

Section 6.4 – Practice Problems

1. Which function listed below, matches the details described in the columns

Graph	A	B	C	D	E	F
Amplitude	2	3	2	3	3	2
Period	π	π	3π	3π	$\frac{4\pi}{3}$	$\frac{2\pi}{3}$
Phase Shift	$\frac{\pi}{3}$	$-\frac{\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{3\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$
Vertical Disp.	-2	2	-2	3	3	-3

$f(x) = 2 \cos \frac{2}{3} \left(x + \frac{2\pi}{3} \right) - 2$ C

$g(x) = 3 \cos \left(\frac{2}{3} x + \frac{\pi}{2} \right) + 3$ D


$h(x) = -2 \sin 2 \left(x - \frac{\pi}{3} \right) - 2$ A


$i(x) = -2 \cos \left(3x - \frac{\pi}{2} \right) - 3$ F


$j(x) = -3 \sin 2 \left(x + \frac{\pi}{6} \right) + 2$ B


$k(x) = 3 \sin \left(\frac{3}{2} x - \frac{\pi}{2} \right) + 3$ E

2. Match the $f(x)$ function with the corresponding $g(x)$ function, such that $f(x) = g(x)$ for all x

a) $f(x) = \sin x$  **A** $g(x) = \cos(-x + \pi)$

b) $f(x) = -\sin x$  **B** $g(x) = -\sin \left(x - \frac{\pi}{2} \right)$

c) $f(x) = \cos x$  **C** $g(x) = \cos \left(x - \frac{\pi}{2} \right)$

d) $f(x) = -\cos x$  **D** $g(x) = \cos \left(x + \frac{\pi}{2} \right)$

Room to write down thoughts and work through ideas

3. State the Amplitude, Period, Phase Shift and Vertical Displacement for the graph of each given function.

a) $y = \frac{1}{3} \sin\left(2x + \frac{\pi}{3}\right) - 1$

$$y = \frac{1}{3} \sin 2\left(x + \frac{\pi}{6}\right) - 1$$

A: $\frac{1}{3}$

PS: $-\frac{\pi}{6}$

P: $\frac{2\pi}{2} = \pi$

VD: -1

b) $y = -\frac{1}{2} \sin \pi\left(x + \frac{3}{4}\right) + 1$

A: $\frac{1}{2}$

PS: $-\frac{3}{4}$

P: $\frac{2\pi}{\pi} = 2$

VD: $+1$

c) $y = -4 \cos \frac{\pi}{3}(x - 1) + 2$

A: 4

PS: $+1$

P: $\frac{2\pi}{\frac{\pi}{3}} \Rightarrow 2\pi \cdot \frac{3}{\pi} = 6$

VD: $+2$

d) $y = -\cos 2\left(\frac{\pi}{6} - x\right)$

$$y = -\cos 2\left(-x + \frac{\pi}{6}\right)$$

$$y = -\cos -2\left(x - \frac{\pi}{6}\right)$$

A: 1

P: π

PS: $\frac{\pi}{6}$

VD: 0

e) $y = 3 \sin\left(\frac{2\pi}{3} - \pi x\right) - 2$

$$y = 3 \sin\left(-\pi x + \frac{2\pi}{3}\right) - 2$$

$$y = 3 \sin\left(-\pi\left(x - \frac{2}{3}\right)\right) - 2$$

A: 3

P: 2

PS: $\frac{2}{3}$

VD: -2

f) $y = \frac{3}{2} \cos 2\left(x + \frac{\pi}{4}\right)$

A: $\frac{3}{2}$

P: π

PS: $-\frac{\pi}{4}$

VD: 0

4. What is the Period of the following functions?

Tan Period is $\frac{\pi}{b}$

a) $y = 2 \tan \frac{1}{3}x$

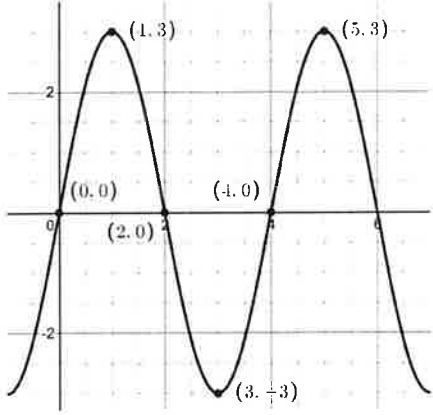
$$\frac{\pi}{\frac{1}{3}} \rightarrow 3\pi$$

b) $y = -2 \tan \frac{\pi}{2}x$

$$\frac{\pi}{\frac{\pi}{2}} \rightarrow \pi \cdot \frac{2}{\pi} = 2$$

5. Write an equation in the form $y = a \sin b(x - c)$ and $y = a \cos b(x - c)$, where c is the smallest positive number and $a > 0, b > 0$

a)



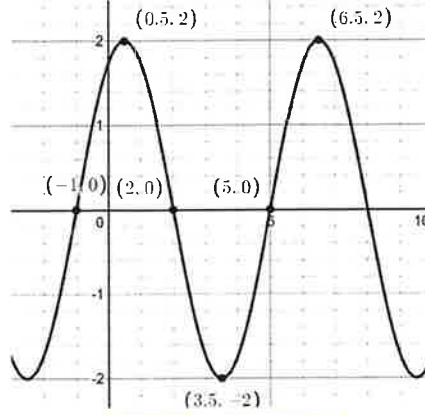
for sin $\rightarrow A:3$ PS:0 VD:0 P:4 so $\frac{2\pi}{b} = 4$
 $b = \frac{\pi}{2}$

$$y = 3 \sin \frac{\pi}{2}(x)$$

for cos $\rightarrow A:3$ PS:+1 VD:0 P:4 so $b = \frac{\pi}{2}$

$$y = 3 \cos \frac{\pi}{2}(x-1)$$

b)

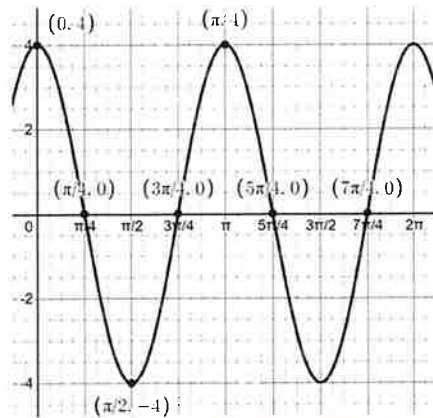


for sin
 $A:2$
 PS: +5
 VD:0
 P:6 so $b = \frac{2\pi}{6} = \frac{\pi}{3}$
 for cos
 $A:2$
 PS: 0.5
 VD:0
 P: $\frac{\pi}{3}$

$$y = 2 \sin \frac{\pi}{3}(x-5)$$

$$y = 2 \cos \frac{\pi}{3}(x-\frac{1}{2})$$

c)



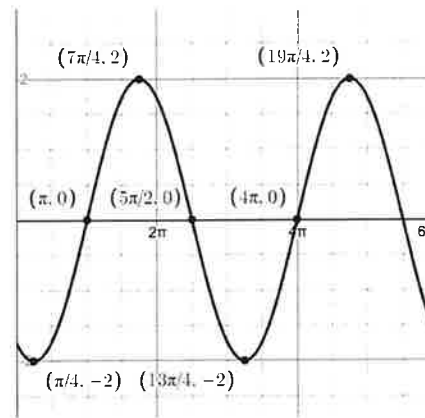
for sin
 $A:4$
 PS: $+\frac{3\pi}{4}$
 VD:0
 P: π so
 $b = \frac{2\pi}{\pi} = 2$

$$y = 4 \sin 2(x - \frac{3\pi}{4})$$

$$y = 4 \cos 2x$$

for cos
 $A:4$
 PS:0
 VD:0
 P: π
 so $b = 2$

d)

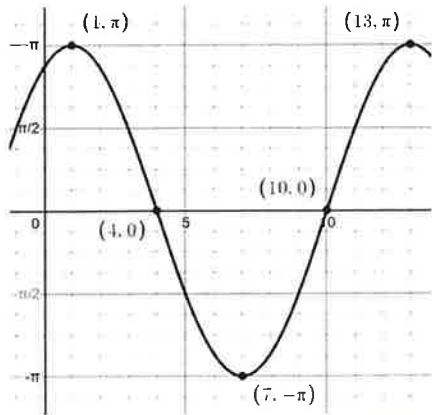


for sin
 $A:2$
 VD:0
 PS: $+\pi$
 P: $3\pi \rightarrow b = \frac{2\pi}{3\pi} = \frac{2}{3}$
 for cos
 All the same
 except
 PS: $+\frac{7\pi}{4}$

$$y = 2 \sin \frac{2}{3}(x - \pi)$$

$$y = 2 \cos \frac{2}{3}(x - \frac{7\pi}{4})$$

e)



$A: \pi$ $V_0: 0$ $P: 12 \Rightarrow b = \frac{2\pi}{12} = \frac{\pi}{6}$

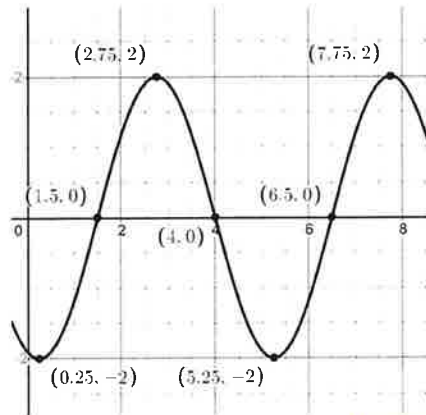
for sin PS: +10

$$y = \pi \sin \frac{\pi}{6}(x-10)$$

for cos PS: +1

$$y = \pi \cos \frac{\pi}{6}(x-1)$$

f)



$A: 2$
 $V_0: 0$
 $P: 5$
 $\Rightarrow b = \frac{2\pi}{5}$

for sin PS: +1.5

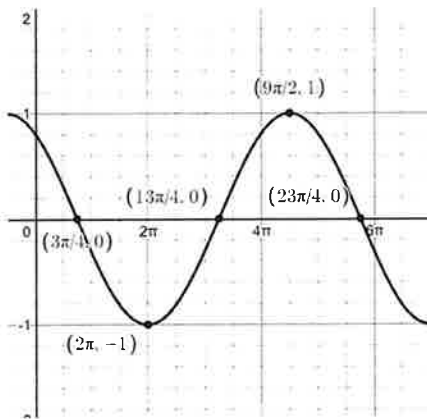
$$y = 2 \sin \frac{2\pi}{5}(x - \frac{3}{2})$$

for cos PS: +2.75

\downarrow
 $2 \times \frac{1}{4}$
 $\frac{1}{4}$

$$y = 2 \cos \frac{2\pi}{5}(x - \frac{11}{4})$$

g)



$A: 1$
 $V_0: 0$
 $P: \frac{20\pi}{4}$
 5π
 $\Rightarrow b = \frac{2\pi}{5\pi}$
 $b = \frac{2}{5}$

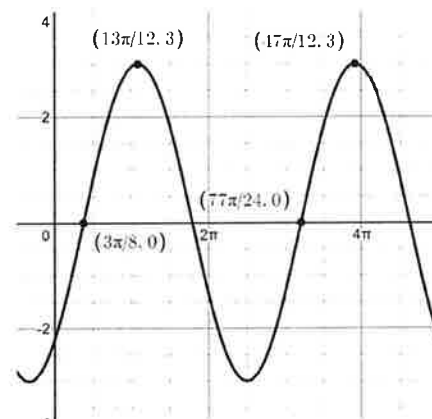
for sin PS: +13pi/4

$$y = \sin \frac{2}{5}(x - \frac{13\pi}{4})$$

for cos PS: +9pi/2

$$y = \cos \frac{2}{5}(x - \frac{9\pi}{2})$$

h)



$A: 3$
 $V_0: 0$
 $P: \frac{17\pi}{6}$
 $b = \frac{2\pi}{17\pi/6}$
 $b = 2\pi \cdot \frac{6}{17\pi} = \frac{12}{17}$

for sin PS: +3pi/8

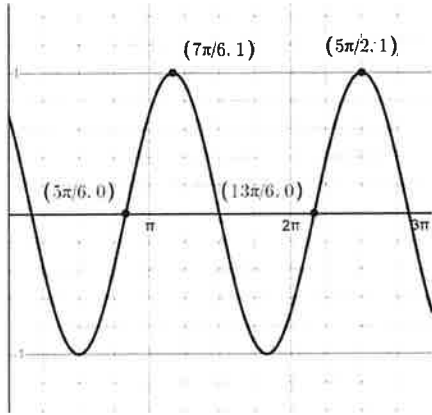
$$y = 3 \sin \frac{12}{17}(x - \frac{3\pi}{8})$$

for cos PS: 13pi/12

$$y = 3 \cos \frac{12}{17}(x - \frac{13\pi}{12})$$

i)

$A: 1$
 $VD: 0$
 $P: \frac{4\pi}{3}$
 $b = \frac{2\pi}{\frac{4\pi}{3}}$
 $\frac{2\pi \cdot 3}{4\pi} = \frac{3}{2}$



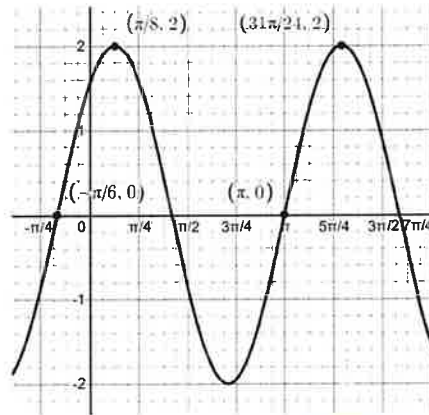
for sin PS: $+\frac{5\pi}{6}$

$$y = \sin \frac{3}{2}(x - \frac{5\pi}{6})$$

for cos PS: $\frac{7\pi}{6}$

$$y = \cos \frac{3}{2}(x - \frac{7\pi}{6})$$

j)



$A: 2$
 $VD: 0$
 $P: \frac{7\pi}{6}$
 $b = \frac{2\pi}{\frac{7\pi}{6}} = \frac{12}{7}$

for sin PS: π

$$y = 2\sin \frac{12}{7}(x - \pi)$$

for cos PS: $\frac{\pi}{8}$

$$y = 2\cos \frac{12}{7}(x - \frac{\pi}{8})$$

6. Accurately sketch at least one full Period of the graph of: $y = -3\sin \frac{\pi}{3}(x + 2) + 1$

$A: 3$ (reflected)

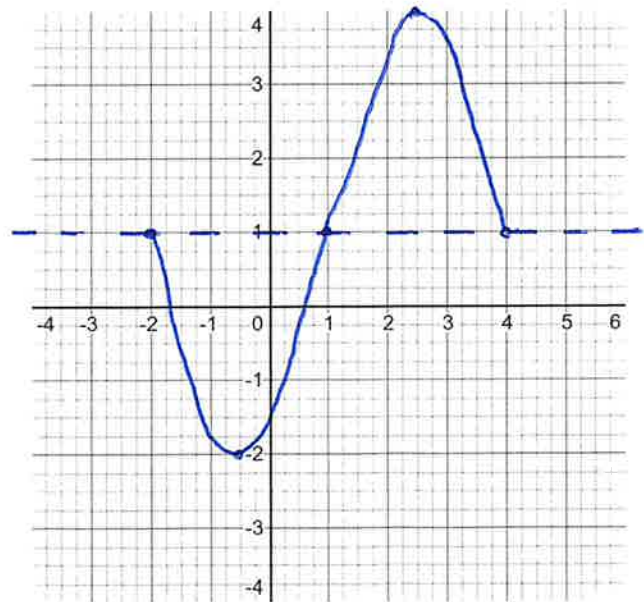
PS: -2

$P: \frac{2\pi}{\frac{\pi}{3}} \rightarrow 6$

Location of Quadrants:

$\frac{6}{4} = \frac{3}{2}$ every $\frac{3}{2}$

VD: $+1$



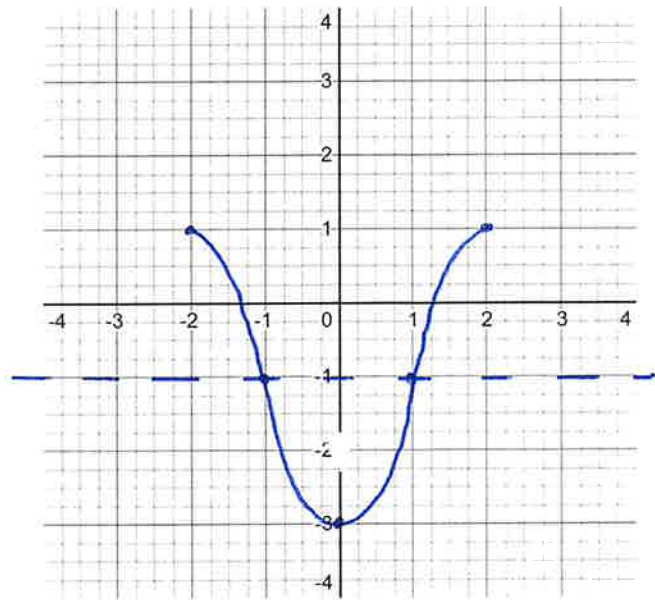
7. Accurately sketch at least one full Period of the graph of: $y = 2 \cos\left(\frac{\pi}{2}x + \pi\right) - 1$

$$y = 2 \cos\left(\frac{\pi}{2}x + \pi\right) - 1$$

$$= 2 \cos\frac{\pi}{2}(x+2) - 1$$

A: 2
 PS: -2
 VD: -1
 P: $\frac{2\pi}{\frac{\pi}{2}} = 4$

Quadrantal points
 occur every
 $\frac{4}{4} = 1$



8. Find a function in the form $y = a \sin bx + c$ where there is a maximum point at (2, 3) and the next closest minimum point is at (6, -7)

max to min represent half a period (x-values) quadrantal every 2

$6 - 2 = 4$ so $P = 8$

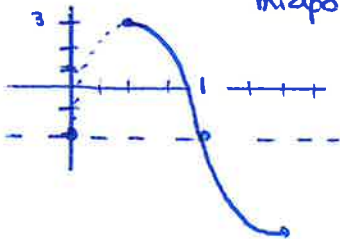
y-values are amplitude

$b = \frac{2\pi}{8} = \frac{\pi}{4}$

midpoint is

$\frac{-7+3}{2} = -2$

VD: -2
 A: 5
 PS: 0



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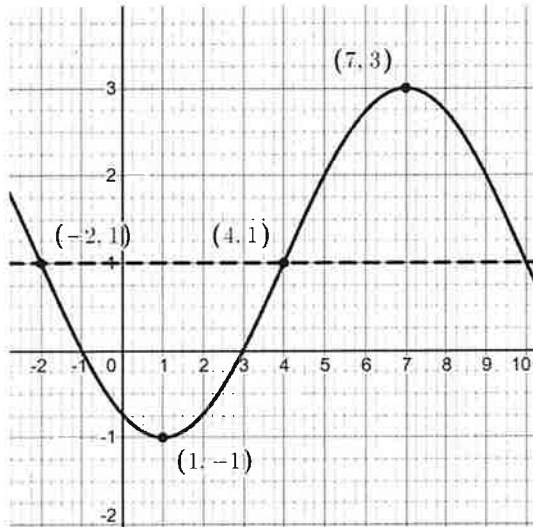
$$y = 5 \sin \frac{\pi}{4}x - 2$$

9. Find a function in the form $y = a \cos bx + c$ where there is a maximum point at (2, 3) and the next closest minimum point is at (6, -7)

consider all values for # 8
 but PS: +2

$$y = 5 \cos \frac{\pi}{4}(x-2) - 2$$

10. a) The graph below describes the function $y = a \sin b(x - c) + d$. Write a sine equation to describe the graph if:
 i) $a > 0$ and ii) $a < 0$



i) $A: 2$ PS: $+4$ VD: $+1$ P: 12
 $b = \frac{\pi}{6}$

$$2 \sin \frac{\pi}{6}(x-4) + 1$$

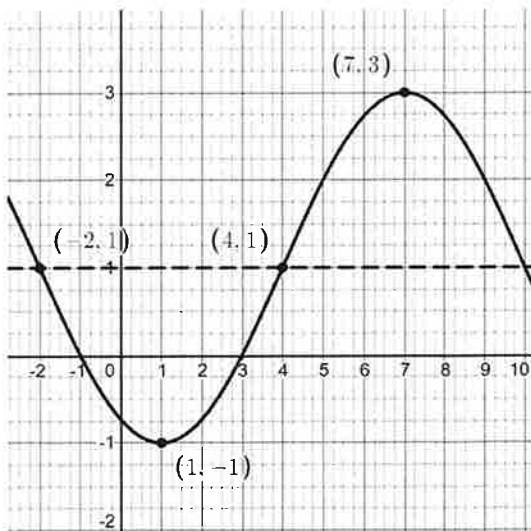
ii) $A: 2$ PS: -2 or $+10$ VD: $+1$ P: 12
 $b = \frac{\pi}{6}$

$$-2 \sin \frac{\pi}{6}(x+2) + 1$$

or

$$-2 \sin \frac{\pi}{6}(x-10) + 1$$

b) The graph can also be described as a function $y = a \cos b(x - c) + d$. Write a cosine equation to describe the graph if:
 i) $a > 0$ and ii) $a < 0$



$A: 2$ VD: 1 P: 12 $b = \frac{\pi}{6}$

iii) PS: $+7$

$$2 \cos \frac{\pi}{6}(x-7) + 1$$

iv) PS: $+1$

$$-2 \cos \frac{\pi}{6}(x-1) + 1$$

See Website for Detailed Answer Key

Pre-Calculus 12

Extra Work Space