Section 7.2 – Practice Problems

Prove the following identities.

Work on one side at a time, or only one side. You cannot algebraically manipulate the initial statement.

1. $\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

$\sin^2 x - (1 - \sin^2 x)$

$\sin^2 x - 1 + \sin^2 x$

$2\sin^2 x - 1 \quad = RHS$

2. $\sin x + \cos x \cot x = \csc x$

$\sin x + \cos x \left(\frac{\cos x}{\sin x}\right)$

$\frac{\sin^2 x + \cos^2 x}{\sin x}$

$\frac{1}{\sin x}$

$\csc x \quad = RHS$

3. $\frac{1}{\cos x} - \cos x = \frac{\sin^2 x}{\cos x}$

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

$\frac{1 - \cos^2 x}{\cos x}$

$\frac{\sin^2 x}{\cos x} \quad = RHS$

4. $\frac{1}{\sec x \tan x} = \csc x - \sin x$

$\frac{1}{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}$

$\frac{1}{\frac{\sin x}{\cos^2 x}}$

$\frac{\cos^2 x}{\sin x}$

$\frac{1 - \sin^2 x}{\sin x}$

$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$

$\csc x - \sin x \quad = RHS$

5. $\dfrac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \cos^4 x$

$\dfrac{(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)}{(1 - \tan^2 x)(1 + \tan^2 x)}$

$\dfrac{1\,(\cos x + \sin x)(\cos x - \sin x)}{(1 - \tan x)(1 + \tan x)\sec^2 x}$

$\dfrac{(\cos x + \sin x)(\cos x - \sin x)}{\left(\dfrac{\cos x - \sin x}{\cos x}\right)\left(\dfrac{\cos x + \sin x}{\cos x}\right)\left(\dfrac{1}{\cos^2 x}\right)}$

$\dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)} \rightarrow \cos^4 x = RHS$

$\cos^4 x$

Flip
Multiply
cancel

6. $\dfrac{\sec^4 x - 1}{\tan^2 x} = 2 + \tan^2 x$

$\dfrac{(\sec^2 x - 1)(\sec^2 x + 1)}{\tan^2 x}$

$\dfrac{\tan^2 x(\sec^2 x + 1)}{\tan^2 x}$

$\sec^2 x + 1$

$2 + \sec^2 x - 1$

$\sec^2 x + 1$

LHS = RHS

7. $\dfrac{\sin x + \cos x}{\csc x + \sec x} = \sin x \cos x$

$\dfrac{\sin x + \cos x}{\frac{1}{\sin x} + \frac{1}{\cos x}}$

$\dfrac{\sin x + \cos x}{\dfrac{\cos x + \sin x}{\sin x \cos x}}$

$\sin x + \cos x \cdot \dfrac{\cos x \sin x}{\sin x + \cos x}$

$\sin x \cos x = RHS$

8. $\dfrac{\cos x + \sin x}{\cos x - \sin x} = \dfrac{1 + \tan x}{1 - \tan x}$

$\dfrac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} \rightarrow \dfrac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}$

$\dfrac{\cos x + \sin x}{\cos x} \cdot \dfrac{\cos x}{\cos x - \sin x}$

LHS $= \dfrac{\cos x + \sin x}{\cos x - \sin x}$

9. $\dfrac{\sec x}{1-\cos x} = \dfrac{\sec x + 1}{\sin^2 x}$

$\dfrac{\frac{1}{\cos x}}{1-\cos x}$

$\dfrac{1}{\cos x(1-\cos x)}$

$\dfrac{1}{\cos x - \cos^2 x}$

$\dfrac{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}{1-\cos^2 x}$

$\dfrac{\frac{1+\cos x}{\cos x}}{1-\cos^2 x}$

$\dfrac{1+\cos x}{\cos x} \cdot \dfrac{1}{1-\cos^2 x}$

$\dfrac{(1+\cos x)}{\cos x\,(1-\cos x)(1+\cos x)}$

$\dfrac{1}{\cos x - \cos^2 x}$

LHS = RHS

10. $\dfrac{\sin\theta + \cos\theta \cot\theta}{\cos\theta \csc\theta} = \sec\theta$

$\dfrac{\sin\theta + \cos\theta \frac{\cos\theta}{\sin\theta}}{\cos\theta \frac{1}{\sin\theta}}$

$\dfrac{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}}{\frac{\sin\theta\cos\theta}{\sin\theta}}$

$\dfrac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} \rightarrow \dfrac{1}{\sin\theta} \cdot \dfrac{\sin\theta}{\cos\theta} = \sec\theta$

RHS

11. $\dfrac{1+\sec\theta}{\sin\theta\tan\theta} = \csc\theta$

$\dfrac{1+\frac{1}{\cos\theta}}{\sin\theta + \frac{\sin\theta}{\cos\theta}}$

$\dfrac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta}}$

$\dfrac{\cos\theta+1}{\sin\theta(\cos\theta+1)}$

$\dfrac{1}{\sin\theta}$

$\csc\theta = RHS$

12. $\dfrac{\sec x}{1-\sin x} = \dfrac{1+\sin x}{\cos^3 x}$ $\rightarrow$ $\dfrac{1+\sin x}{\cos x(\cos^2 x)}$

