Section 7.1 – Practice Problems

1. Match the given statement with a possible identity.

a) $\cot x$	F
b) $\tan x$	C
c) $\sec x$	B
d) $\csc x$	A
e) $\tan^2 x$	E
f) $1 + \tan^2 x$	D
g) $\sin^2 x$	H

A. $\frac{1}{\sin x}$	B. $\frac{1}{\cos x}$
C. $\frac{\sin x}{\cos x}$	D. $\frac{1}{\cos^2 x}$
E. $\frac{1}{\cot^2 x}$	F. $\frac{\cos x}{\sin x}$
G. $\frac{1}{\sin^2 x}$	H. $1 - \cos^2 x$

2. Write with a Common Denominator, then simplify if possible.

a) $\frac{3}{2\sin x} - \frac{4}{\sin^2 x}$

$\frac{3\sin x}{2\sin^2 x} - \frac{4(2)}{2\sin^2 x}$

$\frac{3\sin x - 8}{2\sin^2 x}$

b) $\frac{1}{1-\sin x} + \frac{1}{\sin x}$

$\frac{\sin x}{\sin x(1-\sin x)} + \frac{1(1-\sin x)}{\sin x(1-\sin x)}$

$\frac{\sin x + 1 - \sin x}{\sin x(1-\sin x)}$

$\frac{1}{\sin x(1-\sin x)}$

c) $\dfrac{1+\frac{1}{\tan x}}{\frac{1}{\tan^2 x}}$

$\dfrac{\frac{\tan x}{\tan x}+\frac{1}{\tan x}}{\frac{1}{\tan^2 x}} \rightarrow \dfrac{\frac{1+\tan x}{\tan x}}{\frac{1}{\tan^2 x}}$

$\dfrac{1+\tan x}{\tan x} \cdot \dfrac{\tan^2 x}{1}$

$(1+\tan x)\tan x$

d) $\dfrac{1}{\sin^2 x}-1$

$\dfrac{1}{\sin^2 x}-\dfrac{\sin^2 x}{\sin^2 x}$

$\dfrac{1-\sin^2 x}{\sin^2 x} = \dfrac{\cos^2 x}{\sin^2 x}$

$\cot^2 x$

e) $\sin x+\dfrac{\cos^2 x}{\sin x}$

$\dfrac{\sin^2 x}{\sin x}+\dfrac{\cos^2 x}{\sin x}$

$\dfrac{\sin^2 x+\cos^2 x}{\sin x}$

$\dfrac{1}{\sin x}$

$\csc x$

f) $\dfrac{1}{1+\cos x}+\dfrac{1}{1-\cos x}$

$\dfrac{1(1-\cos x)}{(1+\cos x)(1-\cos x)}+\dfrac{1(1+\cos x)}{(1-\cos x)(1+\cos x)}$

$\dfrac{1-\cos x+1+\cos x}{1-\cos^2 x}$

$\dfrac{2}{\sin^2 x} = 2\csc^2 x$

g) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}$

$\dfrac{(\cos x)(\cos x)}{(1+\sin x)\cos x}+\dfrac{(1+\sin x)(1+\sin x)}{\cos x(1+\sin x)}$

$\dfrac{\cos^2 x+1+2\sin x+\sin^2 x}{\cos x(1+\sin x)}$

$\dfrac{2\sin x+1+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$

$\dfrac{2\sin x+2}{\cos x(1+\sin x)} \rightarrow \dfrac{2(\sin x+1)}{\cos x(\sin x+1)}$

$\dfrac{2}{\cos x} \rightarrow \boxed{2\sec x}$

h) $\tan x-\dfrac{\sec^2 x}{\tan x}$

$\dfrac{\tan^2 x}{\tan x}-\dfrac{\sec^2 x}{\tan x}$

$\dfrac{\tan^2 x-\sec^2 x}{\tan x} \rightarrow \dfrac{\sec^2 x-1-\sec^2 x}{\tan x}$

