## Section 7.2 – Practice Problems

1. Find the derivative of $y$ with respect to $x$ in each of the following.

a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$= \boxed{4\sin(4x)}$

b) $y = \sin(3x + 2\pi)$

$y' = \cos(3x+2\pi) \cdot 3$

$= \boxed{3\cos(3x+2\pi)}$

c) $y = 4\sin(-2x^2 - 3)$

$y' = 4[\cos(-2x^2-3)] \cdot -4x$

$= \boxed{-16x\cos(-2x^2-3)}$

d) $y = -\frac{1}{2}\cos(4 + 2x)$

$y' = -\frac{1}{2}[-\sin(4+2x) \cdot 2]$

$\boxed{\sin(4+2x)}$

e) $y = \sin x^2$

$y' = \cos x^2 \cdot 2x$

$= \boxed{2x\cos x^2}$

f) $y = -\cos x^2$

$y' = -(-\sin x^2) \cdot 2x$

$= \boxed{2x\sin x^2}$

g) $y = \sin^{-2}(x^3)$

$= (\sin x^3)^{-2}$

$y' = -2(\sin x^3)^{-3} \cdot \cos x^3 \cdot 3x^2$

$= \boxed{\dfrac{-6x^2 \cos x^3}{\sin^3 x^3}}$

h) $y = \cos(x^2 - 2)^2$

$y' = -\sin(x^2-2)^2 \cdot 2(x^2-2) \cdot 2x$

$= \boxed{-4x(x^2-2)\sin(x^2-2)^2}$

i) $y = 3\sin^4(2-x)^{-1}$

$= 3(\sin(2-x)^{-1})^4$

$y' = 3\left[4(\sin(2-x)^{-1})^3 \cdot \cos(2-x)^{-1} \cdot -1(2-x)^{-2} \cdot -1\right]$

$3\left[4\sin^3(2-x)^{-1} \cdot \cos(2-x)^{-1} \cdot (2-x)^{-2}\right]$

$\boxed{\dfrac{12\sin^3(2-x)^{-1}\cos(2-x)^{-1}}{(2-x)^2}}$

j) $y = x\cos x$

$y' = x(-\sin x) + \cos x$

$= \boxed{\cos x - x\sin x}$

k) $y = \frac{x}{\sin x}$

$y' = \dfrac{\sin x(1) - x\cos x}{\sin^2 x}$

$= \boxed{\dfrac{\sin x - x\cos x}{\sin^2 x}}$

l) $y = \frac{\sin x}{1+\cos x}$

$y' = \dfrac{(1+\cos x)\cos x - (\sin x(-\sin x))}{(1+\cos x)^2}$

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \dfrac{\cos x + 1}{(1+\cos x)^2} \rightarrow \boxed{\dfrac{1}{1+\cos x}}$

m) $y = (1 + \cos^2 x)^6$

$y' = 6(1+(\cos x)^2)^5 \cdot 2\cos x \cdot -\sin x$

$= -6(1+\cos^2 x)^5 \cdot \underbrace{2\sin x \cos x}_{\text{identity}}$

$\sin 2x$

$\boxed{-6\sin 2x\,(1+\cos^2 x)^5}$

n) $y = \sin\frac{1}{x}$

$y = \sin x^{-1}$

$y' = \cos x^{-1} \cdot -1x^{-2}$

$= -\dfrac{\cos\frac{1}{x}}{x^2} \rightarrow \boxed{-\frac{1}{x^2}\cos\frac{1}{x}}$