$\dfrac{\frac{1}{\cos x}}{1-\sin x}$

$\dfrac{1}{\cos x(1-\sin x)}$

$\dfrac{1}{\cos x(1-\sin x)}$

$\dfrac{1+\sin x}{\cos x(1-\sin^2 x)}$

$\dfrac{1+\sin x}{\cos x(1-\sin x)(1+\sin x)}$

$\dfrac{1}{\cos x(1-\sin x)}$

LHS = RHS

13. $\cos^2 x = \dfrac{1-2\sin^2 x}{1-\tan^2 x}$

$\cos^2 x$

$\dfrac{\cos^2 x - \sin^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$\dfrac{2\cos^2 x - (\sin^2 x + \cos^2 x)}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$\dfrac{2\cos^2 x - 1}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$\dfrac{1-2+2\cos^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$\dfrac{1-2(1-\cos^2 x)}{1-\frac{\sin^2 x}{\cos^2 x}}$

LHS = RHS

14. $\dfrac{\tan x}{\tan x + \sin x} = \dfrac{1-\cos x}{\sin^2 x}$

$\dfrac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + \sin x}$

$\dfrac{\frac{\sin x}{\cos x}}{\frac{\sin x + \sin x \cos x}{\cos x}}$

$\dfrac{\sin x}{\sin x + \sin x \cos x}$

$\dfrac{\cancel{\sin x}}{\cancel{\sin x}(1+\cos x)}$

$\dfrac{1}{1+\cos x}$

$\dfrac{1-\cos x}{1-\cos^2 x}$

$\dfrac{1-\cos x}{(1+\cos x)(1-\cos x)}$

$\dfrac{1}{1+\cos x}$

LHS = RHS

15. $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{1}{\csc\theta + \cot\theta}$

$\dfrac{1}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$

$\dfrac{1}{\frac{1+\cos\theta}{\sin\theta}}$

$\dfrac{\sin\theta}{1+\cos\theta} \cdot \dfrac{(1-\cos\theta)}{(1-\cos\theta)}$

$\dfrac{\sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$

$\dfrac{\cancel{\sin\theta}(1-\cos\theta)}{\sin^{\cancel{2}}\theta}$

$\dfrac{1-\cos\theta}{\sin\theta}$

LHS = RHS

16. $\dfrac{\sec x}{1-\cos x} = \dfrac{\sec x + 1}{\sin^2 x}$

$\dfrac{\frac{1}{\cos x} + (}{\sin^2 x}$

$\dfrac{\frac{1+\cos x}{\cos x}}{\sin^2 x}$

$\dfrac{1+\cos x}{\sin^2 x \cos x}$

$\dfrac{1+\cos x}{(1-\cos^2 x)\cos x}$

$\dfrac{\cancel{1+\cos x}}{(\cancel{1+\cos x})(1-\cos x)\cos x}$

$\dfrac{1}{(1-\cos x)\cos x}$

$\dfrac{1}{(1-\cos x)} \cdot \dfrac{1}{\cos x}$

$\dfrac{\sec x}{1-\cos x}$

LHS = RHS

17. $\dfrac{\sin^2 x - \tan x}{\cos^2 x - \cot x} = \tan^2 x$

$$\frac{\sin^2 x - \frac{\sin x}{\cos x}}{\cos^2 x - \frac{\cos x}{\sin x}}$$

$$\frac{\frac{\sin^2 x \cos x - \sin x}{\cos x}}{\frac{\cos^2 x \sin x - \cos x}{\sin x}}$$

$$\frac{\sin^2 x \cos x - \sin x}{\cos x} \cdot \frac{\sin x}{\cos^2 x \sin x - \cos x}$$

$$\frac{\sin^3 x \cos x - \sin^2 x}{\cos^3 x \sin x - \cos^2 x} \rightarrow \frac{\sin^2 x (\sin x \cos x - 1)}{\cos^2 x (\sin x \cos x - 1)} = \text{RHS}$$

18. $\cos^2 x - \sin^2 x = \dfrac{\cot x - \tan x}{\cot x + \tan x}$

$$\frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \rightarrow \frac{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}}$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \cdot \frac{\sin x \cos x}{\underbrace{\sin^2 x + \cos^2 x}_{1}}$$

$$\text{LHS} = \cos^2 x - \sin^2 x$$

19. $\csc x - \dfrac{\sin x}{1 + \cos x} = \cot x$

$$\frac{1}{\sin x} - \frac{\sin x}{1+\cos x}$$

$$\frac{1+\cos x}{\sin x (1+\cos x)} - \frac{\sin x \sin x}{\sin x (1+\cos x)}$$

$$\frac{1+\cos x - \sin^2 x}{\sin x (1+\cos x)}$$

$$\frac{1+\cos x - [1-\cos^2 x]}{\sin x (1+\cos x)}$$

$$\frac{\cos^2 x + \cos x}{\sin x (1+\cos x)}$$

$$\frac{\cos x (\cos x + 1)}{\sin x (\cos x + 1)}$$

$$\frac{\cos x}{\sin x} \rightarrow \cot x = \text{RHS}$$

20. $\cot x - \tan x = \dfrac{2\cos^2 x - 1}{\sin x \cos x}$

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$\frac{\cos^2 x - [1 - \cos^2 x]}{\sin x \cos x}$$