$\dfrac{-1}{\tan x} = \boxed{-\cot x}$

3. Factor, and then if possible, simply the expressions

a) $1-\sin^2 x$ Diff of Squares

$\boxed{(1-\sin x)(1+\sin x)}$

b) $\sec^2 x-\tan^2 x$ Diff of squares

$\boxed{(\sec x+\tan x)(\sec x-\tan x)}$

c) $\tan^2 x-\tan^2 x\sin^2 x$

$\tan^2 x(1-\sin^2 x)$

$\tan^2 x(\cos^2 x)$

$\dfrac{\sin^2 x(\cos^2 x)}{\cos^2 x} \rightarrow \boxed{\sin^2 x}$

d) $\sec^2 x+\sec^2 x\tan^2 x$

$\sec^2 x(1+\tan^2 x)$

$\sec^2 x(\sec^2 x)$

$\boxed{\sec^4 x}$

e) $\sin^2 x \sec^2 x - \sin^2 x$

$\sin^2 x(\sec^2 x - 1)$

$\sin^2 x(\tan^2 x)$

$\sin^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right)$ noting cancels

f) $\frac{\csc^2 x - 1}{\csc x - 1}$ diff of squares

$\frac{(\csc x - 1)(\csc x + 1)}{(\csc x - 1)}$

$\csc x + 1$

g) $\cot^4 x + 2\cot^2 x + 1$

$(\cot^2 x + 1)(\cot^2 x + 1)$

$(\csc^2 x)(\csc^2 x)$

$\csc^4 x$

h) $1 - 2\sin^2 x + \sin^4 x$

$\sin^4 x - 2\sin^2 x + 1$

$(\sin^2 x - 1)(\sin^2 x - 1)$

$(-\cos^2 x)(-\cos^2 x)$

$\cos^4 x$

i) $\sin^4 x - \cos^4 x$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

1

$\sin^2 x - \cos^2 x$

$(\sin x - \cos x)(\sin x + \cos x)$

j) $\sec^3 x - \sec^2 x - \sec x + 1$

Factor by grouping

$(\sec^3 x - \sec^2 x)(-\sec x + 1)$

$\sec^2 x(\sec x - 1) - 1(\sec x - 1)$

$(\sec^2 x - 1)(\sec x - 1)$

$\tan^2 x(\sec x - 1)$

4. Multiply, then simply using identities.

a) $(\sin x + \cos x)^2$

$(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + 2\sin x \cos x + \cos^2 y$

$\sin^2 x + \cos^2 x + 2\sin x \cos x$

$\boxed{1 + 2\sin x \cos x}$

b) $\sin x\,(\csc x - \sin x)$

$\sin x \csc x - \sin^2 x$

$\sin x \cdot \frac{1}{\sin x} - \sin^2 x$

$1 - \sin^2 x$

$\boxed{\cos^2 x}$

c) $(\csc x - 1)(\csc x + 1)$

$\csc^2 x - 1$

$\boxed{\cot^2 x}$

d) $(2 - 2\cos x)(2 + 2\cos x)$

$4 - 4\cos^2 x$

$4(1 - \cos^2 x)$

$4(\sin^2 x)$

$\boxed{4\sin^2 x}$

e) $(\csc x - \cot x)\,(\csc x + \cot x)$

$\csc^2 x - \cot^2 x$

$\cot^2 x + 1 - \cot^2 x$

$\boxed{1}$

f) $(\tan x + \sec x)\,(\tan x - \sec x)$

$\tan^2 x - \sec^2 x$

$\sec^2 x - 1 - \sec^2 x$

$\boxed{-1}$

5. Rewrite the given expressions in terms of $\sin x$ only.

a) $\sin^2 x - \cos^2 x$

$\sin^2 x - (1 - \sin^2 x)$

$\sin^2 x - 1 + \sin^2 x$

$2\sin^2 x - 1$

b) $\sec^2 x$

$\frac{1}{\cos^2 x} \rightarrow \frac{1}{1 - \sin^2 x}$

c) $\dfrac{\tan x + \sec x}{\cos x}$

$\dfrac{\frac{\sin x}{\cos x} + \frac{1}{\cos x}}{\cos x}$

$\frac{\sin x + 1}{\cos x} \cdot \frac{1}{\cos x} \rightarrow \frac{\sin x + 1}{\cos^2 x}$

$\frac{\sin x + 1}{1 - \sin^2 x} \rightarrow \frac{(1 + \sin x)}{(1 + \sin x)(1 - \sin x)} \rightarrow \boxed{\frac{1}{1 - \sin x}}$

d) $\dfrac{\sin x + \tan x}{1 + \sec x}$

$\dfrac{\sin x + \frac{\sin x}{\cos x}}{1 + \frac{1}{\cos x}} \rightarrow \dfrac{\frac{\sin x \cos x + \sin x}{\cos x}}{\frac{\cos x + 1}{\cos x}}$