o) $y = \sin(\cos x)$

$y' = \cos(\cos x) \cdot -\sin x$

$= \boxed{-\sin x \cos(\cos x)}$

p) $y = \cos^3(\sin x)$

$= (\cos(\sin x))^3$

$y' = 3\cos^2(\sin x) \cdot -\sin(\sin x) \cdot \cos x$

$= \boxed{-3\cos x \sin(\sin x)\cos^2(\sin x)}$

2. Find $\frac{dy}{dx}$ in each of the following.

a) $\sin y = \cos 2x$

$\cos y \frac{dy}{dx} = -\sin 2x \cdot 2$

$\boxed{\frac{dy}{dx} = \frac{-2\sin 2x}{\cos y}}$

b) $x\cos y = \sin(x+y)$

$$x(-\sin y)\frac{dy}{dx} + \cos y = \cos(x+y)\left(1+\frac{dy}{dx}\right)$$

$$x(-\sin y)\frac{dy}{dx} + \cos y = \cos(x+y) + \cos(x+y)\frac{dy}{dx}$$

$$-x\sin y\frac{dy}{dx} - \cos(x+y)\frac{dy}{dx} = \cos(x+y) - \cos y$$

$$\frac{dy}{dx}(-x\sin y - \cos(x+y)) = \cos(x+y) - \cos y$$

$$\frac{dy}{dx} = \frac{\cos(x+y)-\cos y}{-x\sin y - \cos(x+y)}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos y - \cos(x+y)}{x\sin y + \cos(x+y)}}$$

c) $\sin y + y = \cos x + x$

$$\cos y\frac{dy}{dx} + \frac{dy}{dx} = -\sin x + 1$$

$$\frac{dy}{dx}(1+\cos y) = 1-\sin x$$

$$\boxed{\frac{dy}{dx} = \frac{1-\sin x}{1+\cos y}}$$

d) $\sin(\cos x) = \cos(\sin y)$

$$\cos(\cos x)\cdot -\sin x = -\sin(\sin y)\cdot\cos y\frac{dy}{dx}$$

$$-\sin x\cos(\cos x) = \frac{dy}{dx}[-\cos y\sin(\sin y)]$$

$$\boxed{\frac{dy}{dx} = \frac{\sin x\cos(\cos x)}{\cos y\sin(\sin y)}}$$

e) $\sin x\cos y + \cos x\sin y = 1$

$$\sin x(-\sin y)\frac{dy}{dx} + \cos x\cos y + \cos x\cos y\frac{dy}{dx} + -\sin x\sin y = 0$$

$$-\sin x\sin y\frac{dy}{dx} + \cos x\cos y\frac{dy}{dx} = \sin x\sin y - \cos x\cos y$$

$$\frac{dy}{dx}(\cos x\cos y - \sin x\sin y) = \sin x\sin y - \cos x\cos y$$

$$\frac{dy}{dx} = \frac{\sin x\sin y - \cos x\cos y}{-1(\sin x\sin y - \cos x\cos y)} = \boxed{-1}$$

f) $\sin x + \cos 2x = 2xy$

$$\cos x - \sin 2x \cdot 2 = 2\left[x\frac{dy}{dx} + y\right]$$

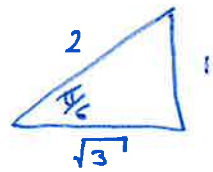
$$\cos x - 2\sin 2x = 2x\frac{dy}{dx} + 2y$$

$$\cos x - 2\sin 2x - 2y = 2x\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x - 2\sin 2x - 2y}{2x}}$$

3. Find the equation of the tangent line to the given curve at the given point.

a)
$y = 2\sin x;\quad \left(\frac{\pi}{6}, 1\right)$




$y' = 2\cos x$ at $x = \frac{\pi}{6}$

$2\cos\frac{\pi}{6}$

$2\left(\frac{\sqrt{3}}{2}\right)$

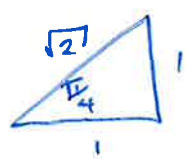
$= \sqrt{3}$

$y - 1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

$$\boxed{y = \sqrt{3}x - \frac{\sqrt{3}\pi}{6} + 1}$$

---

b)
$y = \frac{\sin x}{\cos x};\quad \left(\frac{\pi}{4}, 1\right)$




$$y' = \frac{\cos x \cos x - \sin x(-\sin x)}{(\cos x)^2}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

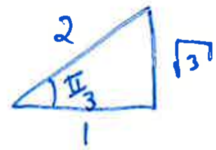
$$\frac{1}{\cos^2 x} \rightarrow \frac{1}{\left(\cos\frac{\pi}{4}\right)^2} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = 2$$

