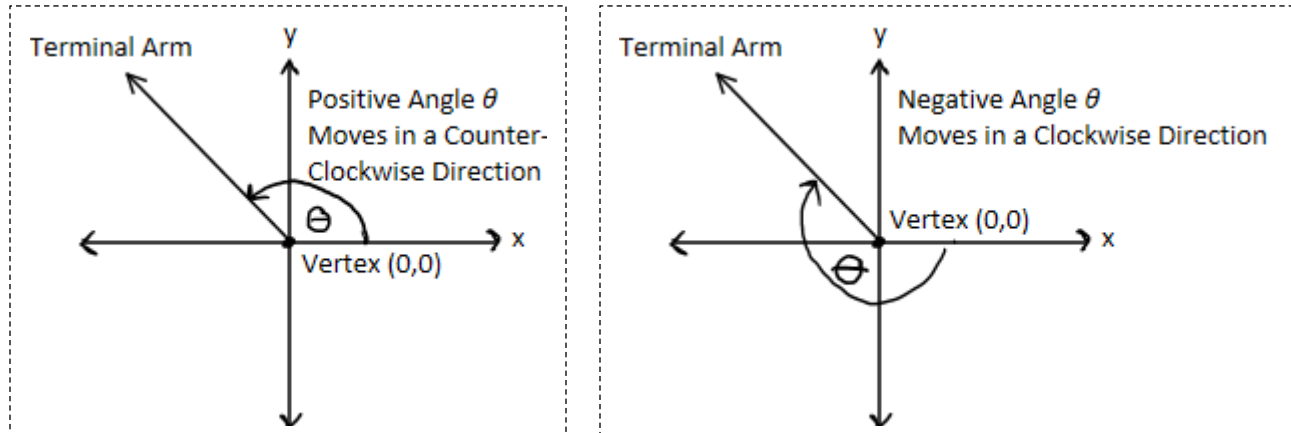


Section 7.1 – Angles and Their Measure

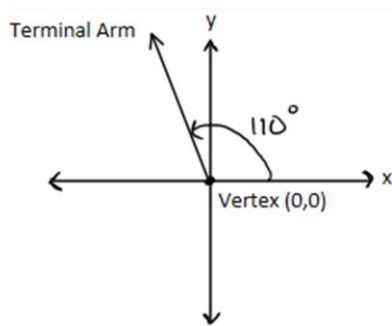
- Trigonometry is all about the study of angles
- An Angle is determine by rotating an arm in a counter-clockwise position at its endpoint
- The endpoint is called the vertex of the angle
- The arm rotating around is called the terminal arm
 - It can rotate in a clockwise direction (negative angle)
 - Or counter clockwise direction (positive angle)



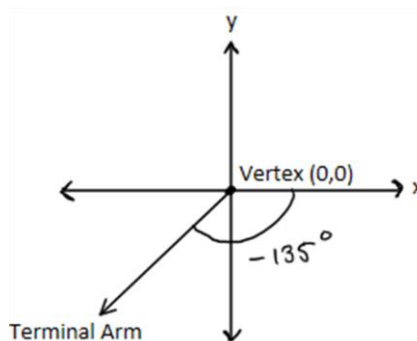
Angles in Standard Position

An angle θ is said to be in **Standard Position** if its **vertex is at the origin** and it **originates from the positive x – axis**. Rotating the terminal arm counter clockwise about the vertex forms a **positive angle θ** , where **rotating clockwise** about the **vertex forms a negative angle θ** .

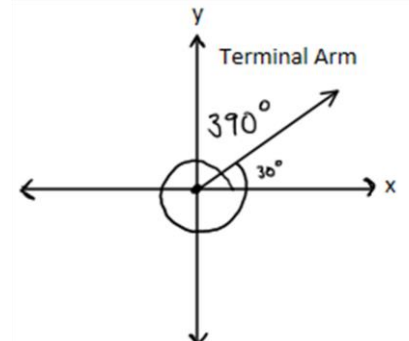
Examples:



- Counter Clockwise
- Positive Direction
- Past the 90° Mark
- An extra 20° into Q2
- Making 110°



