Section 7.2 – Confirming Trigonometric Identities

- **Confirming Identities** involving **proving that one side** of the equation **is equivalent to the other**
- When working through this process there **are many strategies**, but know that there is not only one specific way to do it
- You **cannot algebraically manipulate the equation**, only substitute identities for one another on the individual sides of the equation
- It is considered **PROVEN** when the **left side** is **identical** to the **right side**

Here are some helpful rules:

Rules to Simplify Your Life

1. Change all terms into values of Sine and Cosine if possible
2. Write an expression with a Common Denominator
3. Remember to factor and how to use conjugates
4. Work with one side of the equation at a time, start with the most complicated

Note Often you will only need to work with one side only

Example 1: Prove the Identity $\dfrac{\csc^2\theta - 1}{\csc^2\theta} = \cos^2\theta$

Solution 1: Start with the more complicated side and see if you can get it to be the same as the right side

$$\frac{\csc^2\theta - 1}{\csc^2\theta} = \cos^2\theta$$

$1 + \cot^2\theta = \csc^2\theta$

So

$\csc^2\theta = 1 = \cot^2\theta$

$$\frac{\cot^2\theta}{\csc^2\theta} =$$

Identity #5

Identity #1

$$\frac{\dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin^2\theta}} =$$

Fraction Basics and Cancelling Common Factors

$$\frac{\cos^2\theta}{\sin^2\theta}\cdot\sin^2\theta =$$

$$\cos^2\theta =$$

Example 2: Prove the Identity $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}=2\csc^2\theta$

Solution 2: Start with the more complicated side and see if you can get it to be the same as the right side

$$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=2\csc^2\theta$$

$$\frac{1\cdot(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}+\frac{1\cdot(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}=$$

Common Denominator

$(1-\cos\theta)(1+\cos\theta)$

$$\frac{(1+\cos\theta)}{1-\cos^2\theta}+\frac{(1-\cos\theta)}{1-\cos^2\theta}=$$

Simplify the Top and Bottom

$$\frac{2}{\sin^2\theta}=$$

If: $\sin^2\theta+\cos^2\theta=1$

Then: $1-\cos^2\theta=\sin^2\theta$

$$2\csc^2\theta=$$

Identity #1

Example 3: Prove the Identity $\tan x+\cot x=\sec x\csc x$

Solution 3: Start with the more complicated side and see if you can get it to be the same as the right side

$$\tan x+\cot x=\sec x\csc x$$

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\cos x}\cdot\frac{1}{\sin x}$$

Identity #1 *and* 2

$$\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}=\frac{1}{\sin x\cos x}$$

Common Denominator

$\sin x\cos x$

$$\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=$$

Rearrange Denominator, Order Does Not Matter

$$\frac{1}{\sin x\cos x}=$$

Identity #6

Example 4: Prove the Identity $\csc x + \cot x = \dfrac{\sin x}{1 - \cos x}$

Solution 4: Start with the more complicated side and see if you can get it to be the same as the right side

$$\csc x + \cot x = \frac{\sin x}{1 - \cos x}$$

Change to Sine and Cosine. Write as one Denominator

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$$

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$$

Use the Conjugate

$$\frac{1 + \cos x}{\sin x} = \frac{\sin x\,(1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$$

Multiply Denominators

$$= \frac{\sin x\,(1 + \cos x)}{1 - \cos^2 x}$$

$\sin^2 x + \cos^2 x = 1$

Therefore: $\sin^2 x = 1 - \cos^2 x$

$$= \frac{\sin x\,(1 + \cos x)}{\sin^2 x}$$

Cancel $\sin x$

$$= \frac{1 + \cos x}{\sin x}$$

Example 5: Prove the Identity $\dfrac{\tan^2 x}{1 + \sec x} = \dfrac{1 - \cos x}{\cos x}$

Solution 5: Start with the more complicated side and see if you can get it to be the same as the right side

$$\frac{\tan^2 x}{1 + \sec x} = \frac{1 - \cos x}{\cos x}$$

$\tan^2 x + 1 = \sec^2 x$

Therefore

$\tan^2 x = \sec^2 x - 1$

$$\frac{\sec^2 x - 1}{1 + \sec x} =$$

Difference of Squares

$$\frac{(\sec x + 1)(\sec x - 1)}{1 + \sec x} =$$

Cancel $\sec x + 1$

$\sec x + 1 = 1 + \sec x$

$$\sec x - 1 =$$

Identity #2

$$\frac{1}{\cos x} - 1 =$$

Common Denominator

$$\frac{1 - \cos x}{\cos x} =$$

THERE CAN BE MORE THAN ONE WAY TO ACHIEVE THE RESULT. IT'S LIKE SOLVING A PUZZLE, HAVE FUN!!

Section 7.2 – Practice Problems

Prove the following identities.

Work on one side at a time, or only one side. You cannot algebraically manipulate the initial statement.

1. $\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

2. $\sin x + \cos x \cot x = \csc x$

3. $\dfrac{1}{\cos x} - \cos x = \dfrac{\sin^2 x}{\cos x}$

4. $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$

5. $\dfrac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \cos^4 x$

6. $\dfrac{\sec^4 x - 1}{\tan^2 x} = 2 + \tan^2 x$

7. $\dfrac{\sin x + \cos x}{\csc x + \sec x} = \sin x \cos x$

8. $\dfrac{\cos x + \sin x}{\cos x - \sin x} = \dfrac{1 + \tan x}{1 - \tan x}$

9. $\dfrac{\sec x}{1-\cos x}=\dfrac{\sec x+1}{\sin^2 x}$

10. $\dfrac{\sin\theta+\cos\theta\cot\theta}{\cos\theta\csc\theta}=\sec\theta$

11. $\dfrac{1+\sec\theta}{\sin\theta+\tan\theta}=\csc\theta$

12. $\dfrac{\sec x}{1-\sin x}=\dfrac{1+\sin x}{\cos^3 x}$

13. $\cos^2 x = \dfrac{1 - 2\sin^2 x}{1 - \tan^2 x}$

14. $\dfrac{\tan x}{\tan x + \sin x} = \dfrac{1 - \cos x}{\sin^2 x}$

15. $\dfrac{1 - \cos\theta}{\sin\theta} = \dfrac{1}{\csc\theta + \cot\theta}$

16. $\dfrac{\sec x}{1 - \cos x} = \dfrac{\sec x + 1}{\sin^2 x}$

17. $\dfrac{\sin^2 x - \tan x}{\cos^2 x - \cot x} = \tan^2 x$

18. $\cos^2 x - \sin^2 x = \dfrac{\cot x - \tan x}{\cot x + \tan x}$

19. $\csc x - \dfrac{\sin x}{1 + \cos x} = \cot x$

20. $\cot x - \tan x = \dfrac{2\cos^2 x - 1}{\sin x \cos x}$

21. $\dfrac{1-\sin x}{1+\sin x}=(\sec x-\tan x)^2$

22. $\dfrac{\cos x}{\csc x+1}+\dfrac{\cos x}{\csc x-1}=2\tan x$

23. $\tan x(\csc x+1)=\dfrac{\cot x}{\csc x-1}$

24. $\dfrac{\csc x+\cot x}{\tan x+\sin x}=\cot x\csc x$

25. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

26. $\csc^2\left(\frac{\pi}{2} - x\right) - 1 = \tan^2 x$

See Website for Detailed Answer Key

Extra Work Space