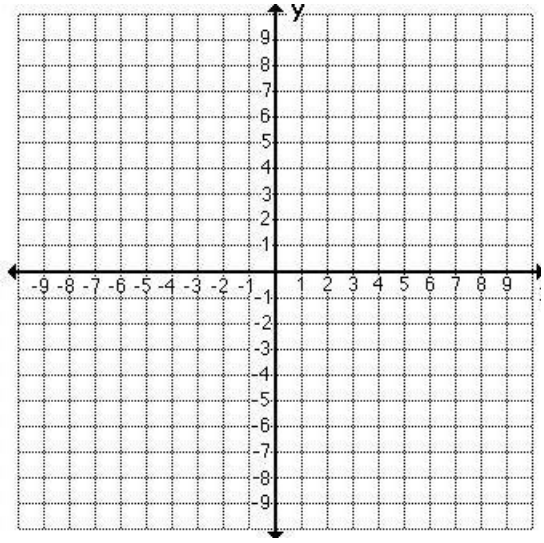
$i)$ $2x - 3y = 12$

x	y

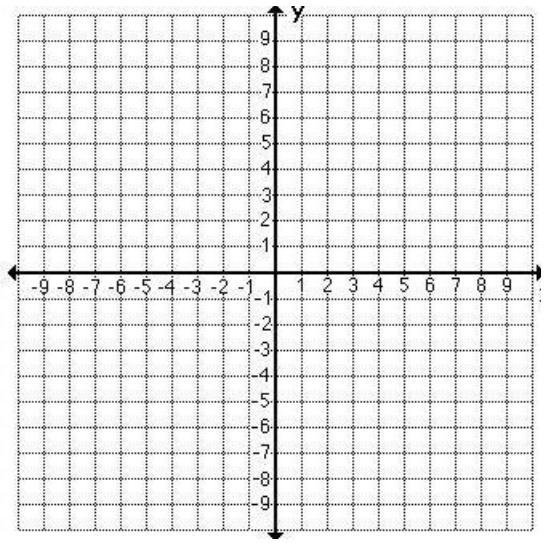


$ii)$ $-4x + 5y = 40$

x	y

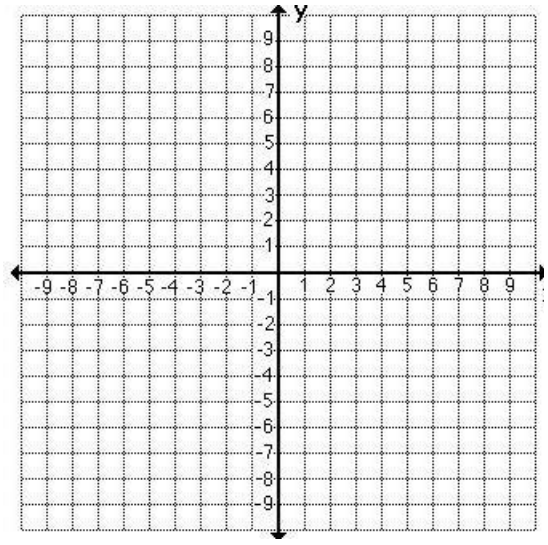


iii) $\frac{2}{3}x - y = 2$



x	y

iv) $-\frac{3}{5}x + \frac{1}{2}y = 3$

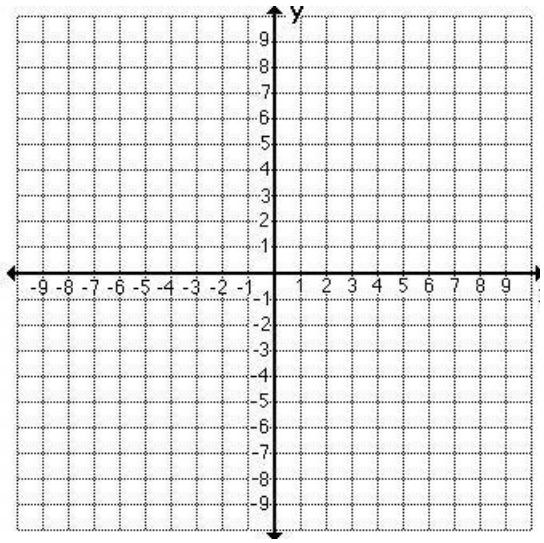


x	y

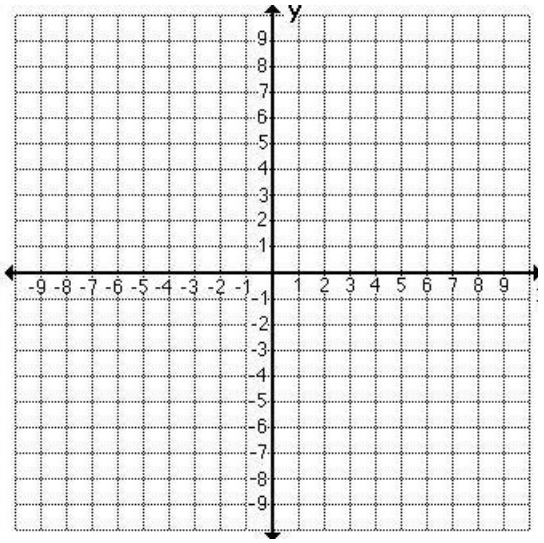
3. How many points are there on a line? Explain your thinking.