$\frac{\sin x \cos x + \sin x}{\cos x} \cdot \frac{\cos x}{\cos x + 1}$

$\frac{\sin x (\cos x + 1)}{(\cos x + 1)}$

$\sin x$

6. Rewrite the given expression in terms of $\cos x$ only

a) $\sin^2 x - \cos^2 x$

$1 - \cos^2 x - \cos^2 x$

$1 - 2\cos^2 x$

b) $(\sec x + 1)(\sec x - 1)$

$\sec^2 x - 1$

$\frac{1}{\cos^2 x} - 1$

$\frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \rightarrow \frac{1 - \cos^2 x}{\cos^2 x}$

c) $\sin x(\csc x - \sin x)$

$\sin x \csc x - \sin^2 x$

$\sin x \cdot \frac{1}{\sin x} - \sin^2 x$

$1 - \sin^2 x$

$\boxed{\cos^2 x}$

d) $\dfrac{\cot x + \csc x}{\sin x}$

$\dfrac{\frac{\cos x}{\sin x} + \frac{1}{\sin x}}{\sin x} \rightarrow \dfrac{\frac{\cos x + 1}{\sin x}}{\sin x}$

$\dfrac{\cos x + 1}{\sin x} \cdot \dfrac{1}{\sin x} \rightarrow \dfrac{\cos x + 1}{\sin^2 x}$

$\dfrac{\cos x + 1}{1 - \cos^2 x} \rightarrow \dfrac{\cos x + 1}{(1 - \cos x)(1 + \cos x)}$

$\boxed{\dfrac{1}{1 - \cos x}}$

7. Rewrite in terms of Sine and Cosine only.

a) $\csc x + \cot x$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$\boxed{\dfrac{1 + \cos x}{\sin x}}$

b) $\sec x + \tan x$

$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$\boxed{\dfrac{1 + \sin x}{\cos x}}$

c) $\dfrac{1}{\tan x + \cot x}$

$\dfrac{1}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} \rightarrow \dfrac{1}{\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}}$

$\dfrac{1}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} \rightarrow 1 \cdot \dfrac{\sin x \cos x}{\sin^2 x + \cos^2 x}$

$\dfrac{1 \cdot \sin x \cos x}{1} \leftarrow$ $\boxed{\sin x \cos x}$

d) $\sec x - \dfrac{\cos x}{1 + \sin x}$

$\dfrac{1}{\cos x} - \dfrac{\cos x}{1 + \sin x} \rightarrow \dfrac{1 + \sin x - \cos^2 x}{\cos x (1 + \sin x)}$

$\dfrac{(1 + \sin x)}{\cos x (1 + \sin x)} - \dfrac{\cos x (\cos x)}{(1 + \sin x)(\cos x)}$

$\dfrac{1 + \sin x - (1 - \sin^2 x)}{\cos x (1 + \sin x)} \rightarrow \dfrac{1 + \sin x - 1 + \sin^2 x}{\cos x (1 + \sin x)}$

$\dfrac{\sin^2 x + \sin x}{\cos x (1 + \sin x)} \rightarrow \dfrac{\sin x (\sin x + 1)}{\cos x (1 + \sin x)} = \boxed{\dfrac{\sin x}{\cos x}}$

8. Determine all restrictions, $0 \le x < 2\pi$.

a) $\dfrac{\cot x}{1+\sin x}$

$\sin x \neq -1$

this occurs at $x = \dfrac{3\pi}{2}$

also $\cot x = \dfrac{\cos x}{\sin x}$

$\sin x \neq 0$

occurs at $0, \pi$

so $x \neq 0, \pi, \dfrac{3\pi}{2}$

b) $\dfrac{\sec x}{1-\cos x}$ $\quad \dfrac{\frac{1}{\cos x}}{1-\cos x}$

$\cos x \neq 0$ or 1

occurs at $0, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$x \neq 0, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

c) $\dfrac{1}{2\cos^2 x + \cos x - 1}$

$\dfrac{1}{(2\cos x - 1)(\cos x + 1)}$

$\cos x \neq \dfrac{1}{2}$ ← reference angle $60°$, $\dfrac{\pi}{3}$, Q1, Q4

$\cos x \neq -1$ → π

$x \neq \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \pi$

d) $\cot x + \tan x$

$\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$

$\sin x \neq 0$

$\cos x \neq 0$

$x \neq 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$

9. Simplify the following trigonometric expressions.

a) $(\sec x \cdot \csc x - \cot x)(\sin x - \csc x)$

$\left(\dfrac{1}{\cos x}\cdot\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right)\left(\sin x - \dfrac{1}{\sin x}\right)$

