Section 6.3 – Practice Problems

1. Find the reference angle for each given angle in standard position

a) $150°$ Q2 $180 - 150 = \boxed{30°}$

b) $-150°$ $-150 + 360 = 210°$ Q3: $210 - 180 = \boxed{30°}$

c) $314°$ Q4 $360 - 314 = \boxed{46°}$

d) $-314°$ $-314 + 360 = \boxed{46°}$

e) $612°$ $612 - 360 = 252°$ Q3 $252 - 180 = \boxed{72°}$

f) $-537°$ $-537 + 2(360) = 183°$ Q3 $183 - 180 = \boxed{3°}$

g) $1100°$ $1100 - 3(360) = \boxed{20°}$

h) $6325°$ $6325 - 17(360) = 205$ Q3: $205 - 180 = \boxed{25°}$

i) $810°$ $810 - 2(360) = 90°$ $\boxed{90°}$

j) $-900°$ $-900 + 3(360) = 180°$ $180°$ has ref angle of $\boxed{0°}$

k) $\frac{7\pi}{6}$ $\frac{7\pi}{6}$ Q3 $\frac{7\pi}{6} - \pi = \boxed{\frac{\pi}{6}}$

l) $-\frac{21\pi}{4}$ $-\frac{21\pi}{4} + 3\left(\frac{8\pi}{4}\right) = \frac{3\pi}{4}$ $2\pi = \frac{8\pi}{4}$ Q2: $\pi - \frac{3\pi}{4} = \boxed{\frac{\pi}{4}}$

m) $-\frac{19\pi}{5}$ $-\frac{19\pi}{5} + 2\left(\frac{10\pi}{5}\right)$ $2\pi = \frac{10\pi}{5}$ $\boxed{\frac{\pi}{5}}$

n) $\frac{24\pi}{7}$ $\frac{24\pi}{7} - \frac{14\pi}{7} = \frac{10\pi}{7}$ $2\pi = \frac{14\pi}{7}$ Q3: $\frac{10\pi}{7} = \pi + \frac{3\pi}{7}$ so $\boxed{\frac{3\pi}{7}}$

o) $\frac{17\pi}{3}$ $\frac{17\pi}{3} - 2\left(\frac{6\pi}{3}\right)$ $2\pi = \frac{6\pi}{3}$ $\frac{5\pi}{3}$ $\frac{5\pi}{3} \Rightarrow$ Q4 $\frac{6\pi}{3} - \frac{5\pi}{3}$ $\boxed{\frac{\pi}{3}}$

p) $\frac{16\pi}{5}$ $\frac{16\pi}{5} - \frac{10\pi}{5}$ $2\pi = \frac{10\pi}{5}$ $\frac{6\pi}{5}$ $\rightarrow$ Q3 $\pi + \frac{\pi}{5} = \frac{6\pi}{5}$ $\boxed{\frac{\pi}{5}}$

1 Rotation is 2π or 360

For Radians consider Denominator Ex: $2\pi = \frac{8\pi}{4}$

2. Determine the exact value of each trigonometric function, no calculator needed.

a) $\sin 120°$ Ref angle $60°$ Q2

$\sin 120 = \frac{\sqrt{3}}{2}$

b) $\cot 135°$ Ref angle $45°$ Q2

$\tan 45° = \frac{1}{1}$ $\cot 45 = \frac{-1}{1}$ -1

c) $\cos 330°$ Ref angle $30°$ Q4

$\cos 330 = \frac{\sqrt{3}}{2}$

d) $\tan 660°$ Ref angle $60°$ Q4

$\tan 660° = -\sqrt{3}$

e) $\csc 1125°$ Ref angle $45°$ Q1

$\csc 1125° = \sqrt{2}$

f) $\sec \frac{\pi}{6}$ Q1 $\frac{\pi}{6} \approx 30°$ $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

so $\sec \frac{\pi}{6} = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

g) $\sin \frac{5\pi}{4}$ Q3 Ref angle $\frac{\pi}{4}$

$\sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

h) $\tan \frac{11\pi}{6}$ Q4 ref angle $\frac{\pi}{6}$

$\tan \frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

i) $\csc \frac{19\pi}{6}$ Ref angle $\frac{\pi}{6}$ in Q3

$\frac{19\pi}{6} - \frac{12\pi}{6} = \frac{7\pi}{6}$

$\csc \frac{19\pi}{6} = -2$

j) $\cot \frac{13\pi}{3}$ Ref angle $\frac{\pi}{3}$ Q1

$\frac{13\pi}{3} - \frac{6\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$