4. Using your algebraic logic, manipulate the STANDARD FORM equations in to SLOPE-INTERCEPT equations and graph them.

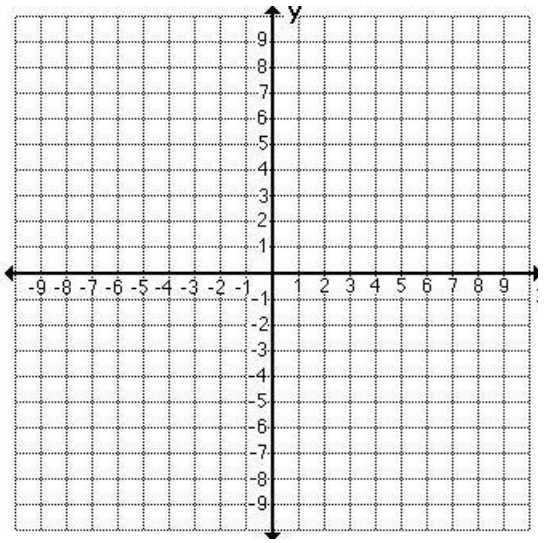
a) $2x + 2y = -4$



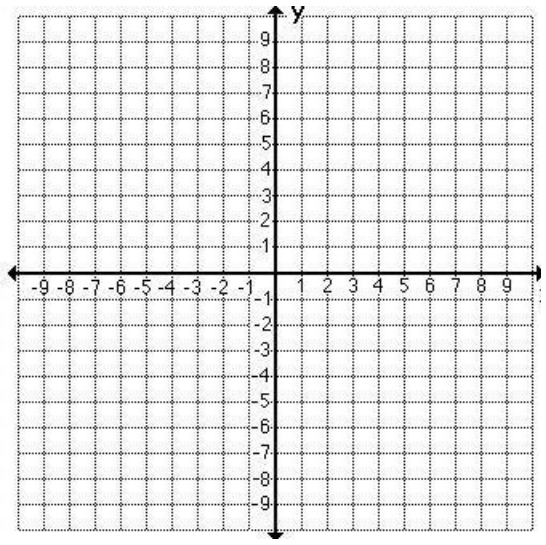
b) $\frac{3}{5}x - \frac{2}{3}y = \frac{2}{3}$



c) $12x - 5y = 10$



d) $-\frac{1}{6}x - \frac{2}{3}y = 2$



5. Using your algebraic logic, manipulate the SLOPE-INTERCEPT to STANDARD FORM, remember that $Ax + By = C$ has NO FRACTIONS and $A > 0$

