Section 6.5 – Practice Problems

1. Assume the harmonic motion of a spring is described by the equation:

$$S = 4\cos\left(\frac{\pi t}{2}\right)$$

S is given in cm and t is in $seconds$. At what point between 0 and 8 $seconds$ is the spring passing through the origin?

$S = 4\cos\frac{\pi}{2}(t)$ Period: $\frac{2\pi}{\frac{\pi}{2}} \rightarrow 2\pi \cdot \frac{2}{\pi} = 4$

$\boxed{\text{occurs } t = 1, 3, 5, 7 \text{ seconds}}$

so $0 \rightarrow 8$ seconds is two Periods

cosine goes through origin at cosine $\theta = 0$ so $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$t=1$ $\frac{\pi}{2}(t) = \frac{\pi}{2}$ $t=3$ $\frac{\pi}{2}(t) = \frac{3\pi}{2}$ $t=5$ $\frac{5\pi}{2}$ $t=7$ $\frac{7\pi}{2}$ $t=9$ $\frac{9\pi}{2}$

outside domain

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2. The voltage E in an electrical circuit is given by:

$$E = 4\cos 60\pi t$$

t is measured in seconds.

a) Find the amplitude and the Period.

$E = 4\cos 60\pi t$ Amplitude $= 4$

b) The reciprocal of the period, is called the frequency. It is the number of periods completed in one second. Find the frequency.

$P = \frac{2\pi}{b} = \frac{2\pi}{60\pi} = \frac{1}{30}$ Period $= \frac{1}{30}$ seconds

Reciprocal is: 30 Frequency is 30 cycles/second

3. The temperature in Inuvik, Northwest Territory is given by:

$$T = 35\sin\left[\left(\frac{2\pi}{365}\right)(x-100)\right] + 27$$

where $x = 1$ is January 1st and $x = 365$ is December 31st. Use Desmos to find what days of the year the temperature was below 0°.

Using Desmos: We are below zero from 1–49 and 333–365

so the first 49 days and the last 32 days.

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4. Sales of snowblowers are seasonal. Suppose sales in Dawson's Creek is approximated by:

$$S = 200 + 200\cos\left[\frac{\pi}{6}(t+2)\right]$$

where t is time in months with $t = 0$ being January. In what months are sales equal to 0?

$0 = 200 + 200\cos\left[\frac{\pi}{6}(t+2)\right] \rightarrow -200 = 200\cos\left[\frac{\pi}{6}(t+2)\right]$

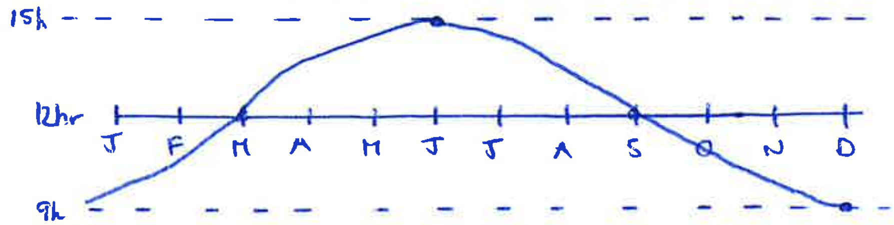
$-1 = \cos\left[\frac{\pi}{6}(t+2)\right]$ $\quad \cos\theta = -1$ at π

so $\pi = \frac{\pi}{6}(t+2) \longrightarrow 6\pi = \pi(t+2) \longrightarrow 6 = t+2$ $\quad \boxed{t = 4}$

$t = 0$ January
$t = 1$ February
$t = 2$ March
$t = 3$ April

$\boxed{t = 4 \text{ is May}}$

5. June 21st is the longest day of the year in Victoria, it is 15 hours long. The shortest day, 9 hours long, is on December 21st and both March 21st and September 21st are 12 hours long. Write a sine equation for the number of daylight hours as a function of the day of the year.