$\cot \frac{13\pi}{3} = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

k) $\cot(-240°)$ Q2 ref angle $60°$

$-240 + 360 = 120°$

$\cot(-240) = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

l) $\sec(-945°)$ Q2 Ref angle $45°$

$-945 + 3(360) = 135°$

$\sec(-945) = -\sqrt{2}$

m) $\cos(-\frac{5\pi}{3})$ Q1 Ref angle $\frac{\pi}{3}$

$-\frac{5\pi}{3} + \frac{6\pi}{3} = \frac{\pi}{3}$

$\cos(-\frac{5\pi}{3}) = \frac{1}{2}$

n) $\tan(-\frac{29\pi}{6})$ Q3 Ref angle $\frac{\pi}{6}$

$-\frac{29\pi}{6} + \frac{12\pi}{6} + \frac{12\pi}{6} + \frac{12\pi}{6} = \frac{7\pi}{6}$

$\tan(-\frac{29\pi}{6}) = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

o) $\sin(-\frac{20\pi}{3})$ Q3 Ref angle $\frac{\pi}{3}$

$-\frac{20\pi}{3} + 4(\frac{6\pi}{3}) = \frac{4\pi}{3}$

$\sin(-\frac{20\pi}{3}) = -\frac{\sqrt{3}}{2}$

p) $\csc(-\frac{27\pi}{4})$ Q3 Ref angle $\frac{\pi}{4}$

$-\frac{27\pi}{4} + 4(\frac{8\pi}{4}) = \frac{5\pi}{4}$

$\csc(-\frac{27\pi}{4}) = -\sqrt{2}$

3. For which value(s) of θ, $0° \le \theta < 360°$ is each of the following trig functions undefined?

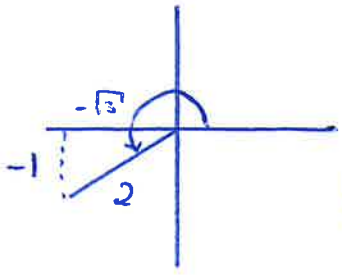
a) $\sin\theta$ undefined when $r = 0$ NONE	b) $\cos\theta$ undefined when $r = 0$ NONE
c) $\tan\theta$ undefined when $x = 0$ $90°$ and $270°$	d) $\cot\theta$ undefined when $y = 0$ $0°$ and $180°$
e) $\sec\theta$ undefined when $x = 0$ $90°$ and $270°$	f) $\csc\theta$ undefined when $y = 0$ $0°$ and $180°$

4. For which value(s) of θ, $0 \le \theta < 2\pi$ is each of the following trig functions undefined? same logic as #3

a) $\sin\theta$ NONE	b) $\cos\theta$ NONE
c) $\tan\theta$ $\frac{\pi}{2}$ and $\frac{3\pi}{2}$	d) $\cot\theta$ 0 and π
e) $\sec\theta$ $\frac{\pi}{2}$ and $\frac{3\pi}{2}$	f) $\csc\theta$ 0 and π

