

Section 7.2 – Confirming Trigonometric Identities

- **Confirming Identities** involving **proving that one side of the equation is equivalent to the other**
- When working through this process there **are many strategies**, but know that there is not only one specific way to do it
- You **cannot algebraically manipulate the equation**, only substitute identities for one another on the individual sides of the equation
- It is considered **PROVEN** when the **left side is identical to the right side**

Here are some helpful rules:

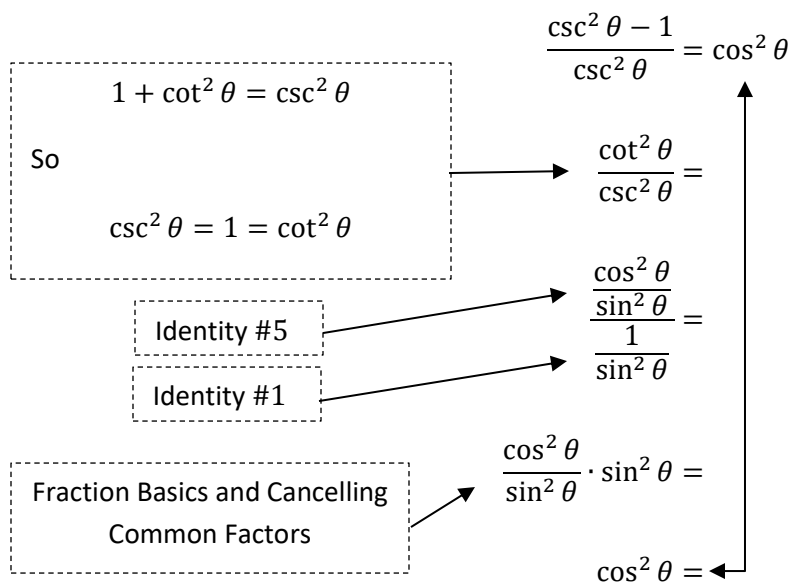
Rules to Simplify Your Life

1. Change all terms into values of Sine and Cosine if possible
2. Write an expression with a Common Denominator
3. Remember to factor and how to use conjugates
4. Work with one side of the equation at a time, start with the most complicated

Note Often you will only need to work with one side only

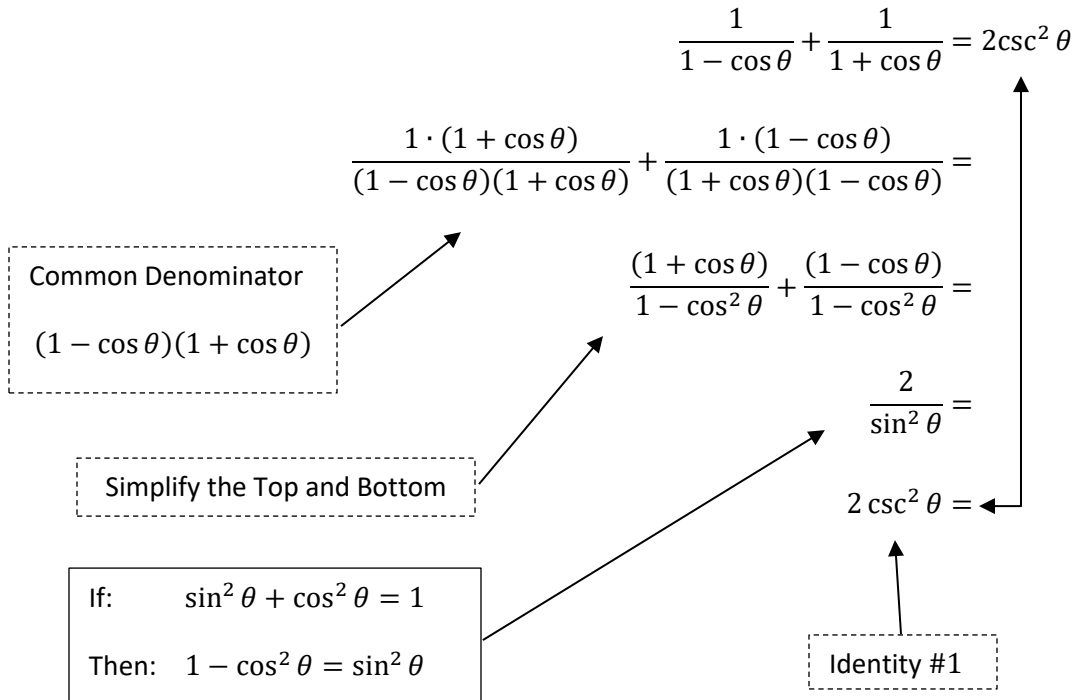
Example 1: Prove the Identity $\frac{\csc^2 \theta - 1}{\csc^2 \theta} = \cos^2 \theta$