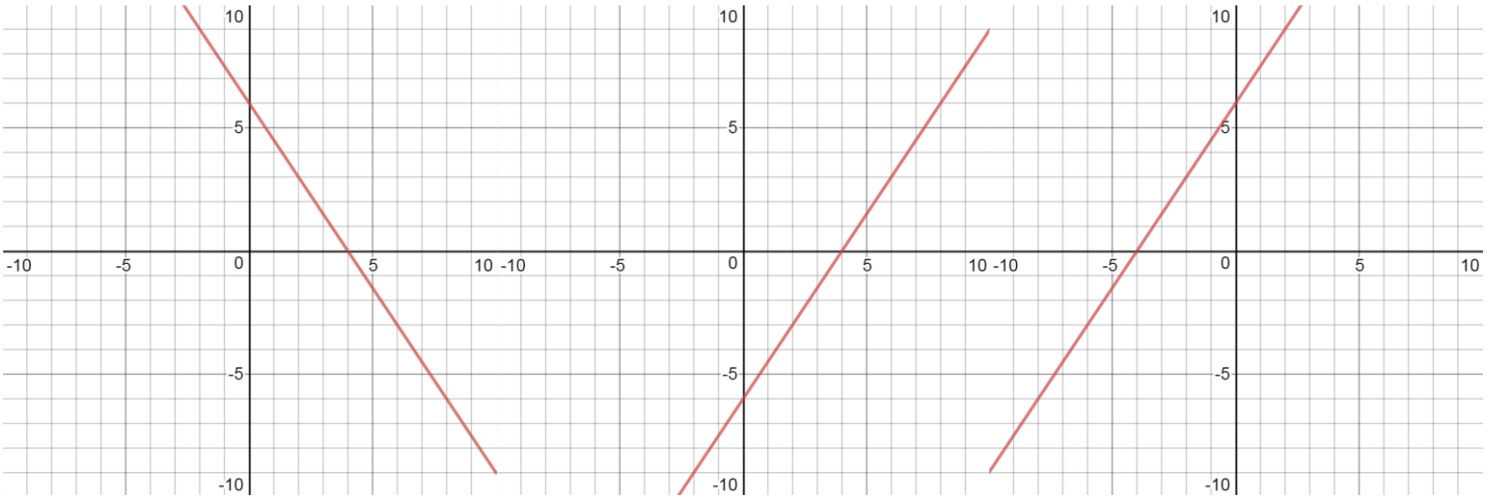
a) $y = \frac{2}{5}x + 6$

b) $y = -7x - 4$

c) $y = 5x - \frac{2}{3}$

d) $-4 + 3x = y$

6. Which graph represents $3x - 2y = 12$ How do you know?



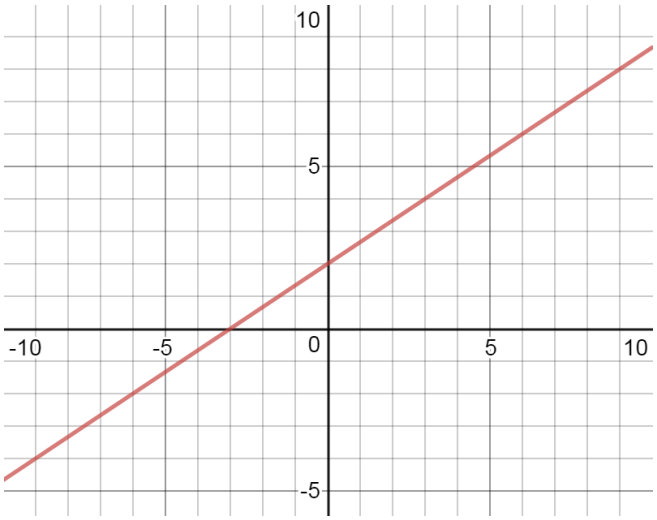
Explanation Goes Here

7. Which equation matches the graph below:

$2x + 3y = 6$

$-2x - 3y = 6$