5. Find the smallest positive θ in degrees (drawings help), for which:

a) $\sin\theta = -\frac{1}{2}$

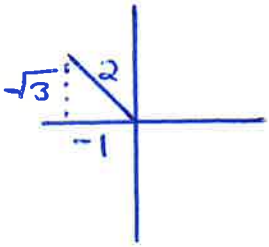


Ref angle $30°$

$\theta = 180 + 30$

$\theta = 210°$

b) $\tan\theta = -\sqrt{3}$

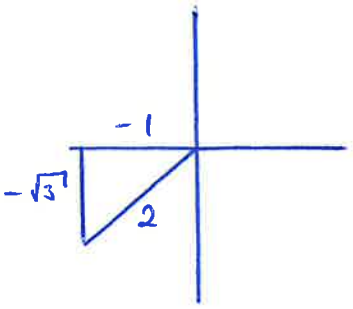


Ref angle $60°$

$\theta = 180 - 60$

$\theta = 120°$

c) $\csc\theta = -\frac{2}{\sqrt{3}}$ ← r, ← y

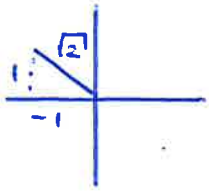


Ref angle 60°

$\theta = 180 + 60$

$\theta = 240°$

d) $\sec\theta = -\frac{\sqrt{2}}{1}$ ← r, ← x

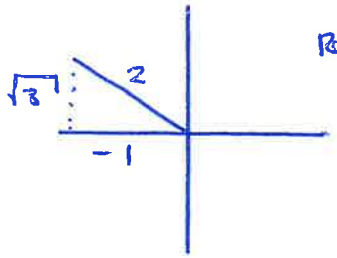


Ref angle 45°

$\theta = 180 - 45$

$\theta = 135°$

e) $\cot\theta = -\frac{1}{\sqrt{3}}$ ← x, ← y

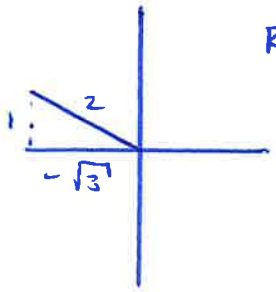


Ref angle 60°

$\theta = 180° - 60°$

$\theta = 120°$

f) $\cos\theta = -\frac{\sqrt{3}}{2}$ ← x, ← r



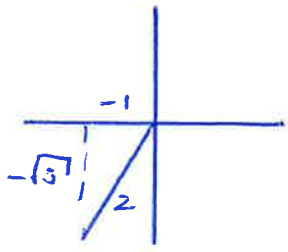
Ref angle 30°

$\theta = 180 - 30$

$\theta = 150°$

6. Find the smallest positive x in radians, for which:

a) $\sin x = -\frac{\sqrt{3}}{2}$

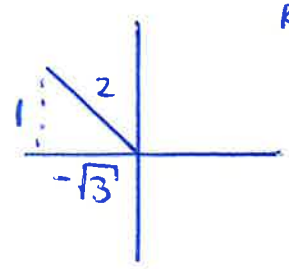


Ref angle $\frac{\pi}{3}$

$x = \pi + \frac{\pi}{3}$

$x = \frac{4\pi}{3}$

b) $\cot x = \frac{-\sqrt{3}}{1}$



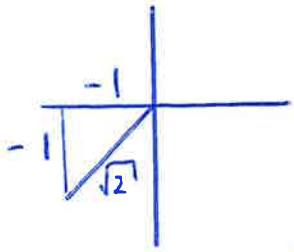
Ref angle $\frac{\pi}{6}$

$x = \pi - \frac{\pi}{6}$

$= \frac{6\pi}{6} - \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

c) $\csc x = \frac{-\sqrt{2}}{1}$ ← r, ← y

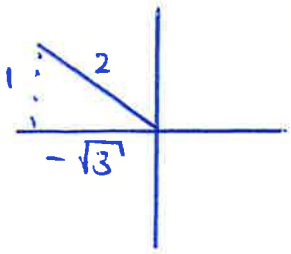


Ref angle $\frac{\pi}{4}$

$x = \pi + \frac{\pi}{4}$

$x = \frac{5\pi}{4}$

d) $\sec x = -\frac{2}{\sqrt{3}}$ ← r, ← x

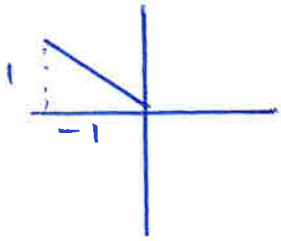


Ref angle: $\frac{\pi}{6}$

$x = \pi - \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

e) $\tan x = \frac{-1}{1}$

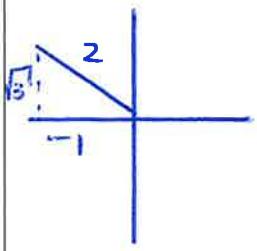


Ref angle $\frac{\pi}{4}$

$x = \pi - \frac{\pi}{4}$