$y - 1 = 2\left(x - \frac{\pi}{4}\right)$

$$\boxed{y = 2x - \frac{\pi}{2} + 1}$$

c)

$$y = \frac{1}{\cos x} - 2\cos x; \quad \left(\frac{\pi}{3}, 1\right)$$




$y = (\cos x)^{-1} - 2\cos x$

$y' = -1(\cos x)^{-2} \cdot -\sin x - 2(-\sin x)$

$= \dfrac{\sin x}{\cos^2 x} + 2\sin x$

$\dfrac{\sin \frac{\pi}{3}}{\left(\cos \frac{\pi}{3}\right)^2} + 2\sin\frac{\pi}{3}$

$\rightarrow \dfrac{\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)^2} + \dfrac{2\sqrt{3}}{2} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

$y - 1 = 3\sqrt{3}\left(x - \dfrac{\pi}{3}\right)$

$\boxed{y = 3\sqrt{3}x - \sqrt{3}\pi + 1}$

d)

$$y = \frac{\cos^2 x}{\sin^2 x} \quad \left(\frac{\pi}{4}, 1\right)$$

$y' = \dfrac{\sin^2 x \cdot 2\cos x \cdot -\sin x - \cos^2 x \cdot 2\sin x \cos x}{\sin^4 x}$

$\dfrac{-2\sin x\cos x \sin^2 x - 2\sin x \cos x \cos^2 x}{\sin^4 x}$

Identity: $2\sin x\cos x$
$= \sin 2x$

$\rightarrow \dfrac{-2\sin x\cos x\,(\sin^2 x + \cos^2 x)}{\sin^4 x}$

$\rightarrow \dfrac{-\sin 2x\,(1)}{\sin^4 x}$ at $x = \frac{\pi}{4}$

$\dfrac{-\sin\frac{\pi}{2}}{\left(\sin\frac{\pi}{4}\right)^4} \rightarrow \dfrac{-1}{\left(\frac{1}{\sqrt{2}}\right)^4}$

$= \dfrac{-1}{\frac{1}{4}} = \boxed{-4}$

$y - 1 = -4x - \pi$

$\boxed{y = -4x + \pi + 1}$

e)

$$y = \sin x + \cos 2x; \quad \left(\frac{\pi}{6}, 1\right)$$

$y' = \cos x - \sin 2x \cdot 2$

$= \cos x - 2\sin 2x$ at $x = \frac{\pi}{6}$

$\rightarrow \cos\frac{\pi}{6} - 2\sin\frac{\pi}{3}$

$\Rightarrow \dfrac{\sqrt{3}}{2} - \dfrac{2\sqrt{3}}{2}$

$\boxed{-\dfrac{\sqrt{3}}{2}}$

$y - 1 = -\dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{6}\right)$

$\boxed{y = -\dfrac{\sqrt{3}}{2}x + \dfrac{\sqrt{3}\pi}{12} + 1}$

f)
$y = \cos(\cos x)$; $\left(\frac{\pi}{2}\right)$

$y' = -\sin(\cos x) \cdot -\sin x$

$= \sin x \sin(\cos x)$ at $x = \frac{\pi}{2}$

$\sin\frac{\pi}{2} \sin(\cos\frac{\pi}{2})$

$1 \cdot \sin 0$

$y' = 0$ so we have a horizontal line

at $\cos(\cos\frac{\pi}{2})$

$\cos 0$

$= 1$

$\boxed{y = 1}$

4. Find the critical numbers, the intervals of increase and decrease, and any maximum or minimum values

a) $y = \sin^2 x,\ -\pi \le x \le \pi$

ct pts: $\frac{\pi}{2}, -\frac{\pi}{2}, -\pi, \pi, 0$

$y' = 2\sin x \cdot \cos x$

$= 2\sin x \cos x$

$= \sin 2x$

$0 = \sin 2x$ at $\sin 0$ and $\pi$

so $2x = 0$ $x = 0$

$2x = 0$ $x = \frac{\pi}{2}$

local max
$f(-\frac{\pi}{2}) = 1$
$f(\frac{\pi}{2}) = 1$

local min
$f(0) = 0$

| Interval | $2\sin x$ | $\cos x$ | $f'(x)$ | $f(x)$ |
|---|---|---|---|---|
| $(-\pi, -\frac{\pi}{2})$ | − | − | + | inc |
| $(-\frac{\pi}{2}, 0)$ | − | + | − | dec |
| $(0, \frac{\pi}{2})$ | + | + | + | inc |
| $(\frac{\pi}{2}, \pi)$ | + | − | − | dec |