$2x - 3y = -6$



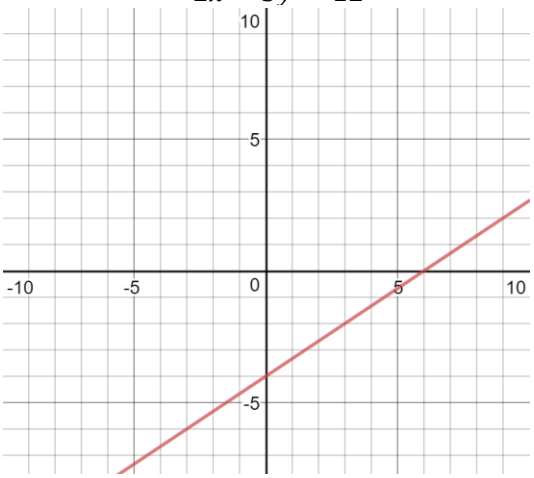
Explanation Goes Here

Answer Key – Section 7.2

1. a) Yes b) No c) No d) Yes

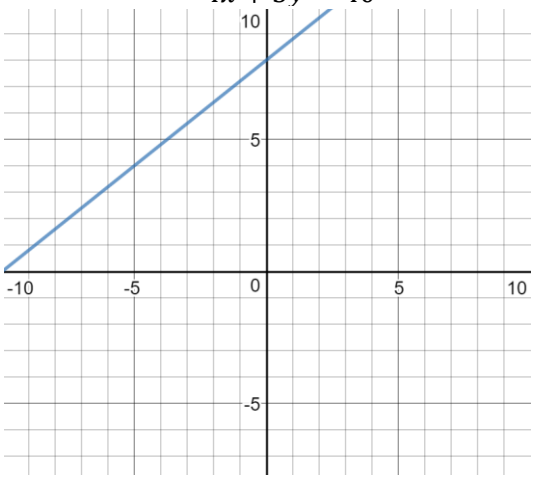
2. i)

$2x - 3y = 12$



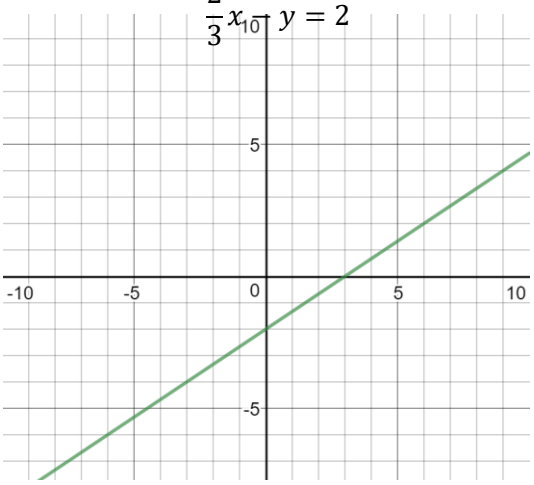
ii)

$-4x + 5y = 40$



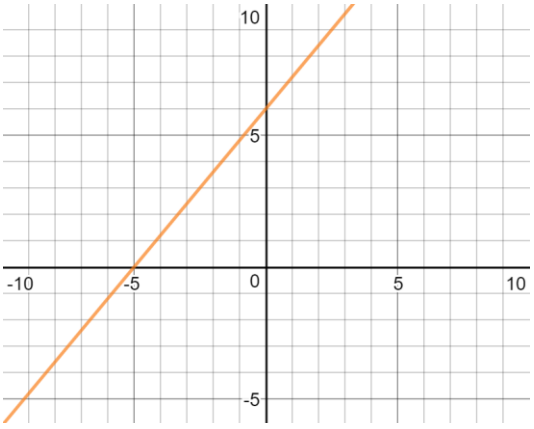
iii)

$\frac{2}{3}x - y = 2$



iv)

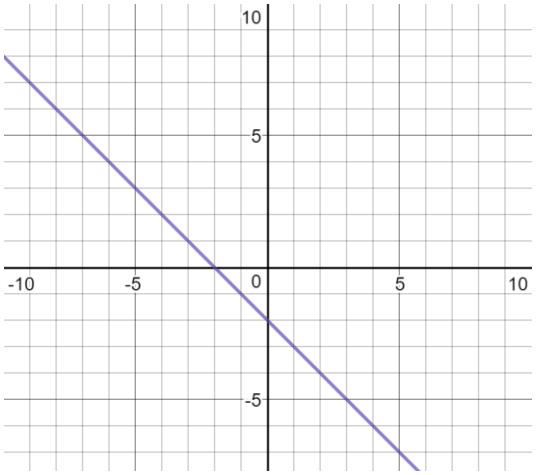
$-4x + 5y = 40$



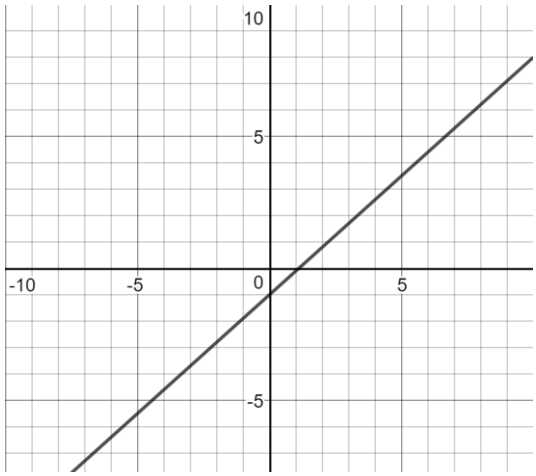
3. Infinte Number of Points on a line

Explanations will Vary

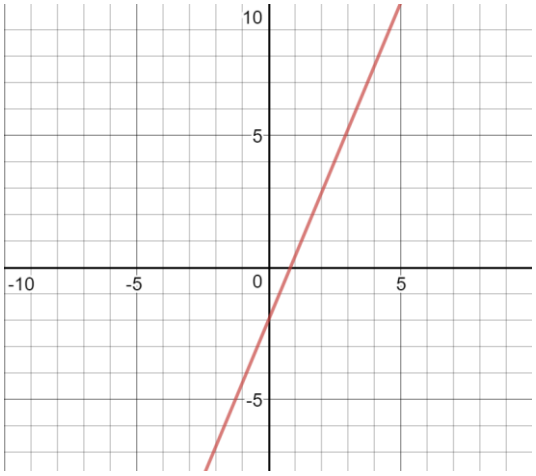
4. a) $2x + 2y = -4 \quad \rightarrow \quad y = -x - 2$



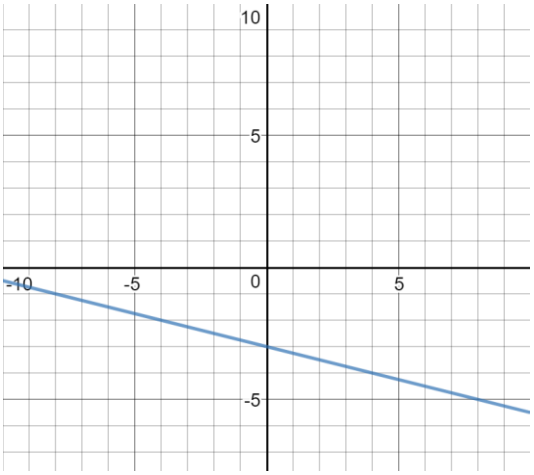
b) $\frac{3}{5}x - \frac{2}{3}y = \frac{2}{3} \quad \rightarrow \quad y = \frac{9}{10}x - 1$



c) $12x - 5y = 10 \quad \rightarrow \quad y = \frac{12}{5}x - 2$



d) $-\frac{1}{6}x - \frac{2}{3}y = 2 \quad \rightarrow \quad y = -\frac{1}{4}x - 3$



5. a) $2x - 5y = -30$ b) $7x + y = -4$ c) $15x - 3y = 2$ d) $3x - y = 4$

6. Second Graph
7. Third Equation

Extra Work Space