$x = \frac{3\pi}{4}$

f) $\cos x = -\frac{1}{2}$



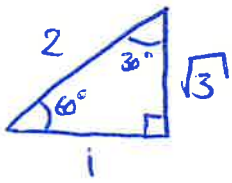
Ref angle $\frac{\pi}{3}$

$x = \pi - \frac{\pi}{3}$

$x = \frac{2\pi}{3}$

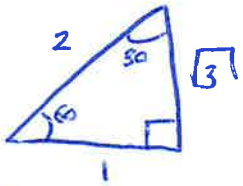
7. Find the exact value of each expression, no calculator needed. Recall $sin^2\theta = (\sin\theta)^2$

a) $\sin 60°$

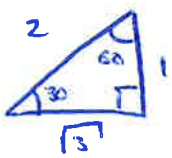


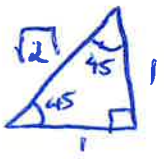
$\frac{\sqrt{3}}{2}$

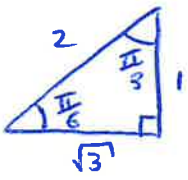
b) $2\sin 30°\cos 30°$

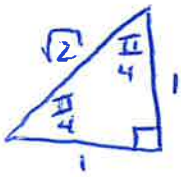


$2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$









c) $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{6}$

$\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4}+\frac{3}{4}=\frac{4}{4}=\boxed{1}$

d) $\sin^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}$

$\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2$

$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=\boxed{1}$

e) $\sec^2 60°-\tan^2 60°$

$\left(\frac{2}{1}\right)^2-\left(\frac{\sqrt{3}}{1}\right)^2$

$\frac{4}{1}-\frac{3}{1}\rightarrow\frac{1}{1}=\boxed{1}$

f) $\csc^2\frac{\pi}{6}-\cot^2\frac{\pi}{6}$

$(2)^2-(\sqrt{3})^2$

$4-3$

$\boxed{1}$

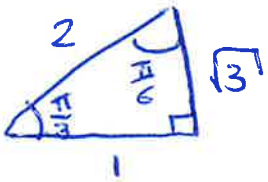
g) $2\sin^2\frac{\pi}{6}$

$2\left(\frac{1}{2}\right)^2$

$2\left(\frac{1}{4}\right)=\boxed{\frac{1}{2}}$

h) $1-\cos\frac{\pi}{3}$

$1-\frac{1}{2}\rightarrow\frac{2}{2}-\frac{1}{2}=\boxed{\frac{1}{2}}$



i) $\tan\frac{\pi}{3}$

$\sqrt{3}$

j) $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

$\dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} \rightarrow \dfrac{\frac{2}{\sqrt{3}}}{\frac{3}{3}-\frac{1}{3}}$

$\frac{2}{\sqrt{3}} \div \frac{2}{3} \rightarrow \frac{2}{\sqrt{3}} \cdot \frac{3}{2} = \frac{3}{\sqrt{3}}$