Solution 1: Start with the more complicated side and see if you can get it to be the same as the right side



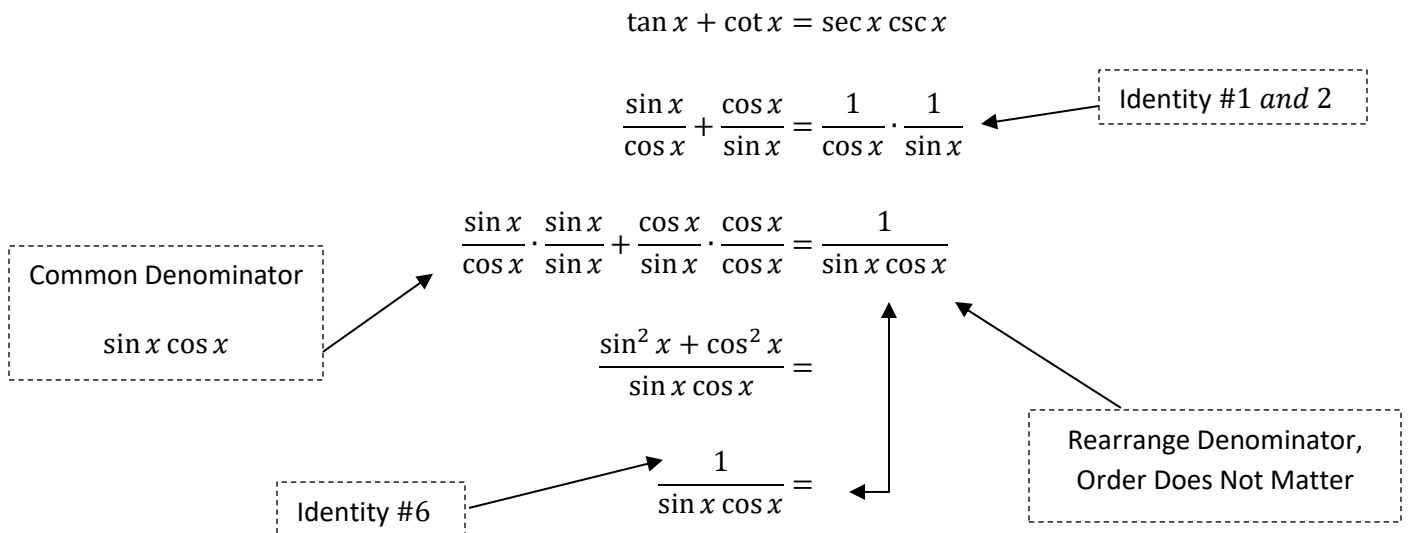
Example 2: Prove the Identity $\frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta} = 2\csc^2 \theta$

Solution 2: Start with the more complicated side and see if you can get it to be the same as the right side



Example 3: Prove the Identity $\tan x + \cot x = \sec x \csc x$

Solution 3: Start with the more complicated side and see if you can get it to be the same as the right side



Example 4: Prove the Identity $\csc x + \cot x = \frac{\sin x}{1 - \cos x}$

Solution 4: Start with the more complicated side and see if you can get it to be the same as the right side

Change to Sine and Cosine. Write as one Denominator

$$\csc x + \cot x = \frac{\sin x}{1 - \cos x}$$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$$

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$$

Use the Conjugate

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x (1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$$