P: 12

Quadrantal: $\frac{12}{4} = 3$

For days: $P = 365$

$P = 2\pi$

A: 3

VD: 12

PS: Jan 1 → March 21st

80 days

$b = \frac{2\pi}{365}$

$y = 3\sin\frac{2\pi}{365}(t - 80) + 12$

6. A healthy adult breathes in and exhales about 0.84 *litres* of air every 4 seconds. The minimal amount of air in the lungs is 0.08 *litres* when $t = 0$. Write a cosine function with $0 \le t \le 8$ and find the time of maximum air capacity in this interval.

A: $\frac{0.84}{2} = 0.42$

PS: NONE

$t = 0$.

Quadrantal every 1

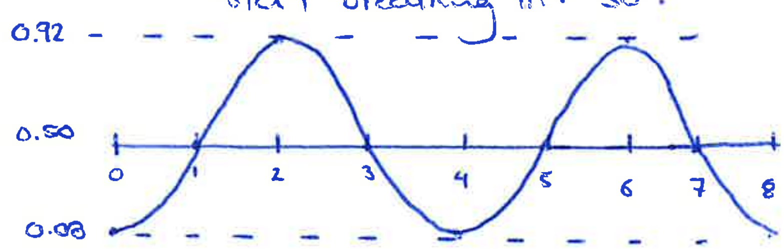
VD: $0.42 + 0.08$

$= 0.50$

$P = 4$

$b = \frac{2\pi}{4} = \frac{\pi}{2}$

start breathing in: so:



Max capacity:

$t = 2$

$t = 6$

7. If the voltage E in an electrical circuit has an amplitude of 110 volts and a period of $\frac{1}{60}$ seconds. And $E = 110$ when $t = 0$, find a periodic equation in terms of cosine that describes this voltage.

$\cos 0 = 1$

A: 110

$P = \frac{1}{60}$ $\quad b = \frac{2\pi}{\frac{1}{60}} = 120\pi$

so $y = 110 \cos 120\pi(t)$

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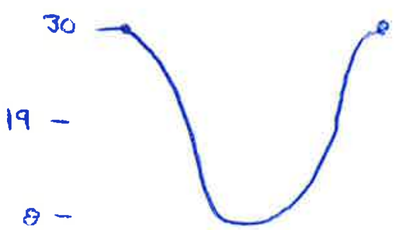
8. The pedals on a bicycle have a maximum height of $30cm$ above the ground and a minimum distance of $8cm$ above the ground. A person pedals at a constant rate of 20 cycles per minute

a) What is the period, in seconds, for this function?

P: 20 cycles/per min



$\frac{20\text{cycles}}{\text{min}} \cdot \frac{1\text{min}}{60\text{sec}} = \frac{1\text{cycle}}{3\text{seconds}}$

Period = 3 seconds

b) Determine the equation for this periodic function

$P = 3$ $\quad b = \frac{2\pi}{3}$ $\quad$ A: 11 $\quad$ PS: None $\quad$ VD: +19

$y = 11 \cos \frac{2\pi}{3}(t) + 19$

9. A Ferris Wheel of radius 25 *meters*, placed 1 *meter* above the ground, varies in a sine wave pattern with respect to time. The Ferris Wheel makes one rotation every 24 *seconds*, with a person sitting 26 *meters* from the ground and rising when it starts to rotate. (Pictures Help)

sin wave

a) Write a sine/cosine function that describes the function from the person's starting point.

$P = 24$ seconds $A = 25$

$b = \frac{2\pi}{24} = \frac{\pi}{12}$ VD $= +1$ $+25$ $= 26$

$y = 25\sin\frac{\pi}{12}(t) + 26$

$2\pi = 24$ seconds

$\pi = 12$ seconds

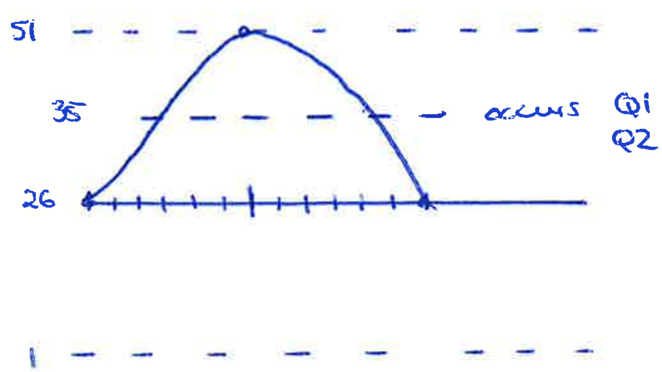
b) How high above the ground would the person be 16 *seconds* after the Ferris Wheel starts moving?