$\sqrt{3}$

8. Find an angle x such that $x \neq y, 0 \leq x < 2\pi$, and $\sin x = \sin y$ looking Q1 Q2 angle for sine or Q3 Q4

a) $y = \frac{\pi}{6}$

$x = \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

b) $y = \frac{7\pi}{4}$

$\frac{7\pi}{4}$ is in Q4 ref angle $\frac{\pi}{4}$

in Q3 $\pi + \frac{\pi}{4}$ $\frac{5\pi}{4}$

c) $y = \frac{11\pi}{6}$ ← in Q4 ref agl $\frac{\pi}{6}$

Q3 $\pi + \frac{\pi}{6}$

$\frac{7\pi}{6}$

d) $y = \frac{4\pi}{3}$

$\frac{4\pi}{3}$ in Q3 ref angle $\frac{\pi}{3}$

Q4 $2\pi - \frac{\pi}{3}$

$\frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

cos angles match Q1 with Q4
Q2 with Q3

9. Find an angle x such that $x \neq y, 0 \leq x < 2\pi$, and $\cos x = \cos y$

a) $y = \frac{\pi}{6}$ Q1

Q4 $2\pi - \frac{\pi}{6}$

$\frac{12\pi}{6} - \frac{\pi}{6}$

$\boxed{\frac{11\pi}{6}}$

b) $y = \frac{7\pi}{4}$ ← Q4 ref angle $\frac{\pi}{4}$

Q1 is $\boxed{\frac{\pi}{4}}$

c) $y = \frac{7\pi}{6}$ ← Q3 ref angle $\frac{\pi}{6}$

Q2 $\pi - \frac{\pi}{6}$

$\boxed{\frac{5\pi}{6}}$

d) $y = \frac{4\pi}{3}$ ← Q3 ref angle $\frac{\pi}{3}$

Q2 $\pi - \frac{\pi}{3}$

$\boxed{\frac{2\pi}{3}}$

10. Find an angle x such that $x \neq y, 0 \leq x < 2\pi$, and $\tan x = \tan y$

tan angles match Q1 and Q3
Q2 and Q4

a) $y = \frac{\pi}{6}$ Q1

so Q3

$\pi + \frac{\pi}{6}$

$\boxed{\frac{7\pi}{6}}$

b) $y = \frac{7\pi}{4}$ ← Q4

ref angle $\frac{\pi}{4}$

Q2 $\pi - \frac{\pi}{4}$

$\boxed{\frac{3\pi}{4}}$

c) $y = \frac{11\pi}{6}$ ← Q4 ref angle $\frac{\pi}{6}$

Q2 $\pi - \frac{\pi}{6}$

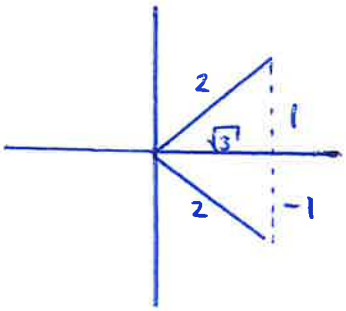
$\frac{5\pi}{6}$

d) $y = \frac{4\pi}{3}$ ← Q3 ref angle $\frac{\pi}{3}$

Q1 $\frac{\pi}{3}$

11. Determine all possible values of x by special angles, $0 \leq x < 2\pi$ Drawings Help!

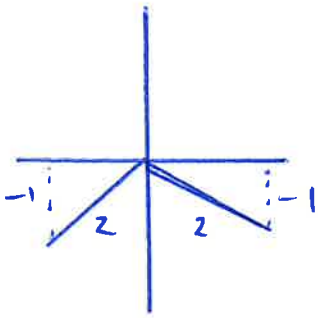
a) $\cos x = \frac{\sqrt{3}}{2}$ Ref angle $\frac{\pi}{6}$



$x = \frac{\pi}{6}$ Q1

$x = \frac{11\pi}{6}$ Q4

b) $\sin x = -\frac{1}{2}$ ref angle $\frac{\pi}{6}$

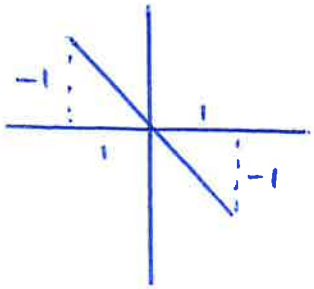


Q3: $x = \frac{7\pi}{6}$

Q4: $x = \frac{11\pi}{6}$

c) $\tan x = -1$

Ref angle $\frac{\pi}{4}$

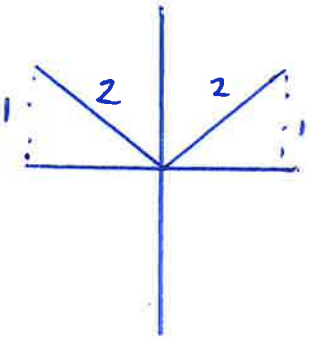


Q2: $\frac{3\pi}{4}$

Q4: $\frac{7\pi}{4}$

d) $\csc x = 2$

Ref angle $\frac{\pi}{6}$

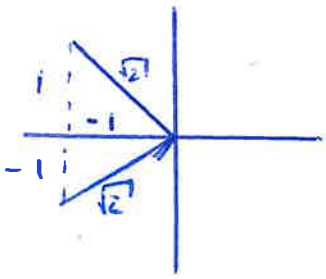


Q1: $\frac{\pi}{6}$

Q2: $\frac{5\pi}{6}$

e) $\sec x = -\sqrt{2}$

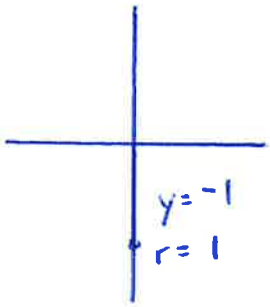
Ref angle $\frac{\pi}{4}$



Q2: $\frac{3\pi}{4}$

Q3: $\frac{5\pi}{4}$

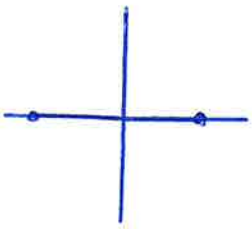
f) $\sin x = -1$



$x = \frac{3\pi}{2}$

g) $\cot x = undefined$

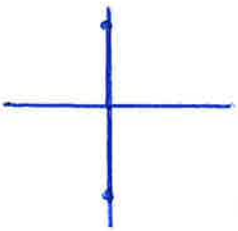
$\frac{x}{y} \leftarrow = 0$



$x = 0$

$x = \pi$

h) $\cos x = 0$

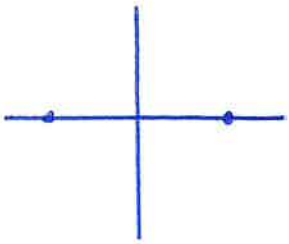


$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

i) $\csc x = undefined$

$\frac{r}{y} \leftarrow 0$

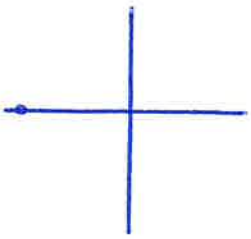


$x = 0$

$x = \pi$

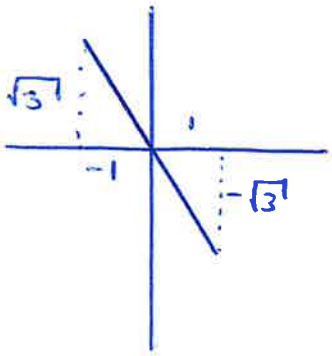
j) $\sec x = -1$

$\frac{r}{x}$ $x = -1$



$x = \pi$

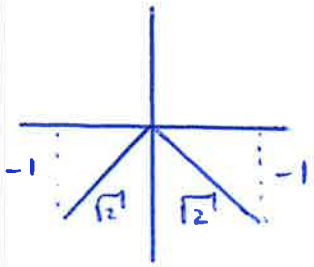
k) $\cot x = -\frac{1}{\sqrt{3}}$ ← x, ← y Ref angle $\frac{\pi}{3}$