Multiply Denominators

$$= \frac{\sin x (1 + \cos x)}{1 - \cos^2 x}$$

$\sin^2 x + \cos^2 x = 1$
Therefore: $\sin^2 x = 1 - \cos^2 x$

$$= \frac{\sin x (1 + \cos x)}{\sin^2 x}$$

Cancel $\sin x$

$$= \frac{1 + \cos x}{\sin x}$$

Example 5: Prove the Identity $\frac{\tan^2 x}{1 + \sec x} = \frac{1 - \cos x}{\cos x}$

Solution 5: Start with the more complicated side and see if you can get it to be the same as the right side

$\tan^2 x + 1 = \sec^2 x$
Therefore
 $\tan^2 x = \sec^2 x - 1$

Cancel $\sec x + 1$
 $\sec x + 1 = 1 + \sec x$

Identity #2

$$\frac{\tan^2 x}{1 + \sec x} = \frac{1 - \cos x}{\cos x}$$

$$\frac{\sec^2 x - 1}{1 + \sec x} =$$

Difference of Squares

$$\frac{(\sec x + 1)(\sec x - 1)}{1 + \sec x} =$$

Common Denominator

$$\sec x - 1 =$$

$$\frac{1}{\cos x} - 1 =$$

$$\frac{1 - \cos x}{\cos x} =$$

THERE CAN BE MORE THAN ONE WAY TO ACHIEVE THE RESULT. IT'S LIKE SOLVING A PUZZLE, HAVE FUN!!

Section 7.2 – Practice Problems

Prove the following identities.

Work on one side at a time, or only one side. You cannot algebraically manipulate the initial statement.

1. $\sin^2 x - \cos^2 x = 2 \sin^2 x - 1$

2. $\sin x + \cos x \cot x = \csc x$

3. $\frac{1}{\cos x} - \cos x = \frac{\sin^2 x}{\cos x}$

4. $\frac{1}{\sec x \tan x} = \csc x - \sin x$

$$5. \frac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \cos^4 x$$

$$6. \frac{\sec^4 x - 1}{\tan^2 x} = 2 + \tan^2 x$$

$$7. \frac{\sin x + \cos x}{\csc x + \sec x} = \sin x \cos x$$

$$8. \frac{\cos x + \sin x}{\cos x - \sin x} = \frac{1 + \tan x}{1 - \tan x}$$

$$9. \frac{\sec x}{1 - \cos x} = \frac{\sec x + 1}{\sin^2 x}$$

$$10. \frac{\sin \theta + \cos \theta \cot \theta}{\cos \theta \csc \theta} = \sec \theta$$

$$11. \frac{1 + \sec \theta}{\sin \theta + \tan \theta} = \csc \theta$$

$$12. \frac{\sec x}{1 - \sin x} = \frac{1 + \sin x}{\cos^3 x}$$

$$13. \cos^2 x = \frac{1 - 2\sin^2 x}{1 - \tan^2 x}$$

$$14. \frac{\tan x}{\tan x + \sin x} = \frac{1 - \cos x}{\sin^2 x}$$

$$15. \frac{1 - \cos \theta}{\sin \theta} = \frac{1}{\csc \theta + \cot \theta}$$

$$16. \frac{\sec x}{1 - \cos x} = \frac{\sec x + 1}{\sin^2 x}$$

$$17. \frac{\sin^2 x - \tan x}{\cos^2 x - \cot x} = \tan^2 x$$

$$18. \cos^2 x - \sin^2 x = \frac{\cot x - \tan x}{\cot x + \tan x}$$

$$19. \csc x - \frac{\sin x}{1 + \cos x} = \cot x$$

$$20. \cot x - \tan x = \frac{2 \cos^2 x - 1}{\sin x \cos x}$$

$$21. \frac{1 - \sin x}{1 + \sin x} = (\sec x - \tan x)^2$$

$$22. \frac{\cos x}{\csc x + 1} + \frac{\cos x}{\csc x - 1} = 2 \tan x$$

$$23. \tan x(\csc x + 1) = \frac{\cot x}{\csc x - 1}$$

$$24. \frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$$

$$25. \frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$$

$$26. \csc^2\left(\frac{\pi}{2} - x\right) - 1 = \tan^2 x$$

See Website for Detailed Answer Key

Extra Work Space