$$\frac{\cos^2 x - 1 + \cos^2 x}{\sin x \cos x}$$

$$\frac{2\cos^2 x - 1}{\sin x \cos x} = \text{RHS}$$

21. $\dfrac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$

$(\sec x - \tan x)(\sec x - \tan x)$

$\sec^2 x - 2\sec x \tan x + \tan^2 x$

$\dfrac{1}{\cos^2 x} - \dfrac{2\sin x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$

$\dfrac{\sin^2 x - 2\sin x + 1}{\cos^2 x}$

$\dfrac{(\sin x - 1)(\sin x - 1)}{1 - \sin^2 x}$

$\dfrac{(\sin x - 1)(\sin x - 1)}{(1-\sin x)(1+\sin x)}$

$\dfrac{(1-\sin x)(1-\sin x)}{(1-\sin x)(1+\sin x)}$

LHS

$\dfrac{1-\sin x}{1+\sin x}$

22. $\dfrac{\cos x}{\csc x + 1} + \dfrac{\cos x}{\csc x - 1} = 2\tan x$

$\dfrac{\cos x}{\frac{1}{\sin x} + \frac{\sin x}{\sin x}} + \dfrac{\cos x}{\frac{1}{\sin x} - \frac{\sin x}{\sin x}}$

$\dfrac{\cos x}{\frac{1+\sin x}{\sin x}} + \dfrac{\cos x}{\frac{1-\sin x}{\sin x}}$

$\dfrac{\sin x \cos x}{1+\sin x} + \dfrac{\sin x \cos x}{1-\sin x}$

$\dfrac{\sin x \cos x (1-\sin x) + \sin x \cos x (1+\sin x)}{1 - \sin^2 x}$

$\dfrac{\sin x \cos x - \sin^2 x \cos x + \sin x \cos x + \sin^2 x \cos x}{\cos^2 x}$

$\dfrac{2\sin x \cos x}{\cos^2 x} \rightarrow \dfrac{2\sin x}{\cos x} \rightarrow 2\tan x = \text{RHS}$

23. $\tan x(\csc x + 1) = \dfrac{\cot x}{\csc x - 1}$

$\tan x \csc x + \tan x$

$\dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} + \dfrac{\sin x}{\cos x}$

$\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$

$\dfrac{1+\sin x}{\cos x} \cdot \dfrac{1-\sin x}{1-\sin x}$

$\dfrac{1-\sin^2 x}{\cos x (1-\sin x)}$

$\dfrac{\cos^2 x}{\cos x (1-\sin x)}$

$\dfrac{\cos x}{1-\sin x}$

$\dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \frac{\sin x}{\sin x}}$

$\dfrac{\frac{\cos x}{\sin x}}{\frac{1-\sin x}{\sin x}}$

$\dfrac{\cos x}{1-\sin x}$

LHS = RHS

24. $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

$\dfrac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \sin x}$

$\dfrac{1+\cos x}{\sin x}$

$\dfrac{\sin x + \cos x \sin x}{\cos x}$

$\dfrac{1+\cos x}{\sin x} \cdot \dfrac{\cos x}{\sin x + \cos x \sin x}$

$\dfrac{\cos x + \cos^2 x}{\sin^2 x + \cos x \sin^2 x}$

$\dfrac{\cos x (1+\cos x)}{\sin x (\sin x + \cos x \sin x)}$

$\dfrac{\cos x}{\sin x} \cdot \dfrac{(1+\cos x)}{(1+\cos x)} \cdot \dfrac{1}{(\sin x)} \rightarrow \cot x \csc x = \text{RHS}$

25. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

$$\frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{\sin^2 x + \cos^2 x - \sin^2 y - \cos^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{1 - (\sin^2 y + \cos^2 y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{1 - 1}{(\sin x + \sin y)(\cos x + \cos y)}$$

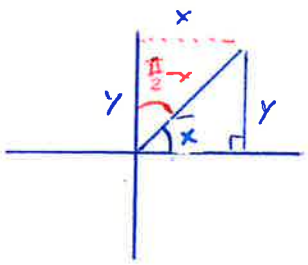
$\boxed{0}$ = RHS

26. $\csc^2\left(\frac{\pi}{2} - x\right) - 1 = \tan^2 x$

↓

$\sec^2 x - 1 =$

$\tan^2 x$ = RHS



$\csc\left(\frac{\pi}{2} - x\right) = \frac{r}{x}$

$\sec x = \frac{r}{x}$

$\csc \frac{\pi}{2} - x = \sec x$

See Website for Detailed Answer Key

Extra Work Space