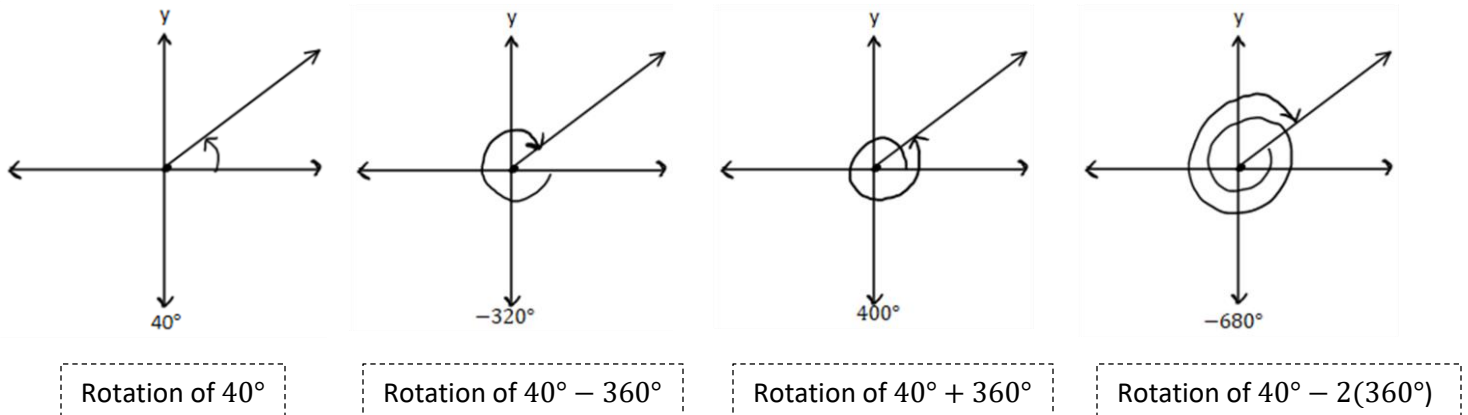
- Clockwise
- Negative Direction
- Past the -90° Mark
- An extra -45° into Q3
- Making -135°



- Counter Clockwise
- Positive Direction
- Past the 360° Mark
- An extra 30° into Q1
- Making 390°

Co-Terminal Angles

- Angles in Standard Position that have the same terminal side are called Co-Terminal angles
- Think about adding or subtracting an entire rotation 360° , from the given angle
- There are an infinite number of Co-Terminal angles, we can add or subtract multiples of 360°



Example 1: If $\theta = 120^\circ$, in Standard Position, find two positive and two negative angles that are co-terminal with θ

Solution 1: There are an infinite number of solutions, so add or subtract 360° to your hearts content

$$120^\circ + 360^\circ = 480^\circ$$

$$120^\circ + 2(360^\circ) = 840^\circ$$

$$120^\circ + 8(360^\circ) = 3000^\circ$$

$$120^\circ - 360^\circ = -240^\circ$$

$$120^\circ - 2(360^\circ) = -600^\circ$$

$$120^\circ - 8(360^\circ) = -2760^\circ$$

Example 2: Find the smallest positive co-terminal angle for

a) 2692°

b) -1940°

Solution 2:

a) Divide 2692 by 360 to see how many full rotations have occurred

$$\frac{2692}{360} = 7.47777\dots$$

7 full rotations: $7 \cdot 360 = 2520^\circ$

$$2692^\circ - 2520^\circ = 172^\circ$$

b) Divide 1940 by 360 to see how many full rotations in the negative direction

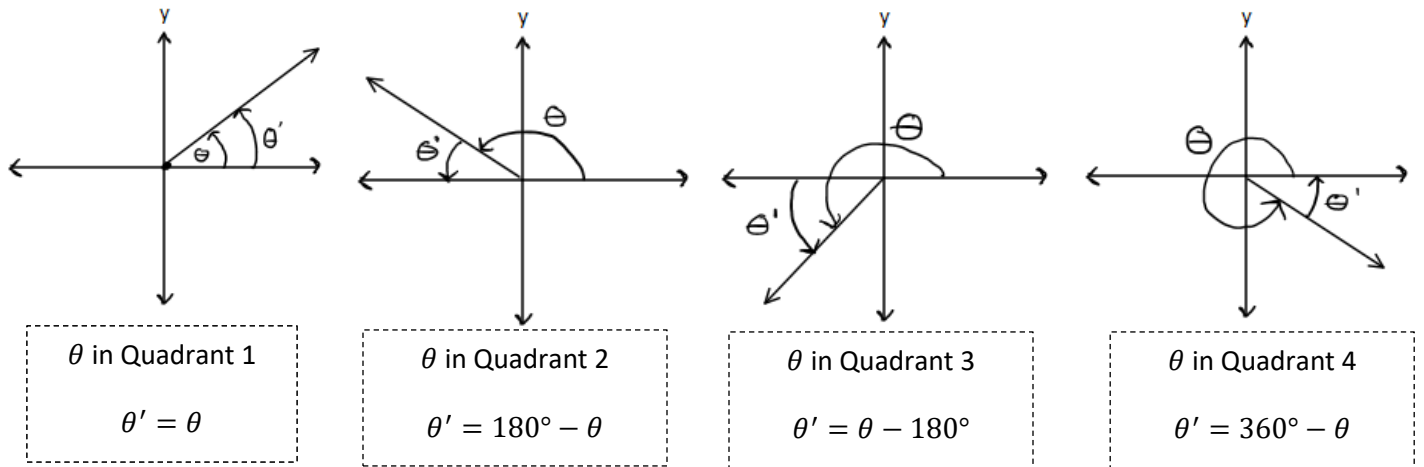
$$\frac{-1940}{360} = -5.3888\dots$$

Since we went negative we will need 6 rotations to get back to the positive. $6 \cdot 360 = 2160^\circ$

$$-1940^\circ + 2160^\circ = 220^\circ$$

Reference Angles

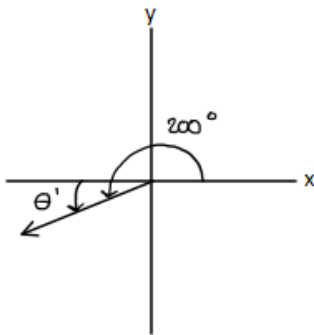
- For an angle θ in **Standard Position**, the **reference angle** is the **positive acute angle θ'** that is **formed** with the **terminal side of θ and the x - axis**.
- Read that again...
- A reference angle is between 0° and 90° : $0^\circ \leq \theta' \leq 90^\circ$
- This will start to make a lot more sense in applications in Section 7.2



Example 3: Find the reference angle for: a) 200° b) 300° c) -200°

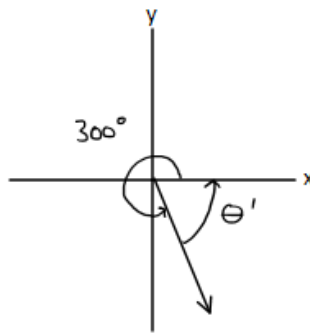
Solution 3:

a)



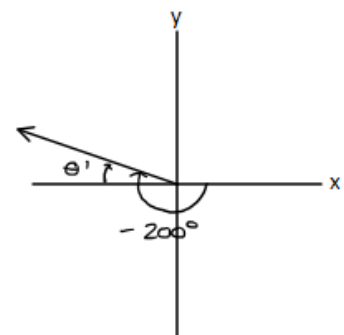
$$\begin{aligned} \theta' &= \theta - 180^\circ \\ \theta' &= 200^\circ - 180^\circ \\ \theta' &= 20^\circ \end{aligned}$$

b)



$$\begin{aligned} \theta' &= 360^\circ - \theta \\ \theta' &= 360^\circ - 300^\circ \\ \theta' &= 60^\circ \end{aligned}$$

c)



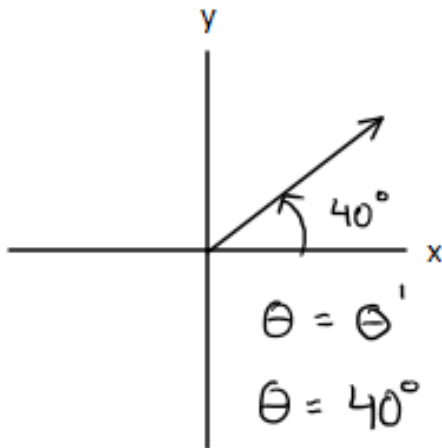
The Co-Terminal Angle of -200° is 160°

$$\begin{aligned} \theta' &= 180^\circ - \theta \\ \theta' &= 180^\circ - 160^\circ \\ \theta' &= 20^\circ \end{aligned}$$

