# Section 6.5 – Graphing Trigonometric Functions

- When we graph Trigonometric Functions, we call them Periodic Functions.
- This wave motion follows a consistent and time dispersed pattern, we call this **Harmonic Motion**.
- Some examples of Harmonic Motion include:

Pendulums, Ferris Wheels, Tides, Rotation of the Earth, and Radio Waves… to name but a few.

**Example 1:**

A weight attached to a spring causes the spring to stretch and contract rhythmically. The function of the distance of the spring from its rest position with respect to time is given by: $d = 3\sin(8\pi t)$

a) How many cycles per second happen?
b) Graph the spring function.
c) At what time does the first maximum and minimum extremes occur?

**Solution 1:**

a) Cycles per second means check the Period over a one second time frame.

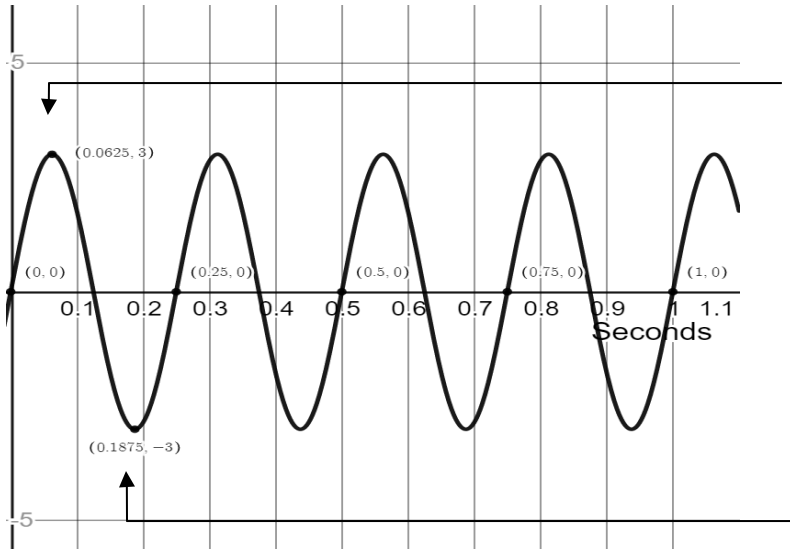
$$d = 3\sin(8\pi t) \quad \rightarrow \quad 3\sin 8\pi(t)$$

$$Period = \frac{2\pi}{|b|} = \frac{2\pi}{8\pi} = \frac{1}{4}$$

So, the wave completes **one full cycle in a quarter of a second**.

This means it completes **4 cycles/second.**

b)




c)

First Max happens at:

$0.0625\ seconds$

First Min happens at:

$0.1875\ seconds$

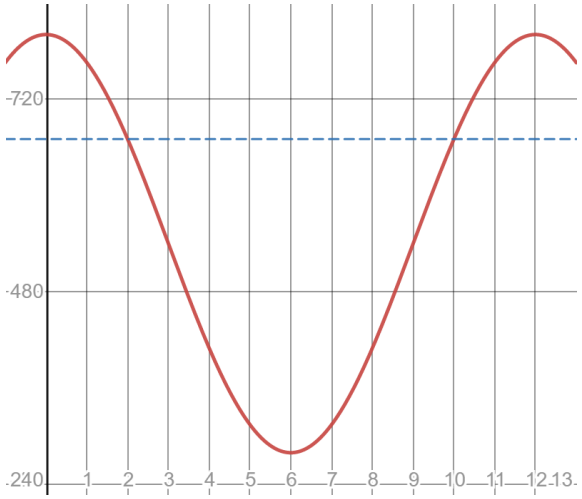
**Example 2:** The sales of snow shovels is iseasonal. The sales can be described by the function:

$$S = 540 + 260\cos\frac{\pi t}{6}$$

$t$ is the time in months, $t = 1$ means January and $t = 12$ means December. Graph the function and described at which months the sales are above $670\ units\ sold$.

**Solution 2:** First let's determine the Period, Amplitude, and Vertical Displacement.

Vertical Displacement $= 540$ Amplitude $= 260$ Period $= \frac{2\pi}{\frac{\pi}{6}} = 12$




$$540 + 260\cos\frac{\pi t}{6} > 670$$

$$260\cos\frac{\pi t}{6} > 130$$

$$\cos\frac{\pi t}{6} > \frac{1}{2}$$

Cosine equal one half at:

$$\frac{\pi}{3} \quad and \quad \frac{5\pi}{3}$$

$$\frac{\pi}{6}t > \frac{\pi}{3} \qquad \frac{\pi}{6}t > \frac{5\pi}{3}$$

$$t = 2 \qquad t = 10$$

So, February and October have sales exactly $670$ units. November, December, and January are above.

**Example 3:** A Ferris Wheel has a radius of $40m$ and is fixed $5m$ off the ground and it rotates every 90 seconds. Find a Cosine Function that gives the height $h$, after $t$ seconds of motion for a rider. If the rider gets in at the bottom of the Ferris Wheel, at what time have they first reached a height of $60m$. Display the results using Desmos.

**Solution 3:**

$$y = -40\cos\left(\frac{\pi}{45}t\right) + 45$$

Since we start at the bottom.