$\left(\dfrac{1}{\cos x \sin x} - \dfrac{\cos^2 x}{\cos x \sin x}\right)\left(\dfrac{\sin^2 x}{\sin x} - \dfrac{1}{\sin x}\right)$

$\left(\dfrac{1-\cos^2 x}{\cos x \sin x}\right)\left(\dfrac{\sin^2 x - 1}{\sin x}\right)$

$\left(\dfrac{\sin^2 x}{\cos x \sin x}\right)\left(\dfrac{-\cos^2 x}{\sin x}\right) = -\cos x$

b) $\dfrac{\frac{\cot x + 1}{\cot x} - 1}{\frac{\cot x - 1}{\cot x} - 1}$ → $\dfrac{\frac{\cot x + 1}{\cot x} - \frac{\cot x}{\cot x}}{\frac{\cot x - 1}{\cot x} - \frac{\cot x}{\cot x}}$

$\dfrac{\cot x + 1 - \cot x}{\cot x} \div \dfrac{\cot x - 1 - \cot x}{\cot x}$

$\dfrac{1}{\cot x} \div \dfrac{-1}{\cot x} \rightarrow \dfrac{1}{\cot x} \cdot \dfrac{\cot x}{-1}$

-1

c) $\dfrac{\tan^2 x}{\cos^2 x + \sin^2 x + \tan^2 x}$

$\cos^2 x + \sin^2 x = 1$

$\dfrac{\tan^2 x}{1 + \tan^2 x} \rightarrow \dfrac{\tan^2 x}{\sec^2 x}$

$\dfrac{\sin^2 x}{\cos^2 x} \div \dfrac{1}{\cos^2 x}$

$\dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{\cos^2 x}{1}$

$\boxed{\sin^2 x}$

d) $\dfrac{\cos x \cdot \tan x + \sin x}{2 \tan x}$

$\dfrac{\cos x \cdot \frac{\sin x}{\cos x} + \sin x}{2 \tan x}$

$\sin x + \sin x \div 2 \tan x$

$2 \sin x \div \dfrac{2 \sin x}{\cos x}$

$\cancel{2 \sin x} \cdot \dfrac{\cos x}{\cancel{2 \sin x}}$

$\boxed{\cos x}$

e) $\dfrac{1 - \sec^2 x}{\sec^2 x} - \cos^2 x$

$\dfrac{-\tan^2 x}{\sec^2 x} - \cos^2 x$

$\dfrac{-\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} - \cos^2 x \rightarrow \dfrac{-\sin^2 x}{\cos^2 x} \cdot \dfrac{\cos^2 x}{1} - \cos^2 x$

$-\sin^2 x - \cos^2 x$

$-1(\sin^2 x + \cos^2 x)$

$-1(1) = \boxed{-1}$

f) $\dfrac{\sec x - \cos x}{\csc x - \sin x}$

$\dfrac{\frac{1}{\cos x} - \cos x}{\frac{1}{\sin x} - \sin x} \rightarrow \dfrac{\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}}{\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}}$

$\dfrac{1 - \cos^2 x}{\cos x} \div \dfrac{1 - \sin^2 x}{\sin x}$

$\dfrac{\sin^2 x}{\cos x} \div \dfrac{\cos^2 x}{\sin x}$

$\dfrac{\sin^2}{\cos x} \cdot \dfrac{\sin x}{\cos^2 x} \rightarrow \dfrac{\sin^3 x}{\cos^3 x} \rightarrow \boxed{\tan^3 x}$

g) $\dfrac{\cot x(\sin x + \tan x)}{\csc x + \cot x}$

$\dfrac{\frac{\cos x}{\sin x}\left(\sin x + \frac{\sin x}{\cos x}\right)}{\frac{1}{\sin x} + \frac{\cos x}{\sin x}} \rightarrow \dfrac{\cos x + 1}{\frac{1 + \cos x}{\sin x}}$

