Section 6.4 – Solving Quadratic Equations

- The factor method is definitely the most efficient method of solving quadratics
- But not all quadratics can be factored easily if at all
- What we get are two methods depending on the situation
    - The Square Root Method
    - The Quadratic Equation

Solving Quadratic Equations by the Square Root Method

- The square root method in mainly used when $b = 0$ in the equation $ax^2 + bx + c = 0$
- Isolate the x^2 on the left side and square root the other side

Example:

$x^2 - 16 = 0$

$(x + 4)(x - 4) = 0$

$x + 4 = 0$ or $x - 4 = 0$

$x = -4 \quad x = 4$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

The procedure on the right is the **SQUARE ROOT METHOD**

Solving a Quadratic Equation of the form $ax^2 + c = 0$

Step 1: Isolate the x^2 on the left side of the equation and the constant on the right
Step 2: Take the **square root of both sides**, the square root of the **constant has to be** $\pm$
Step 3: Simplify if possible
Step 4: Check the solution in the original equation

The **Square Root Property** is defined as follows:

The Square Root Property

The equation $x^2 = n$ has exactly 2 real solutions

$$x = \sqrt{n} \text{ and } x = -\sqrt{n} \quad \text{if } no > 0. \text{ The solution are written } x = \pm n$$

$$x = 0 \text{ if } n = 0$$

$$x = \emptyset \text{ if } n < 0 \text{ (we can't take the square root of a negative)}$$

Example: Solve $4x^2 - 9 = 0$

Solution:

$4x^2 = 9$ - Add nine to both sides

$x^2 = \frac{9}{4}$ - Divide both sides by 4

$x = \pm\sqrt{\frac{9}{4}}$ - Square Root both sides (remember $\pm$)

$= \pm\frac{3}{2}$

Solutions are: $\frac{3}{2}$ ***and*** $-\frac{3}{2}$

Example: Solve $2x^2 + 7 = 0$

Solution:

$2x^2 = -7$ - Subtract seven from both sides

$x^2 = -\frac{7}{2}$ - Divide both sides by 2

$x = \pm\sqrt{-\frac{7}{2}}$ - Ø denotes the '***empty set***'

$x = \emptyset$ - We **can't square root a negative**

Solutions are: ***None***

Example: Solve $4x^2 - 7 = 0$

Solution:

$4x^2 = 7$ - Add seven to both sides

$x^2 = \frac{7}{4}$ - Divide both sides by 4

$x = \pm\sqrt{\frac{7}{4}}$ - Square Root both sides, and simplify

$x = \pm\frac{\sqrt{7}}{2}$

Solutions are: $\frac{\sqrt{7}}{2}$ ***and*** $-\frac{\sqrt{7}}{2}$

Example: Solve $(x + 2)^2 = 10$

Solution:

$x + 2 = \pm\sqrt{10}$ - Square Root both sides first, the 'squared term' is isolated to start

$x = -2 \pm \sqrt{10}$ - Subtract both sides by 2

Solutions are: $-2 + \sqrt{10}$ ***and*** $-2 - \sqrt{10}$

Example: Solve $9(x-1)^2 = 13$

Solution:

$(x-1)^2 = \frac{13}{9}$ - Divide both sides by 9 to isolate the 'squared term'

$x-1 = \pm\sqrt{\frac{13}{9}}$ - Square Root both sides

$x-1 = \pm\frac{\sqrt{13}}{3}$ - Add 1 to both sides

$x = 1 \pm \frac{\sqrt{13}}{3}$ - Simplify

Solutions are: $1+\frac{\sqrt{13}}{3}$ ***and*** $1-\frac{\sqrt{13}}{3}$

Example: Solve $4(x+3)^2 + 11 = 0$

Solution:

$4(x+3)^2 = -11$ - Subtract 11 and Divide both sides by 4 to isolate the 'squared term'

$(x+3)^2 = -\frac{11}{4}$ - Square Root both sides

$x+3 = \pm\sqrt{-\frac{11}{4}}$ - **Can't Square Root a Negative**

Solutions are: ***None***

$x = \emptyset$

Solving Quadratic Equations with the Quadratic Equation

- When quadratics **do not factor easily** we can use the **Quadratic Equation**
- It works every time and will give us **NO SOLUTION** if it cannot be factored

Quadratic Equation

The solution to the quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use The Quadratic Equation to Solve the Following Quadratics

Example: Solve $2x^2 + x - 6 = 0$ $\quad a = 2,\ b = 1\ \ c = -6$

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1 - -48}}{4}$$

$$x = \frac{-1 \pm \sqrt{49}}{4}$$

$$x = \frac{-1 \pm 7}{4}$$

$$x = \frac{-1+7}{4} \text{ and } x = \frac{-1-7}{4}$$

$$x = \frac{6}{4} = \frac{3}{2} \qquad x = \frac{-8}{4} = -2$$

$$x = \left\{-2, \frac{3}{2}\right\}$$

Example: Solve $3x^2 + 2x - 4 = 0$ $\quad a = 3,\ b = 2,\ c = -4$

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - -48}}{6}$$

$$x = \frac{-2 \pm \sqrt{52}}{6}$$

$$x = \frac{-2 \pm \sqrt{4 \cdot 13}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{13}}{6}$$

$$x = \frac{2(-1 \pm \sqrt{13})}{6}$$

$$x = \frac{-1 \pm \sqrt{13}}{3}$$

$$x = \frac{-1+\sqrt{13}}{3} \text{ and } x = \frac{-1-\sqrt{13}}{3}$$

Example: Solve $2x^2 - 3x = -4 \Rightarrow 2x^2 - 3x + 4 = 0$ $\quad a = 2,\ b = -3,\ c = 4$

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 - 32}}{4}$$

$$x = \frac{3 \pm \sqrt{-23}}{4}$$

$x = \phi$

no solutions

Section 6.4 – Practice Questions

Solve by the Square Root Method

1. $x^2 = 16$

2. $3y^2 = 13$

3. $5z^2 - 12 = 0$

4. $8x^2 + 3 = 11x^2$

5. $x^2 + 4 = 1$

6. $(x - 2)^2 = 16$

7. $(2x - 1)^2 = 12$

8. $(3x + 2)^2 = 18$

9. $\frac{(5x-2)^2}{3} = 3$

10. $-2(4x-1)^2 + 18 = 0$

11. $\frac{4(x^2-1)+4}{x^2+3} = 1$

12. $\frac{1}{(x-1)^2} - \frac{1}{2} = -\frac{1}{4}$

13. $(x^2+6x+9) = 5$

14. $\left(x-\frac{3}{2}\right)^2 = \frac{7}{4}$

15. $\left(x+\frac{2}{3}\right)^2=\frac{5}{9}$

16. $\left(x+\frac{1}{2}\right)^2-\frac{5}{4}=0$

Solve using the Quadratic formula. Give exact answers to 2 decimal places.

17. $y^2-4y-3=0$

18. $x^2+5x-3=0$

19. $3z^2-6z+4=0$

20. $\frac{3}{2}x^2-2=x$

21. $5z(z+2) = -4$

22. $12x = 3x^2 - 8$

23. $(x-2)(x-3) = 8$

24. $\frac{2}{x+2} + \frac{1}{x} = 1$

25. $\frac{2}{y} = \frac{3}{y^2} + 2$

26. $\frac{1}{x} - \frac{2}{x-4} = 2$

27. $5.13x^2 - 7.27x - 4.32 = 0$

28. $\sqrt{3}x^2 = 8\sqrt{2}x - 4\sqrt{3}$

Solve each quadratic equation by the method of your choice

29. $\frac{9(3x-5)^2}{4} = 1$

30. $\frac{25(2x+1)^2}{9} = 0$

31. $4t^2 + 25 = 20t$

32. $25t^2 - 25t + 6 = 0$

33. $(x+3)(x-2) = -4$

34. $2x(x+3) = 10$