**Reach $60m$ at 28 $seconds$.**

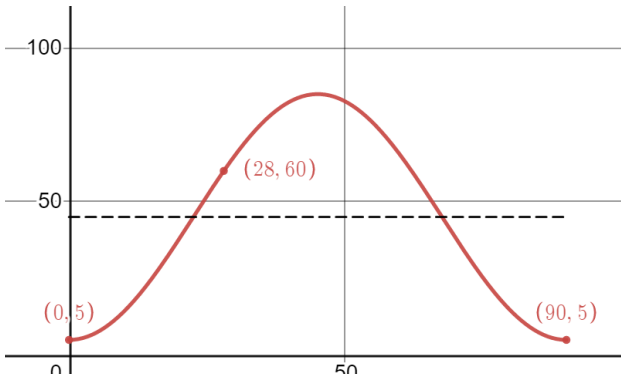
VD $= 45m$

Amp $= 40$

Period $=$

$$\frac{2\pi}{b} = 90$$

$$b = \frac{\pi}{45}$$



# Section 6.5 – Practice Problems

1. Assume the harmonic motion of a spring is described by the equation:

$$S = 4\cos\left(\frac{\pi t}{2}\right)$$

$S$ is given in $cm$ and $t$ is in $seconds$. At what point between $0\ and\ 8\ seconds$ is the spring passing through the origin?

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2. The voltage $E$ in an electrical circuit is given by:

$$E = 4\cos 60\pi t$$

$t$ is measured in seconds.

   a) Find the amplitude and the Period.

   b) The reciprocal of the amplitude, is called the frequency. It is the number of periods completed in one second. Find the frequency.

3. The temperature in Inuvik, Northwest Territory is given by:

$$T = 35 \sin\left[\left(\frac{2\pi}{365}\right)(x - 100)\right] + 27$$

where $x = 1$ is January 1st and $x = 365$ is December 31st. Use Desmos to find what days of the year the temperature was below $0°$.

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4. Sales of snowblowers are seasonal. Suppose sales in Dawson's Creek is approximated by:

$$S = 200 + 200 \cos\left[\frac{\pi}{6}(t + 2)\right]$$

where $t$ is time in months with $t = 0$ being January. In what months are sales equal to $0$?

5. June 21st is the longest day of the year in Victoria, it is $15$ hours long. The shortest day, $9$ hours long, is on December 21st and both March 21st and September 21st are $12$ hours long. Write a sine equation for the number of daylight hours as a function of the day of the year.

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6. A healthy adult breathes in and exhales about $0.84\ litres$ of air every $4$ seconds. The minimal amount of air in the lungs is $0.08\ litres$ when $t = 0$. Write a cosine function with $0 \leq t \leq 8$ and find the time of maximum air capacity in this interval.

7. If the voltage $E$ in an electrical circuit has an amplitude of $110$ volts and a period of $\frac{1}{60}$ seconds. And $E = 110$ when $t = 0$, find a periodic equation in terms of cosine that describes this voltage.

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8. The pedals on a bicycle have a maximum height of $30cm$ above the ground and a minimum distance of $8cm$ above the ground. A person pedals at a constant rate of $20$ cycles per minute

   a) What is the period, in seconds, for this function?

   b) Determine the equation for this periodic function

9. A Ferris Wheel of radius $25\ meters$, placed $1\ meter$ above the ground, varies in a sine wave pattern with respect to time. The Ferris Wheel makes one rotation every $24\ seconds$, with a person sitting $26\ meters$ from the ground and rising when it starts to rotate. (Pictures Help)

   a) Write a sine/cosine function that describes the function from the person's starting point.

   b) How high above the ground would the person be $16\ seconds$ after the Ferris Wheel starts moving?

   c) How many seconds on each rotation is a person more than $35\ meters$ in the air?

10. Tides are a periodic rise and fall of the ocean water due to the gravitational effect of the Moo low tide of $4.2\ meters$ in Sidney happens at $4{:}\ 30am$ and the next high tide of $11.8\ meters$ happens at $11{:}\ 30am$ the same day.

    a) Write a sine/cosine function that describes the function in question.

    b) How high is the tide at $1{:}\ 15pm$ on the same day?

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11. A spring modelling a sinusoidal function sits at $1.6\ meters$ above the ground. If the mass on eh spring is pulled $1.1\ meters$ below its resting position and then is released, it requires $0.5\ seconds$ to move from the maximum position to its minimum position (Assume a perfect vacuum where friction and air resistance are neglected).

    a) Write an equation in terms of cosine that describes this periodic function.

    b) What height is the spring $2.3$ seconds after being released?

12. A tsunami, a very fast-moving body of water, effects the rise and fall of a vast quantity of wat
First, the water will move down from its starting point, move an equal distance above its start
point, and then settle back to where it began. The tsunami that took out Atlantis was $16\ min$
in length, had an amplitude of $8\ meters$. The normal depth at Atlantean Beach Resort was
$6\ meters$.

a) What is the maximum and minimum height of the water caused by the tsunami?

b) Write a periodic model of the tsunami when it first reaches Atlantean Beach Resort?

c) If you were in a boat on the ocean, how would the tsunami affect you?

**See Website for Detailed Answer Key**

**Extra Work Space**