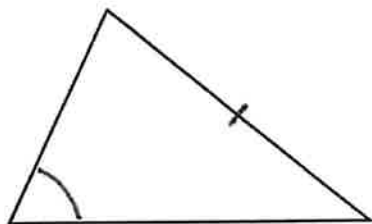


Section 7.2 – The Law of Sines

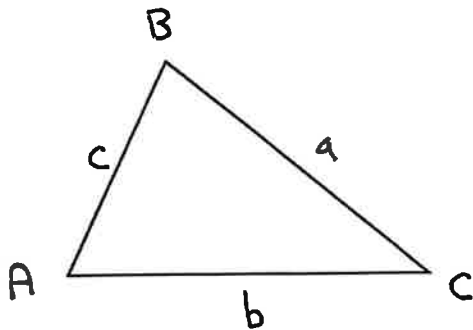
- The Law of Sines allows you to solve oblique triangles (triangles that are not right angle triangles)
- Use the Law of Sines when you are given an angle and the length of its opposite side.



The Law of Sines

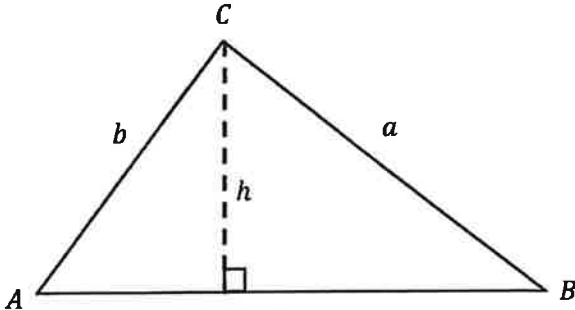
If $\triangle ABC$ is a triangle with sides $a, b,$ and $c,$ then:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

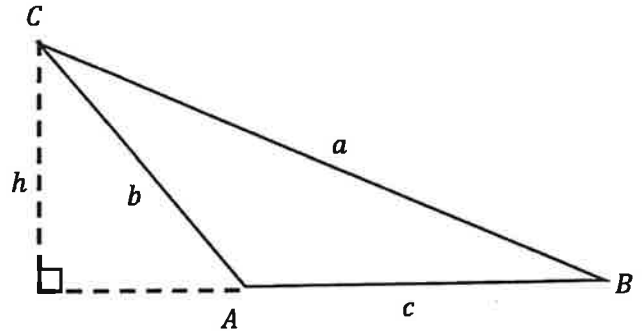


Derivation:

- ΔABC can be an acute or obtuse oblique triangle



➤ Let h be the altitude of either triangle



So,

$$\sin A = \frac{h}{b} \rightarrow b \sin A = h \quad \text{and} \quad \sin B = \frac{h}{a} \rightarrow a \sin B = h$$

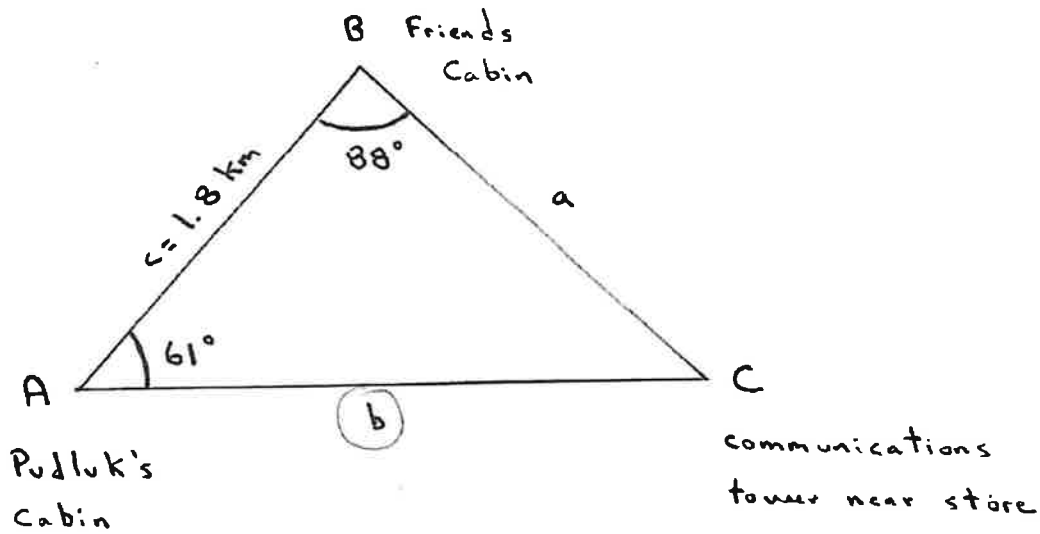
Set them equal to one another we get: $b \sin A = a \sin B$ or $\frac{\sin A}{a} = \frac{\sin B}{b}$

Similarly, if you move the vertex from vertex B to side AC, you end up with,

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Example 1: Determine an Unknown Side Length

Pudluk's family and his friend own cabins on the Kalit River in Nunavut. Pudluk and his friend wish to determine the distance from Pudluk's cabin to the store on the edge of town. They know that the distance between their cabins is 1.8 km. Using a transit, they estimate the measures of the angles between their cabins and the communications tower near the store, as shown in the diagram. Determine the distance from Pudluk's cabin to the store, to the nearest tenth of a kilometer.



$$\angle C = 180^\circ - 88^\circ - 61^\circ \quad \angle C = 31^\circ$$

$$\frac{1.8}{\sin 31^\circ} = \frac{b}{\sin 88^\circ}$$

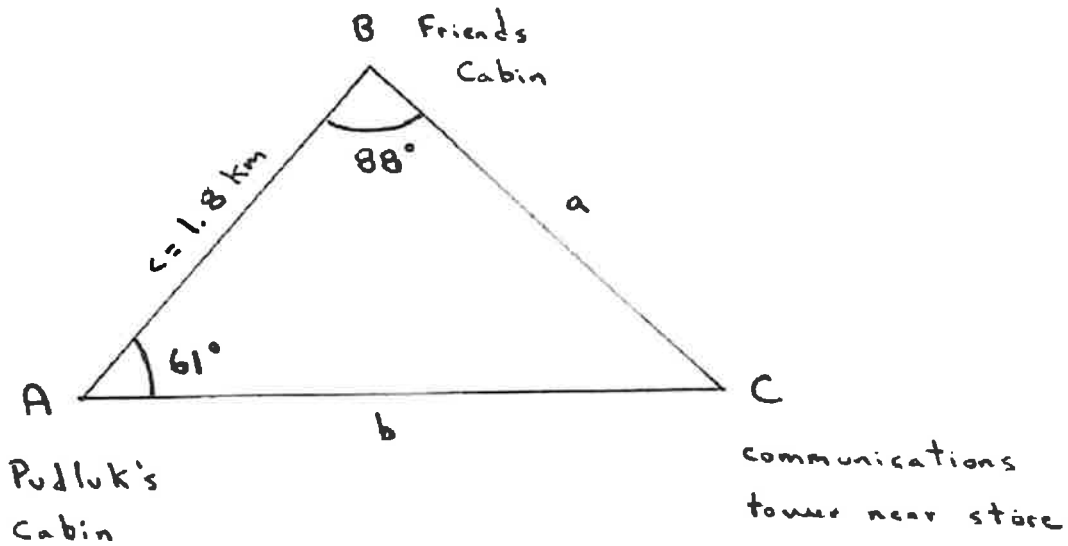
$$b = \frac{1.8 \sin 88^\circ}{\sin 31^\circ}$$

$$b = 3.492$$

Distance to store is approx 3.5 km

Your Turn

Determine the distance from Pudluk's friend's cabin to the store.



$$\frac{1.8}{\sin 31^\circ} = \frac{a}{\sin 61^\circ}$$

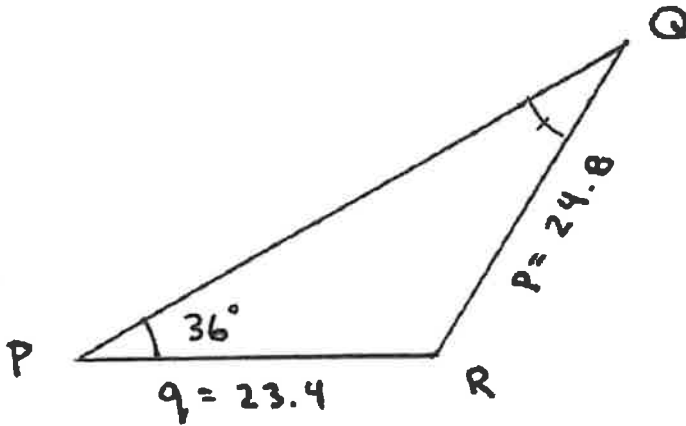
$$a = \frac{1.8 \sin 61^\circ}{\sin 31^\circ}$$

$$a = 3.0566\dots$$

$$a = 3.1 \text{ km}$$

Example 2: Determine an Unknown Angle Measure

In $\triangle PQR$, $P = 36^\circ$, $p = 24.8$ m, and $q = 23.4$ m. Determine the measure of angle R , to the nearest degree.



$$\frac{\sin 36^\circ}{24.8} = \frac{\sin Q}{23.4}$$

$$\sin Q = \frac{23.4 \sin 36^\circ}{24.8}$$

$$\sin Q = 0.5546$$

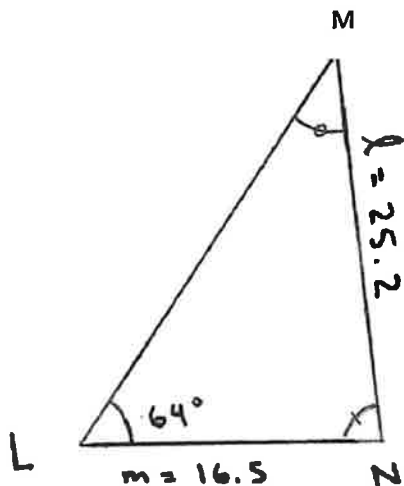
$$\begin{aligned} \angle Q &= \sin^{-1}(0.5546) \\ &= 33.7^\circ \end{aligned}$$

$$\begin{aligned} \angle R &= 180^\circ - 36^\circ - 33.7^\circ \\ &= 110.3^\circ \end{aligned}$$

$$\angle R = 110^\circ$$

Your Turn

In $\triangle LMN$, angle $L = 64^\circ$, $l = 25.2$ cm, and $m = 16.5$ cm. Determine the measure of angle N , to the nearest degree.