$t = 26$

$y = 25\sin\frac{\pi}{12}(16) + 26$

$= 4.35$m

c) How many seconds on each rotation is a person more than 35 *meters* in the air?



$P = 24$ so Quadrantal every 6 seconds

$35 = 25\sin\frac{\pi}{12}(t) + 26$

$9 = 25\sin\frac{\pi}{12}t$ $\frac{9}{25} = \sin\frac{\pi}{12}t$

$\sin^{-1}\left(\frac{9}{25}\right) = \frac{\pi}{12}t$ and symmetry π or 12 seconds − 1.4 second

$t = 1.4$ seconds $t = 10.6$ secs $t = 10.6$ seconds total time $\boxed{9.2 \text{ seconds}}$

10. Tides are a periodic rise and fall of the ocean water due to the gravitational effect of the Moon. low tide of 4.2 *meters* in Sidney happens at $4{:}30am$ and the next high tide of 11.8 *meters* happens at $11{:}30am$ the same day.

a) Write a sine/cosine function that describes the function in question.

PS: -4.5 ↑ shifted from 12am ↓ 4:30am

A: $\frac{11.8-4.2}{2} = 3.8$

VD: $11.8-3.8 = 8$

P: 11:30 → 4:30 7 hours that's half

Period = 14 hrs

$b = \frac{2\pi}{14} = \frac{\pi}{7}$

start low tide so reflected cosine function

$\boxed{y = -3.8\cos\frac{\pi}{7}(x-4.5)+8}$

b) How high is the tide at $1{:}15pm$ on the same day?

1:15pm → 13.25

$y = -3.8\cos\frac{\pi}{7}(13.25-4.5)+8$

$\boxed{y = 10.69m}$

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11. A spring modelling a sinusoidal function sits at 1.6 *meters* above the ground. If the mass on eh spring is pulled 1.1 *meters* below its resting position and then is released, it requires 0.5 *seconds* to move from the maximum position to its minimum position (Assume a perfect vacuum where friction and air resistance are neglected).

a) Write an equation in terms of cosine that describes this periodic function.

VD: 1.6

A: 1.1

P: $0.5 \cdot 2 = 1$ second

$b = 2\pi$

Since pulled down then released, reflected cosine function

$\boxed{y = -1.1\cos 2\pi t + 1.6}$

b) What height is the spring 2.3 seconds after being released?

$y = -1.1\cos 2\pi(2.3)+1.6$

$\boxed{y = 1.94m}$

12. A tsunami, a very fast-moving body of water, effects the rise and fall of a vast quantity of wat
First, the water will move down from its starting point, move an equal distance above its start
point, and then settle back to where it began. The tsunami that took out Atlantis was $16\ min$
in length, had an amplitude of $8\ meters$. The normal depth at Atlantean Beach Resort was
$6\ meters$.

a) What is the maximum and minimum height of the water caused by the tsunami?

A: 8

Normal point 6 m

Max: 14 meters

Min: $6-8=-2$ so 0 meters

b) Write a periodic model of the tsunami when it first reaches Atlantean Beach Resort?

Reflected sine wave

A: 8 PS: 0 VO: +6 P: 16min

$b = \frac{2\pi}{16} = \frac{\pi}{8}$

$$\boxed{y = -8\sin\frac{\pi}{8}t + 6}$$

c) If you were in a boat on the ocean, how would the tsunami affect you?

Due to such a long period length the boat would float on top and hardly be affected by the rise and fall.

See Website for Detailed Answer Key

Extra Work Space