$x = \frac{2\pi}{3}$ Q2

$x = \frac{5\pi}{3}$ Q4

l) $\csc x = -\frac{\sqrt{2}}{1}$ ← r, ← y Ref angle $\frac{\pi}{4}$



Q3: $\frac{5\pi}{4}$

Q4: $\frac{7\pi}{4}$

12. Find the exact values of $\sin 3x$ and $\sin\left(\frac{x}{3}\right)$ for the given values of x.

a) $x = 0$

$\sin 3(0) \rightarrow \sin 0 = 0$

$\sin \frac{0}{3} \rightarrow \sin 0 = 0$

b) $x = \frac{\pi}{2}$

$\sin 3\left(\frac{\pi}{2}\right) = \sin \frac{3\pi}{2} = -1$

$\sin \frac{\frac{\pi}{2}}{3} = \sin \frac{\pi}{6} = \frac{1}{2}$

c) $x = -\frac{\pi}{2}$

$\sin 3\left(-\frac{\pi}{2}\right) = \sin -\frac{3\pi}{2}$

$= \sin \frac{\pi}{2} = 1$

$\sin \frac{-\frac{\pi}{2}}{3} = \sin -\frac{\pi}{6} = \sin \frac{5\pi}{6}$

$= -\frac{1}{2}$

d) $x = -\pi$

$\sin(-3\pi) \rightarrow \sin -\pi \Rightarrow \sin \pi = 0$

$\sin -\frac{\pi}{3} \rightarrow \sin \frac{5\pi}{3} \rightarrow$ Q4 Ref angle $\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

13. Find the exact values of $\cos 3x$ and $\cos\left(\frac{x}{3}\right)$ for the given values of x.

a) $x = 0$

$\cos(3)(0) = \cos 0 = 1$

$\cos\left(\frac{0}{3}\right) = \cos 0 = 1$

b) $x = \frac{\pi}{2}$

$\cos \frac{3\pi}{2} = 0$

$\cos \frac{\frac{\pi}{2}}{3} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

c) $x = -\frac{\pi}{2}$

$\cos -\frac{3\pi}{2} \rightarrow \cos \frac{\pi}{2} = 0$

$\cos \frac{-\frac{\pi}{2}}{3} \rightarrow \cos -\frac{\pi}{6} \rightarrow \cos \frac{11\pi}{6}$

$\frac{\sqrt{3}}{2}$

Q4 ref angle $\frac{\pi}{6}$

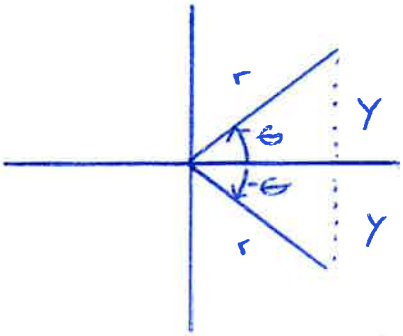
d) $x = -\pi$

$\cos -3\pi \rightarrow \cos \pi = -1$

$\cos -\frac{\pi}{3} \rightarrow \cos \frac{5\pi}{3} = \frac{1}{2}$

14. Choose a variety of angles for $\sin\theta$ and $\sin(-\theta)$. How and when do the value between the two differ or stay the same

consider this



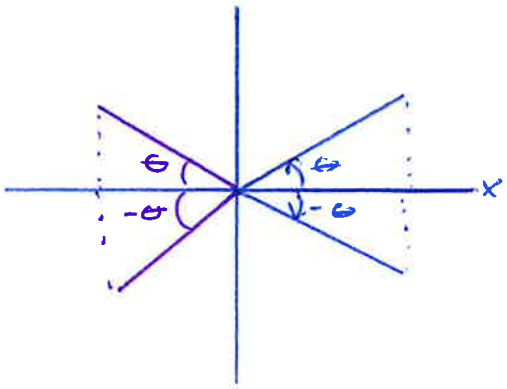
with respect to y we have a reflection in the x-axis

so: $\sin(-\theta) = -\sin\theta$

See Website for Detailed Answer Key

15. Choose a variety of angles for $\cos\theta$ and $\cos(-\theta)$. How and when do the value between the two differ or stay the same

consider this



for cosine the rotation does not affect the x-value

so: $\cos(-\theta) = \cos\theta$

Extra Work Space