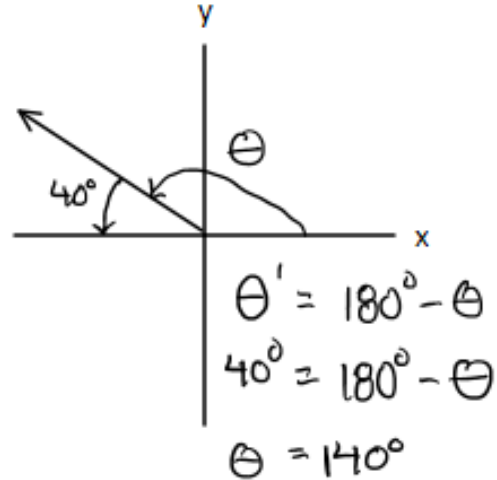
Example 4: Determine the smallest positive angle in Q 1, 2, 3, 4 that has a reference angle of 40°

Solution 4:

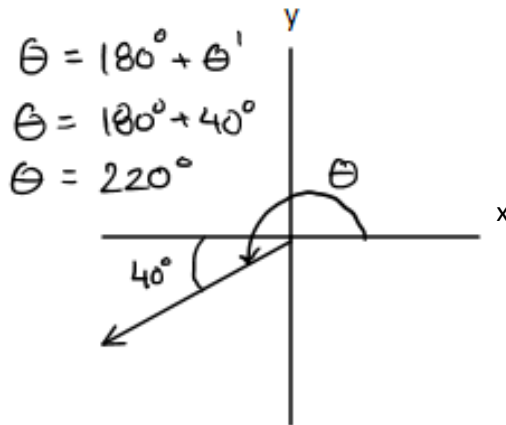
Quadrant 1



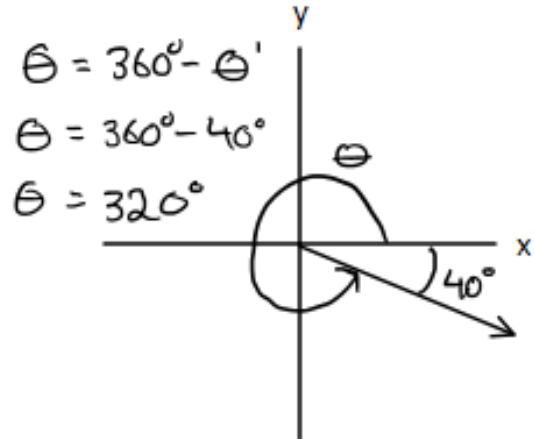
Quadrant 2



Quadrant 3



Quadrant 4



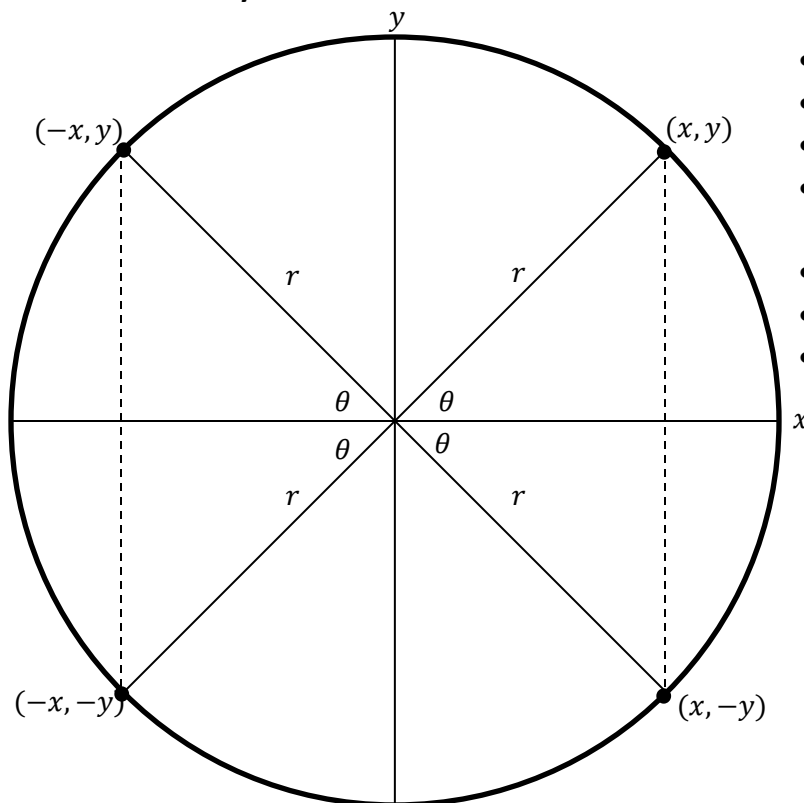
Example 5: Determine the Reference Angle for:

- a) 612° b) -420° c) 844° d) 6425°

Solution 5:

- a) $612^\circ \rightarrow 612^\circ - 360^\circ = 252^\circ$; 252° is in Q3 so: $\theta' = 252^\circ - 180^\circ = 72^\circ$ **Ref Angle: 72°**
 b) $-420^\circ \rightarrow 2(360^\circ) - 420^\circ = 300^\circ$; 300° is in Q4 so: $\theta' = 360^\circ - 300^\circ = 60^\circ$ **Ref Angle: 60°**
 c) $844^\circ \rightarrow 844^\circ - 2(360^\circ) = 124^\circ$; 124° is in Q2 so: $\theta' = 180^\circ - 124^\circ = 56^\circ$ **Ref Angle: 56°**
 d) $6425^\circ \rightarrow 6425^\circ - 18(360^\circ) = 55^\circ$; 55° is in Q1 so: $\theta' = 55^\circ$ **Ref Angle: 55°**

The Coordinate System



- Note: *radius* $r > 0$
- By Pythagorean Theorem: $x^2 + y^2 = r^2$
- So all the triangles must be equal
- So reference angles to the Standard Position must be equal
- $(-x, y)$ is (x, y) reflected over the y – axis
- $(-x, -y)$ is (x, y) reflected over both axes
- $(x, -y)$ is (x, y) reflected over the x – axis

Example 6: Find all angles, $0^\circ \leq \theta \leq 360^\circ$, that have a reference angle of 20°

Solution 6:

Q1: it is also 20°

Q2: $180^\circ - 20^\circ = 160^\circ$

Q3: $180^\circ + 20^\circ = 200^\circ$

Q4: $360^\circ - 20^\circ = 340^\circ$

Example 7: If Quadrant 2 and 4 have the same reference angle, and the Standard Position Angle in Quadrant 2 is 165° , what is the Standard Position Angle in Quadrant 4?

Solution 7:

If Q2 has an angle in Standard Position of 165° , then its Reference Angle is:

$$180^\circ - 165^\circ = 15^\circ$$

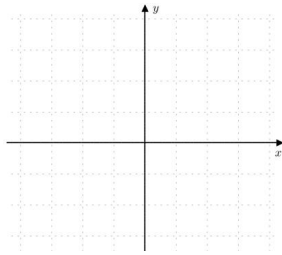
So then the Standard Position Angle in Q4 is:

$$360^\circ - 15^\circ = 345^\circ$$

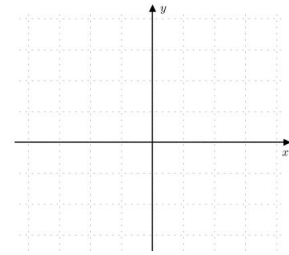
Section 7.1 – Practice Problems

Sketch the angles in Standard Position

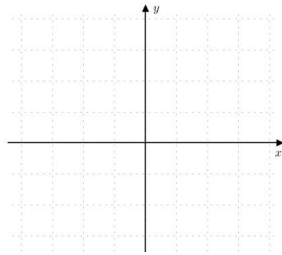
1. 110°



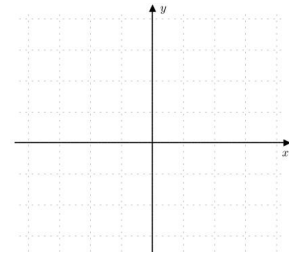
2. -110°



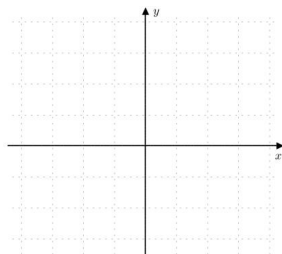
3. 220°



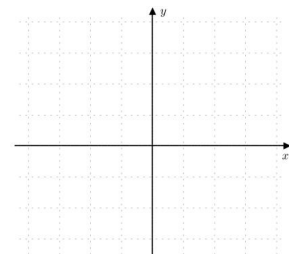
4. -220°



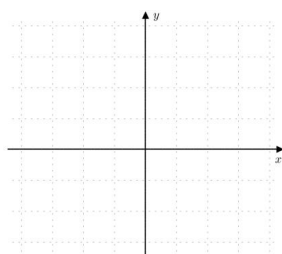
5. 290°



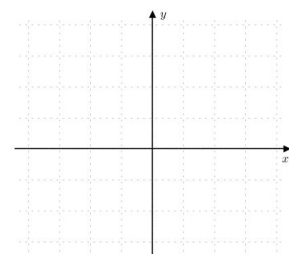
6. -290°



7. 400°



8. -400°



Find the angle of the smallest positive measure co-terminal to the given angle.

9. -30°

10. -96°

11. -197°

12. -314°

13. -127°

14. 405°

15. 502°

16. 437°

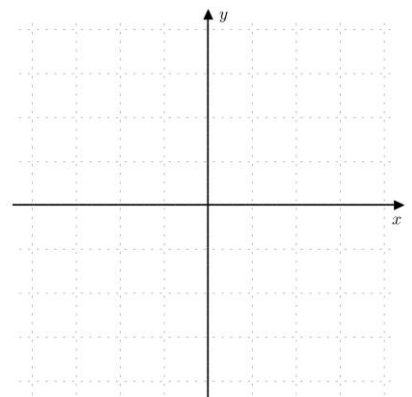
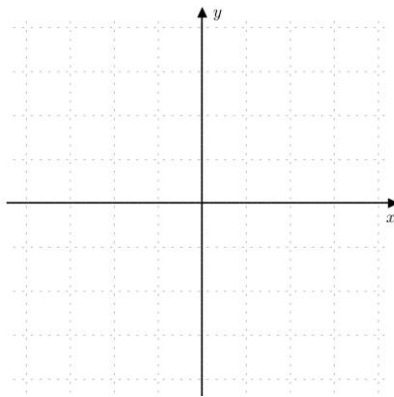
17. 615°

18. -475°

Draw an angle θ in Standard Position, where θ is the smallest measure, and the given point is on the end of the terminal arm. Then determine the length of the terminal arm.

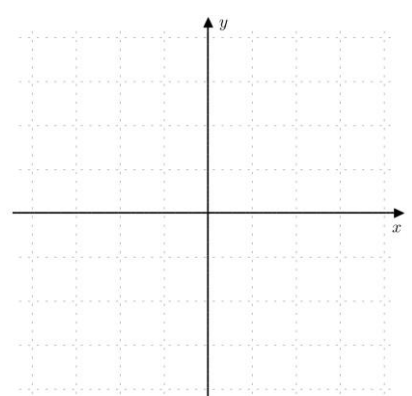
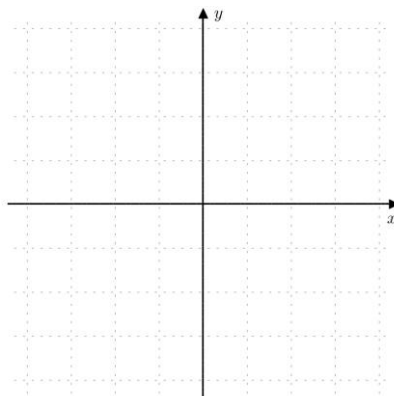
19. $(-4, 3)$

20. $(3, 4)$



21. $(5, -12)$

22. $(-12, -5)$



Find the reference angle

23. 32°

24. -32°

25. 113°

26. -113°

27. 218°

28. -218°

29. 304°

30. -304°

31. 832°

32. -1213°

Find all the angles, $0^\circ \leq \theta \leq 360^\circ$, that have reference angles of the following.

33. 37°

34. 71°

35. 0°

36. 90°

Answer Key – Section 7.1

1. <i>See Website</i>
2. <i>See Website</i>
3. <i>See Website</i>
4. <i>See Website</i>
5. <i>See Website</i>
6. <i>See Website</i>
7. <i>See Website</i>
8. <i>See Website</i>
9. 330°
10. 264°
11. 163°
12. 46°
13. 233°
14. 45°
15. 142°
16. 77°
17. 255°
18. 245°

19. <i>See Website</i>
20. <i>See Website</i>
21. <i>See Website</i>
22. <i>See Website</i>
23. 32°
24. 32°
25. 67°
26. 67°
27. 38°
28. 38°
29. 56°
30. 56°
31. 68°
32. 47°
33. $37^\circ, 143^\circ, 217^\circ, 323^\circ$
34. $71^\circ, 109^\circ, 251^\circ, 289^\circ$
35. $0^\circ, 180^\circ$
36. $90^\circ, 270^\circ$

Extra Work Space