b) $y = \cos x - \sin x, -\pi \le x \le \pi$

ct pts: $-\pi, -\frac{\pi}{4}, \frac{3\pi}{4}, \pi$

$y' = -\sin x - \cos x$

$\rightarrow -1(\sin x + \cos x)$

ct pt when $\sin x = -\cos x$

occurs in Q4 and Q2

reference angle of $\frac{\pi}{4}$

Q4: $-\frac{\pi}{4}$

Q2: $\frac{3\pi}{4}$

| Interval | $-\sin x - \cos x$ | $f'(x)$ | $f(x)$ |
|---|---|---|---|
| $(-\pi, -\frac{\pi}{4})$ | + | + | inc |
| $(-\frac{\pi}{4}, \frac{3\pi}{4})$ | − | − | dec |
| $(\frac{3\pi}{4}, \pi)$ | + | + | inc |

local max
$f(-\frac{\pi}{4}) = \frac{2}{\sqrt{2}}$

local min
$f(\frac{3\pi}{4}) = -\frac{2}{\sqrt{2}}$

5. Determine the concavity and find the inflection points.

a) $y = 2\cos x + \sin 2x,\ 0 \le x \le 2\pi$ ← just means 1 rotation

$y' = -2\sin x + \cos 2x \cdot 2$

$= -2\sin x + 2\cos 2x$

$= 2(\cos 2x - \sin x)$

$y'' = 2[-\sin 2x \cdot 2 - \cos x]$

$= -4\sin 2x - 2\cos x$

$0 = -4\sin 2x - 2\cos x$

$0 = \cos x + 2\sin 2x$

$\cos x + 2(2\sin x\cos x)$

$\cos x + 4\sin x\cos x = 0$

$\cos x(1 + 4\sin x) = 0$

occurs when $\cos x = 0$

so $x = \frac{\pi}{2}, \frac{3\pi}{2}$

occurs when $\sin x = -\frac{1}{4}$ ← in Q3 Q4

ref angle of 0.2527

So

$\pi + 0.2527 = 3.39$

$2\pi - 0.2527 \quad 6.03$

| Interval | $f''(x)$ | $f(x)$ |
|---|---|---|
| $(0, \frac{\pi}{2})$ | − | co |
| $(\frac{\pi}{2}, 3.39)$ | + | cu |
| $(3.39, \frac{3\pi}{2})$ | − | co |
| $(\frac{3\pi}{2}, 6.03)$ | + | cu |
| $(6.03, 2\pi)$ | − | co |

Inflection

$f(\frac{\pi}{2}) = 0 \quad f(3.39) = -1.45$

$f(\frac{3\pi}{2}) = 0 \quad f(6.03) = 1.45$

b) $y = 4\sin^2 x - 1,\ -\pi \le x \le \pi$

$y' = 4[2\sin x \cdot \cos x]$

$= 8\sin x\cos x$

$= 4(2\sin x\cos x)$

$= 4\sin 2x$

$y'' = 4\cos 2x \cdot 2$

$= 8\cos 2x$

$0 = 8\cos 2x$

$0 = \cos 2x$ when $2x = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{3\pi}{2}$

$x = \frac{\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}$

| Interval | $\cos 2x$ | $f(x)$ |
|---|---|---|
| $(-\pi, -\frac{3\pi}{4})$ | + | cu |
| $(-\frac{3\pi}{4}, -\frac{\pi}{4})$ | − | co |
| $(-\frac{\pi}{4}, \frac{\pi}{4})$ | + | cu |
| $(\frac{\pi}{4}, \frac{3\pi}{4})$ | − | co |
| $(\frac{3\pi}{4}, \pi)$ | + | cu |

$f(-\frac{3\pi}{4}) = 1$

$f(-\frac{\pi}{4}) = 1$

$f(\frac{\pi}{4}) = 1$

$f(\frac{3\pi}{4}) = 1$