Section 6.3 – Quadratic Equations

Definition of a Quadratic Equation

An equation that can be written in the form: $ax^2 + bx + c = 0$, where $a, b, and\ c$ are Real Numbers with $a \neq 0$. The values of x that satisfy the equation are called the **solutions or roots of the equation.**

For the quadratic equation $x^2 - 2x - 3 = 0$, the solutions would be -1 and 3

Check $x = -1$

$x^2 - 2x - 3 = 0$

$(-1)^2 - 2(-1) - 3 \stackrel{?}{=} 0$

$1 + 2 - 3 \stackrel{?}{=} 0$

$0 = 0$

Check $x = 3$

$x^2 - 2x - 3 = 0$

$(3)^2 - 2(3) - 3 \stackrel{?}{=} 0$

$9 - 6 - 3 \stackrel{?}{=} 0$

$0 = 0$

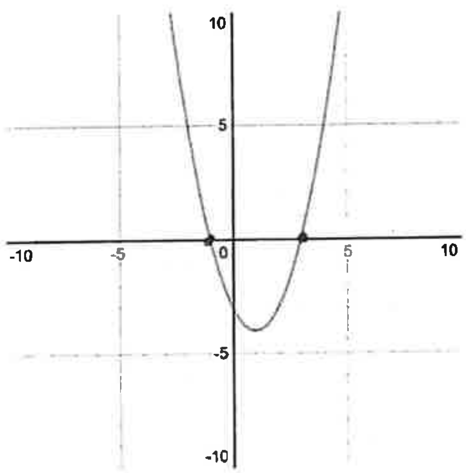
Soln

$x = -1$ and 3

$x = \{-1, 3\}$

Quadratic equations can be solved by graphing the corresponding function $f(x) = ax^2 + bx + c$ and determining the x-coordinates of the **x-intercepts** on the graph. These x-coordinates are referred to as the **zeros** of the function. The **zeros** of a function are the **solutions or roots** of the corresponding equation.

<u>**Example 1:**</u> Solve $x^2 - 2x - 3 = 0$ by graphing using Desmos.



Corresponding Function

$f(x) = x^2 - 2x - 3$

$\shortparallel$

$0 = x^2 - 2x - 3$

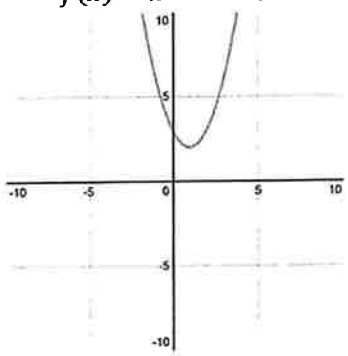
The values of x where $f(x) = 0$ will solve this equation

$f(x) = 0$ at the x-intercepts on the graph.

$x = -1$ and $x = 3$

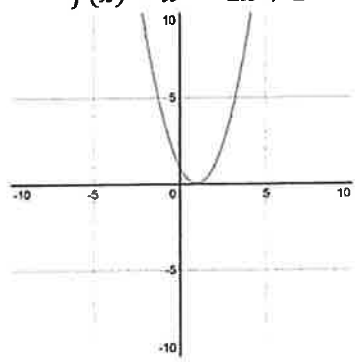
A quadratic function can cross the x-axis either $0, 1, or\ 2\ times$, **therefore the corresponding quadratic equation can have 0, 1, or 2 solutions.**

$f(x) = x^2 - 2x + 3$



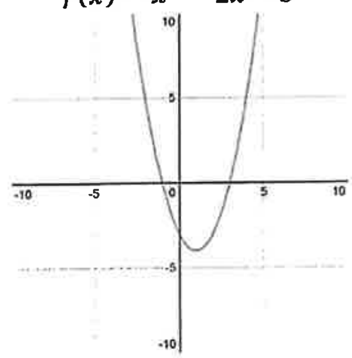
No zeros
(No real roots)

$f(x) = x^2 - 2x + 1$



One zero
(Double root/two roots equal)

$f(x) = x^2 - 2x - 3$



Two zeros
(Two unequal real roots)

Factoring Quadratics in the Form $x^2 + bx + c$

Consider this: $(x\ +\ a)(x\ +\ b)\ = x^2 + bx\ +\ ax\ +\ ab$

$$x^2 + (b\ + a)x\ +\ ab$$

- By looking at this we see that:
  - The first term is the product of x and x
  - The **coefficient of the middle term** is the **sum** of a and b
  - The last term is the **product** of a and b
- This leads us to the **general rule:**

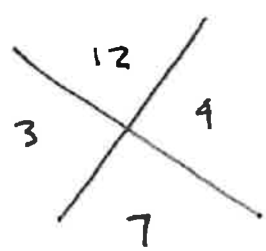
When factoring $x^2 + bx\ +\ c$, look for **two factors of c**, that **multiply** to the **coefficient of the last term,** and **add** to the **coefficient of the middle term.**

Example: Factor $x^2 + 7x\ +\ 12$

Solution: What two numbers **add to 7** and **multiply to 12**?