$\dfrac{(\cos x + 1)\cdot \sin x}{(1 + \cos x)}$

$\boxed{\sin x}$

h) $\dfrac{\sec x - \cos x}{\tan x}$

$\dfrac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} \rightarrow \dfrac{1 - \cos^2 x}{\cos x} \div \dfrac{\sin x}{\cos x}$

$\dfrac{1 - \cos^2 x}{\cos x} \cdot \dfrac{\cos x}{\sin x}$

$\dfrac{\sin^2 x}{\sin x} = \boxed{\sin x}$

i) $\dfrac{\sec^2 x(1 + \csc x) - \tan x(\sec x + \tan x)}{\csc x(1 + \sin x)}$

$\dfrac{\sec^2 x + \sec^2 x \csc x - \tan x \sec x - \tan^2 x}{\csc x + \csc x \sin x}$

$\dfrac{\tan^2 x + 1 + \sec^2 x \csc x - \tan x \sec x - \tan^2 x}{\csc x + \csc x \sin x}$

$\dfrac{1 + \sec^2 x \csc x - \tan x \sec x}{\frac{1}{\sin x} + \frac{1}{\sin x}\cdot \sin x}$

$\dfrac{1 + \frac{1}{\cos^2 x}\cdot\frac{1}{\sin x} - \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}}{\frac{1 + \sin x}{\sin x}}$

$\dfrac{1 + \frac{1}{\sin x \cos^2 x} - \frac{\sin x}{\cos^2 x}}{\frac{1 + \sin x}{\sin x}} \rightarrow \dfrac{\sin x \cos^2 x + 1 - \sin^2 x}{\sin x \cos^2 x}\cdot\dfrac{\sin x}{1 + \sin x} \rightarrow \dfrac{\sin x \cos^2 x + 1 - (1 - \cos^2 x)}{\cos^2 x(1 + \sin x)} \rightarrow \dfrac{\cos^2 x + \sin x \cos^2 x}{\cos^2 x(1 + \sin x)}$

$\dfrac{\cos^2 x(1 + \sin x)}{\cos^2 x(1 + \sin x)} = 1$

j) $\dfrac{\csc^2 x + \sec^2 x}{\csc x \sec x} \rightarrow \dfrac{1 + \cot^2 x + 1 + \tan^2 x}{\frac{1}{\sin x}\cdot\frac{1}{\cos x}}$

$\dfrac{2 + \frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\sin x \cos x}} \rightarrow \dfrac{\frac{2\sin^2 x \cos^2 x + \cos^4 x + \sin^4 x}{\sin^2 x \cos^2 x}}{\frac{1}{\sin x \cos x}}$

$\dfrac{2\sin^2 x \cos^2 x + \cos^4 x + \sin^4 x}{\sin^2 x \cos^2 x}\cdot \sin x \cos x$

$\dfrac{\sin^4 x + 2\sin^2 x \cos^2 x + \cos^4 x}{\sin x \cos x}$ ← Perfect Square Trinomial

$\dfrac{(\sin^2 x + \cos^2 x)(\sin^2 x + \cos^2 x)}{\sin x \cos x} \rightarrow \dfrac{1}{\sin x \cos x} = \boxed{\csc x \sec x}$

$\dfrac{(1)(1)}{\sin x \cos x}$

k) $\dfrac{\cos x + \cot x}{1 + \csc x}$

$$\frac{\cos x + \frac{\cos x}{\sin x}}{1 + \frac{1}{\sin x}}$$

$$\frac{\frac{\sin x \cos x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\sin x} + \frac{1}{\sin x}}$$

$$\frac{\frac{\sin x \cos x + \cos x}{\sin x}}{\frac{\sin x + 1}{\sin x}}$$

$$\frac{\sin x \cos x + \cos x}{\sin x + 1}$$

$$\frac{\cos x(\sin x + 1)}{(\sin x + 1)}$$

$$\boxed{\cos x}$$

l) $\dfrac{\sec x}{\tan x + \cot x}$

$$\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

$$\frac{\frac{1}{\cos x}}{\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}}$$

$$\frac{\frac{1}{\cos x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}}$$

$$\frac{1}{\cos x} \cdot \frac{\sin x \cos x}{\underbrace{\sin^2 x + \cos^2 x}_{1}}$$

$$\frac{\sin x \cos x}{\cos x}$$

$$\boxed{\sin x}$$

See Website for Detailed Answer Key

Extra Work Space