$$\frac{\sin 64^\circ}{25.2} = \frac{\sin M}{16.5}$$

$$\sin M = \frac{16.5 \sin 64^\circ}{25.2}$$

$$\sin M = 0.5885$$

$$\angle M = \sin^{-1}(0.5885)$$

$$\angle M = 36.1$$

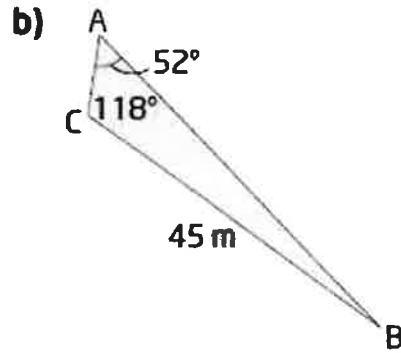
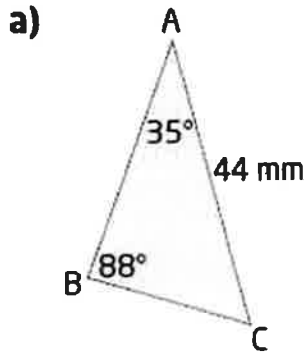
$$\angle N = 180^\circ - 64^\circ - 36.1^\circ$$

$$= 79.9^\circ$$

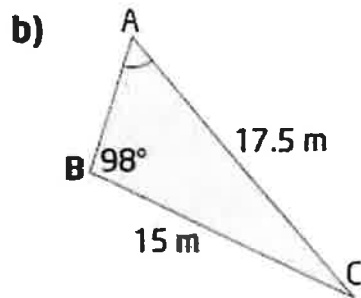
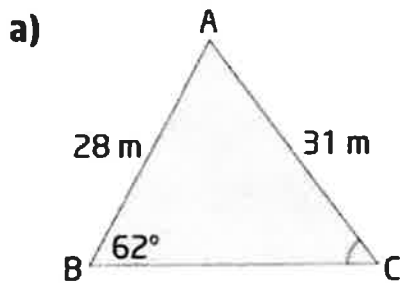
$$= 80.^\circ$$

Section 7.2 – Practice Questions

1. Determine the length of AB in each

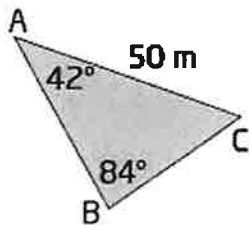


2. Determine the value of the marked unknown angle in each.

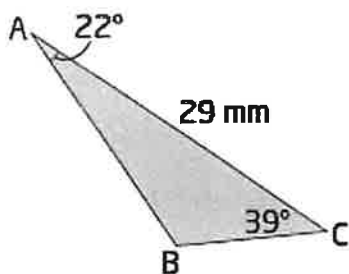


3. Determining the length of all three sides and the measures of all three angles is called solving a triangle. Solve each triangle.

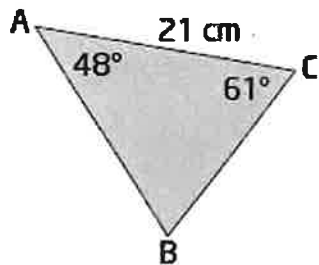
a)



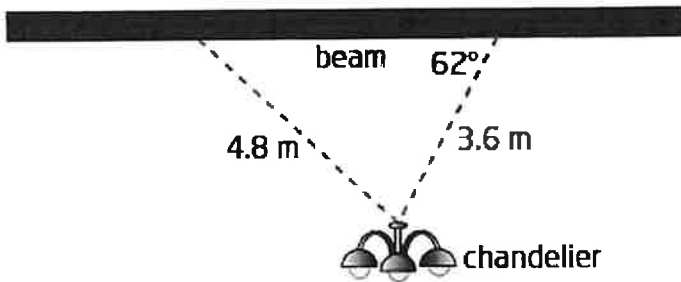
b)



c)



4. A chandelier is suspended from a horizontal beam by two support chains. What angle does the 4.8 m long chain make with the beam?



5. The chemical formula for water, H_2O , tells you that one molecule of water is made up of two atoms of hydrogen and one atom of oxygen bonded together. The nuclei of the atoms are separated by the distance shown, in angstroms. An angstrom is a unit of length used in chemistry. Determine the distance in angstrom between the two hydrogen atoms.