$x^2 + 7x + 12$

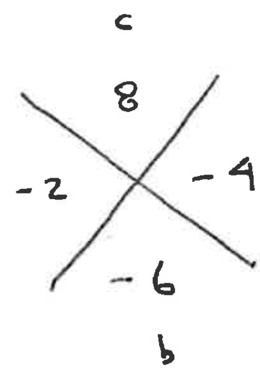
$= (x + 3)(x + 4)$



Example: Factor $x^2 + 8 - 6x$

Solution: First arrange the polynomial in descending order of powers

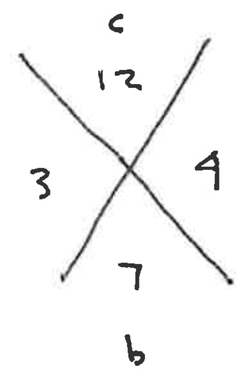
$$x^2 + 8 - 6x$$
$$= x^2 - 6x + 8$$
$$= (x-2)(x-4)$$



Example: Factor $5x^2 + 35x + 60$

Solution: always factor GCF

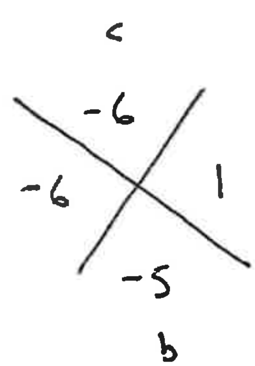
$$5x^2 + 35x + 60$$
$$= 5(x^2 + 7x + 12)$$
$$= 5(x+3)(x+4)$$



Example: Factor $-x^2 + 5x + 6$

Solution: First factor out -1, so that the coefficient of x^2 becomes $+1$.

$$-x^2 + 5x + 6$$
$$= -1(x^2 - 5x - 6)$$
$$= -1(x-6)(x+1)$$



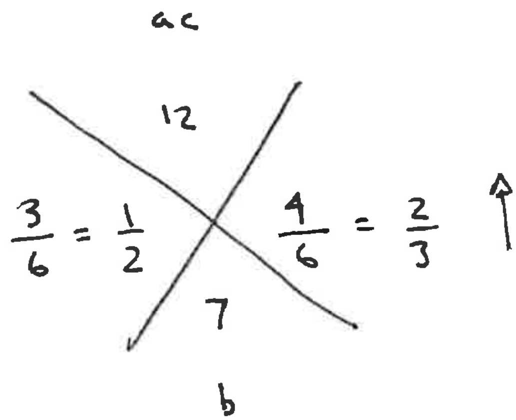
Factoring Quadratics in the Form $ax^2 + bx + c$

The AC method

Example: Factor $\overset{a}{6x^2} + \overset{b}{7x} + \overset{c}{2}$

$$6x^2 + 7x + 2$$

$$= (2x+1)(3x+2)$$



Principle of Zero Products:

$$AB = 0$$

$$A = 0 \quad \text{or} \quad B = 0$$

Combining the principle of zero products with factoring, we can solve quadratic equations by factoring.

Example: Solve the equation $x^2 - x - 6 = 0$

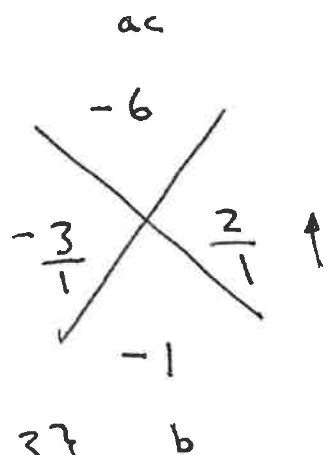
Solution:

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 3 \qquad x = -2$$



$$x = \{-2, 3\}$$

Example: Solve the equation $3x^2 + 9x = 0$

Solution:

$$3x^2 + 9x = 0$$

$$3x(x+3) = 0$$

$$3x = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 0 \qquad x = -3$$

$$x = \{-3, 0\}$$

Example: Solve the equation $6x^2 - 7x - 5 = 0$

Solution:

$$6x^2 - 7x - 5 = 0$$

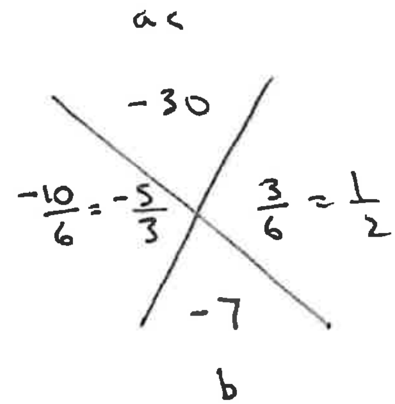
$$(3x-5)(2x+1) = 0$$

$$3x - 5 = 0 \quad \text{or} \quad 2x + 1 = 0$$

$$3x = 5 \qquad 2x = -1$$

$$x = \frac{5}{3} \qquad x = -\frac{1}{2}$$

$$x = \left\{ -\frac{1}{2}, \frac{5}{3} \right\}$$



Example: Solve the equation $x(3x + 1) = 2$

Solution:

$$x(3x+1) = 2$$

$$3x^2 + x = 2$$

$$3x^2 + x - 2 = 0$$

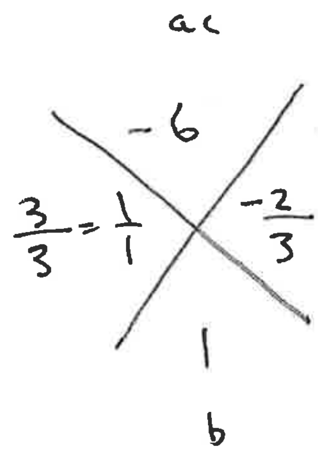
$$(x+1)(3x-2) = 0$$

$$x + 1 = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$x = -1 \quad \text{or} \quad 3x = 2$$

$$x = \frac{2}{3}$$

$$x = \left\{ -1, \frac{2}{3} \right\}$$



Example: $\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{x^2-4x-5}$

Solution:

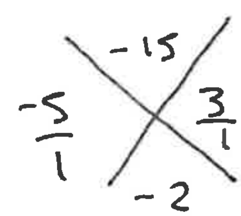
$$\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{(x-5)(x+1)}$$

$$(x-5)(x+1)\left[\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{(x-5)(x+1)}\right]$$

$$(x+1)x - (x-5)3 = 30$$

$$x^2 + x - 3x + 15 = 30$$

$$x^2 - 2x - 15 = 0$$



$$(x-5)(x+3) = 0$$

$$x-5=0 \quad \text{or} \quad x+3=0$$

$$x=5 \quad \text{or} \quad x=-3$$

Check with original soln $\Rightarrow$ reject $x=5$

Soln $x=-3$

Practice Questions

4 - 15

Section 6.3 – Practice Questions

1. Match the zeros of each function on the left with the solutions on the right

$f(x) = (x-1)^2$	________________	$\emptyset$
$g(x) = x^2 - 1$	________________	1
$h(x) = x^2 + 1$	________________	$-1, 1$

2. Match the zeros of each function on the left with the solutions on the right

		No Roots
$f(x) = (2x-1)^2$	________________	$\frac{1}{2}$
$g(x) = 4x^2 - 1$	________________	$-\frac{1}{2}, \frac{1}{2}$
$h(x) = 4x^2 + 1$	________________	

3. Match the $x-intercepts$ of each function on the left with the solution on the right

$f(x) = 4x^2 - 9$		*No intercepts*
$g(x) = 4x^2 + 9$		$-\frac{3}{2}$
$h(x) = 4x^2 - 12x + 9$		$\frac{3}{2}$
$i(x) = 4x^2 + 12x + 9$		$-\frac{3}{2}, \frac{3}{2}$
$j(x) = 9x^2 - y^2$		$-\frac{y}{3}$
$k(x) = 9x^2 + y^2$		$\frac{y}{3}$
$l(x) = 9x^2 - 6xy + y^2$		$\frac{y}{3}, -\frac{y}{3}$
$l(x) = 9x^2 + 6xy + y^2$		

Solve each quadratic equation by factoring. Check your solutions.

4. $x^2 - 3x = 0$

Check Solutions:

5. $2z^2 - 32 = 0$

Check Solutions:

6. $2x^2 - x - 6 = 0$

Check Solutions:

7. $6x^2 - 11x = -3$

Check Solutions:

8. $(2x - 1)(3x + 2) = 24$

Check Solutions:

9. $5x^2 = 8x$

Check Solutions:

10. $12y^2 - 4y = 5$

Check Solutions:

11. $2x^2 + 12x = -10$

Check Solutions:

12. $3x^2 - 8x = 9 - 2x$

Check Solutions:

13. $10x^2 - 23x = -12$

Check Solutions:

14. $\frac{3}{x-1} + x = 5$

Check Solutions:

15. $\frac{1}{x} - x = \frac{8}{3}$

Check Solutions: