Section 6.2 – Practice Problems

1. Find the missing value of the right-angle triangle with sides a, b and hypotenuse c $a^2+b^2=c^2$

a) $a = 5, b = 12, c = ?$

$5^2+12^2=c^2$

$25+144=c^2$

$169=c^2$

$c=13$

b) $a = 2\ b = 3\ c = ?$

$2^2+3^2=c^2$

$4+9=c^2$

$13=c^2$

$c=\sqrt{13}$

c) $a = 15, c = 17, b = ?$

have c so $c^2-a^2=b^2$

$17^2-15^2=b^2$

$289-225=b^2$

$64=b^2$

$b=8$

d) $b = 2\sqrt{2}, c = 3, a = ?$

$c^2-b^2=a^2$

$3^2-(2\sqrt{2})^2=a^2$

$9-4(2)=a^2$

$9-8=a^2$

$a=1$

e) $c = 3\sqrt{5}, b = 6, a = ?$

$c^2-b^2=a^2$

$(3\sqrt{5})^2-6^2=a^2$

$45-36=a^2$

$9=a^2$

$a=3$

f) $c = \sqrt{17}, a = 2\sqrt{2}, b = ?$

$c^2-a^2=b^2$

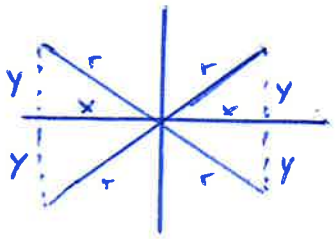
$\sqrt{17}^2-(2\sqrt{2})^2=b^2$

$17-8=b^2$

$9=b^2$

$b=3$

2. Determine the Quadrant in which θ is found, given the following information.



$\sin\theta=\frac{y}{r}$ $\tan\theta=\frac{y}{x}$

$\cos\theta=\frac{x}{r}$

a) $\sin\theta > 0, \quad \sec\theta > 0$

$\frac{y}{r}$ $\frac{r}{x}$

Q1,Q2 Q1,Q4

Q1

b) $\tan\theta < 0, \quad \cos\theta > 0$

Q2,Q4 Q1,Q4

Q4

$\cot\theta=\frac{x}{y}$

$\sec\theta=\frac{r}{x}$

$\csc\theta=\frac{r}{y}$

c) $\csc\theta > 0,\ \cot\theta < 0$

Q1, Q2 Q2, Q4

Q2

d) $\cos\theta < 0,\ \csc\theta < 0$

Q2, Q3 Q3, Q4

Q3

e) $\sin\theta < 0,\ \tan\theta < 0$

Q3, Q4 Q2, Q4

Q4

f) $\cot\theta > 0,\ \sec\theta < 0$

Q1, Q3 Q2, Q3

Q3

g) $\tan\theta < 0,\ \csc\theta > 0$

Q2, Q4 Q1 Q2

Q2

h) $\cos\theta > 0,\ \sec\theta < 0$

Q1, Q4 Q2, Q3

NO SOLUTION

i) $\sin\theta < 0,\ \cot\theta < 0$

Q3, Q4 Q2, Q4

Q4

j) $\tan\theta < 0,\ \sec\theta > 0$

Q2, Q4 Q1, Q4

Q4

3. Find the value of the indicated function

a) If $\csc\theta = \frac{2}{1}$, $\sin\theta = ?$

$\csc\theta = \frac{r}{y}$ $\sin\theta = \frac{y}{r} = \frac{1}{2}$

b) If $\cos\theta = -\frac{2}{3}$, $\sec\theta = ?$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x} = -\frac{3}{2}$

c) If $\tan\theta = \frac{-5}{1}$, $\cot\theta = ?$

$-\frac{1}{5}$

d) If $\sin\theta = \frac{-0.23}{1}$, $\csc\theta = ?$

$\frac{1}{-0.23} = -4.35$

e) If $\sec\theta = \frac{2.35}{1}$, $\cos\theta = ?$

$\frac{r}{x}$ $\frac{1}{2.35} = 0.426$

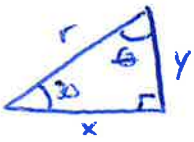
f) If $\cot\theta = \frac{-2.4}{1}$, $\tan\theta = ?$

$\frac{1}{-2.4} = -0.42$

All corresponding angles add to 90° or $\frac{\pi}{2}$

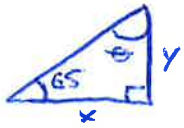
4. Find the acute angle θ, given the following information for the trigonometric functions

a) $\sin 30^\circ = \cos\theta$ so $\theta = ?$



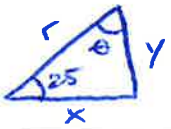
$\theta = 60^\circ$

b) $\tan 65^\circ = \cot\theta$ so $\theta = ?$



$\theta = 25^\circ$

c) $\sec 25^\circ = \csc\theta$ so $\theta = ?$



$\theta = 65^\circ$

d) $\cos\frac{\pi}{4} = \sin\theta$ so $\theta = ?$

$\theta = \frac{\pi}{4}$

e) $\cot\frac{\pi}{6} = \tan\theta$ so $\theta = ?$

$\theta = \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{3\pi}{6} - \frac{\pi}{6}$

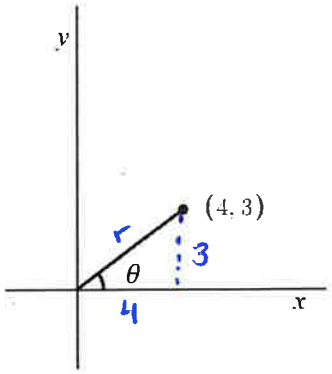
$\frac{2\pi}{6} = \frac{\pi}{3}$

f) $\csc\frac{\pi}{3} = \sec\theta$ so $\theta = ?$

$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

5. Given the point on the Terminal Arm in Standard Position, Evaluate all six trigonometric functions

a)



$3^2 + 4^2 = r^2$

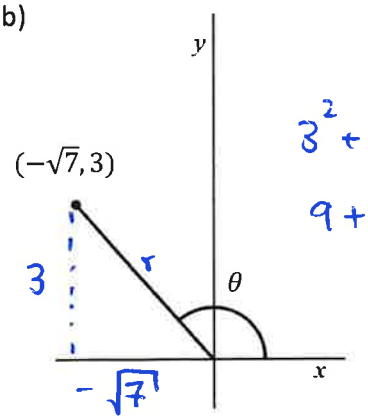
$9 + 16 = r^2$

$25 = r^2$

$5 = r$

$\sin\theta =$ $3/5$	$\cos\theta =$ $4/5$	$\tan\theta =$ $3/4$
$\csc\theta =$ $5/3$	$\sec\theta =$ $5/4$	$\cot\theta =$ $4/3$

b)



$3^2 + (-\sqrt{7})^2 = r^2$

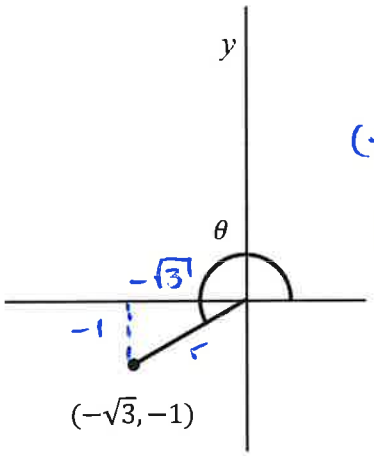
$9 + 7 = r^2$

$16 = r^2$

$4 = r$

$\sin\theta =$ $3/4$	$\cos\theta =$ $-\sqrt{7}/4$	$\tan\theta =$ $-3/\sqrt{7}$
$\csc\theta =$ $4/3$	$\sec\theta =$ $-4/\sqrt{7}$	$\cot\theta =$ $-\sqrt{7}/3$

c)



$(-1)^2 + (-\sqrt{3})^2 = r^2$

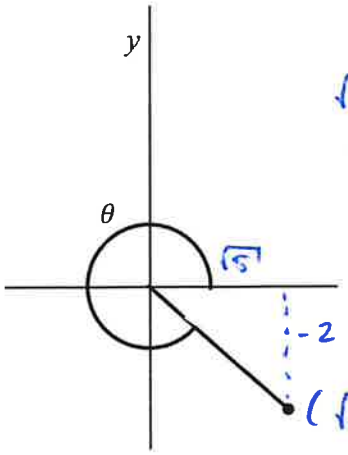
$1 + 3 = r^2$

$4 = r^2$

$2 = r$

$\sin\theta =$ $-1/2$	$\cos\theta =$ $-\sqrt{3}/2$	$\tan\theta =$ $1/\sqrt{3}$
$\csc\theta =$ -2	$\sec\theta =$ $-2/\sqrt{3}$	$\cot\theta =$ $\sqrt{3}$

d)



$(\sqrt{5})^2 + (-2)^2 = r^2$

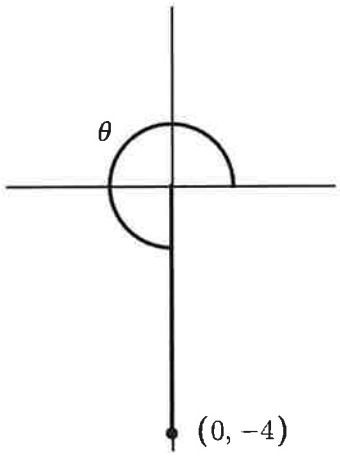
$5 + 4 = r^2$

$9 = r^2$

$3 = r$

$\sin\theta = -\frac{2}{3}$	$\cos\theta = \frac{\sqrt{5}}{3}$	$\tan\theta = -\frac{2}{\sqrt{5}}$
$\csc\theta = -\frac{3}{2}$	$\sec\theta = \frac{3}{\sqrt{5}}$	$\cot\theta = -\frac{\sqrt{5}}{2}$

e)



$r^2 = 0^2 + (-4)^2$

$r^2 = 16$

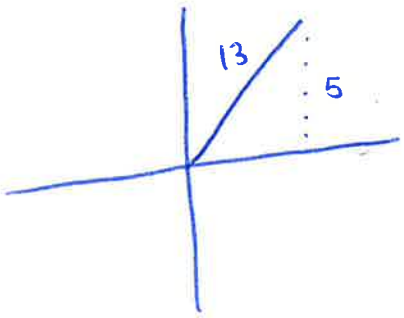
$r = 4$

$\sin\theta = -1$	$\cos\theta = 0$	$\tan\theta =$ undefined
$\csc\theta = -1$	$\sec\theta =$ undefined	$\cot\theta = 0$

6. Given the one trigonometric function, find the other 5.

a) $\sin\theta = \frac{5}{13}$ θ *is in* $Q1$

$\sin\theta = \frac{y}{r}$



$13^2 - 5^2 = x^2$

$169 - 25 = x^2$

$144 = x^2$

$12 = x$

$\sin\theta = \frac{5}{13}$	$\cos\theta = \frac{12}{13}$	$\tan\theta = \frac{5}{12}$
$\csc\theta = \frac{13}{5}$	$\sec\theta = \frac{13}{12}$	$\cot\theta = \frac{12}{5}$

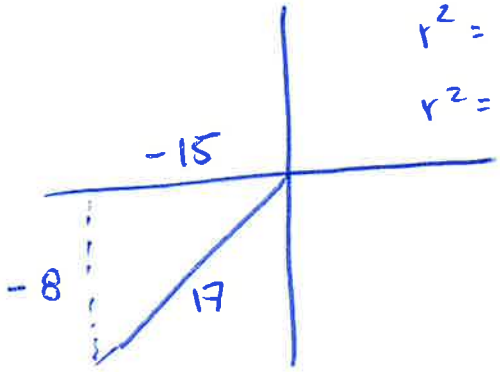
b) $\tan\theta = \frac{8}{15}$ θ *is in* $Q3$

$r^2 = (-8)^2 + (-15)^2$

$r^2 = 64 + 225$

$r^2 = 289$

$r = 17$



$\sin\theta = \frac{-8}{17}$	$\cos\theta = \frac{-15}{17}$	$\tan\theta = \frac{8}{15}$
$\csc\theta = \frac{-17}{8}$	$\sec\theta = \frac{-17}{15}$	$\cot\theta = \frac{15}{8}$

c) $\sec\theta = \frac{3}{2}$ θ *is in Q4*

r

related to cosine

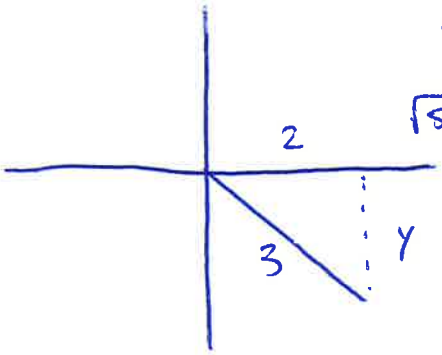
x

$3^2 - 2^2 = y^2$

$9 - 4 = y^2$

$5 = y^2$

$\sqrt{5} = y$ but in Q4 $\rightarrow -\sqrt{5}$



$\sin\theta = -\frac{\sqrt{5}}{3}$	$\cos\theta = \frac{2}{3}$	$\tan\theta = -\frac{\sqrt{5}}{2}$
$\csc\theta = -\frac{3}{\sqrt{5}}$	$\sec\theta = \frac{3}{2}$	$\cot\theta = -\frac{2}{\sqrt{5}}$

d) $\csc\theta = 3$ $\tan\theta < 0$

$\frac{r}{y}$ and positive

so Q1 or Q2 but tan < 0

Q2

$3^2 - 1^2 = x^2$

$9 - 1 = x^2$

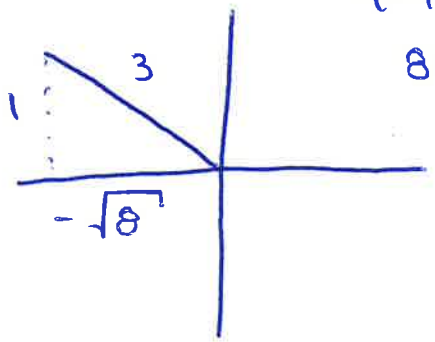
$8 = x^2$

$x = \sqrt{8}$

but Q2

so

$-\sqrt{8}$



$\sin\theta = \frac{1}{3}$	$\cos\theta = -\frac{\sqrt{8}}{3}$	$\tan\theta = -\frac{1}{\sqrt{8}}$
$\csc\theta = 3$	$\sec\theta = -\frac{3}{\sqrt{8}}$	$\cot\theta = -\sqrt{8}$

$\frac{24}{10} = \frac{12}{5}$ ✓

e) $\cot\theta = -2.4 \quad \sin\theta > 0$

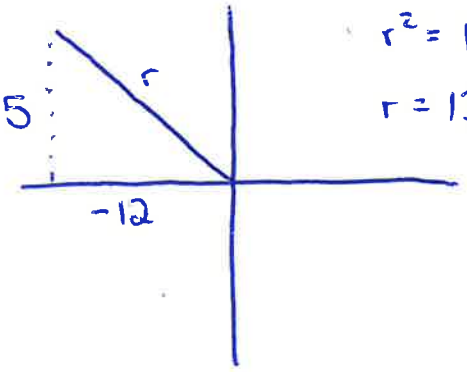
this has to be neg → x, y ; ↓ Q1, Q2

$r^2 = 5^2 + (-12)^2$

$r^2 = 25 + 144$

$r^2 = 169$

$r = 13$



$\sin\theta = \frac{5}{13}$	$\cos\theta = -\frac{12}{13}$	$\tan\theta = -\frac{5}{12}$
$\csc\theta = \frac{13}{5}$	$\sec\theta = -\frac{13}{12}$	$\cot\theta = -2.4$

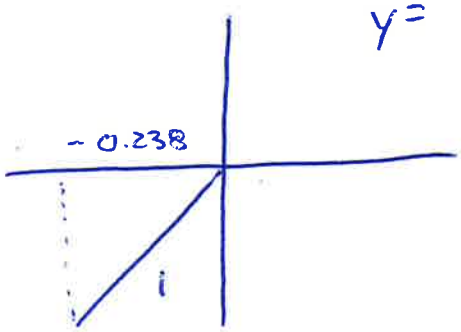
f) $\cos\theta = -0.238 \quad \tan\theta > 0$

↑ Q2 or Q3 ; ↑ Q1, Q3

$y^2 = 1^2 - (-0.238)^2$

$y^2 = 0.943356$

$y = 0.971$ but Q3 so negative -0.971



$\sin\theta = -0.971$	$\cos\theta = -0.238$	$\tan\theta = 4.080$
$\csc\theta = \frac{1}{-0.971} = -1.030$	$\sec\theta = -4.202$	$\cot\theta = 0.2451$

7. Find the six trigonometric functions of θ if θ is an angle created by the Terminal Arm in Standard Position and is located on the cartesian plane according to the given function.

a) $3x + 5y = 0, \quad x \geq 0$

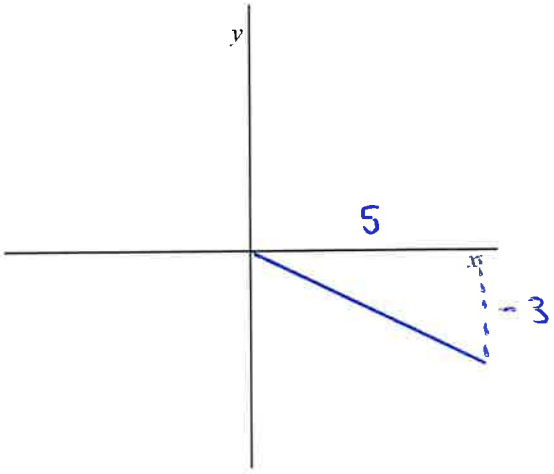
$5y = -3x \rightarrow y = -\frac{3}{5}x$

$5^2 + (-3)^2 = r^2$

$25 + 9 = r^2$

$34 = r^2$

$\sqrt{34} = r$



$\sin\theta = \frac{-3}{\sqrt{34}}$	$\cos\theta = \frac{5}{\sqrt{34}}$	$\tan\theta = -\frac{3}{5}$
$\csc\theta = \frac{\sqrt{34}}{-3}$	$\sec\theta = \frac{\sqrt{34}}{5}$	$\cot\theta = -\frac{5}{3}$

b) $2x - 3y = 0, \quad y \leq 0$

$-3y = -2x$

$y = \frac{2}{3}x$

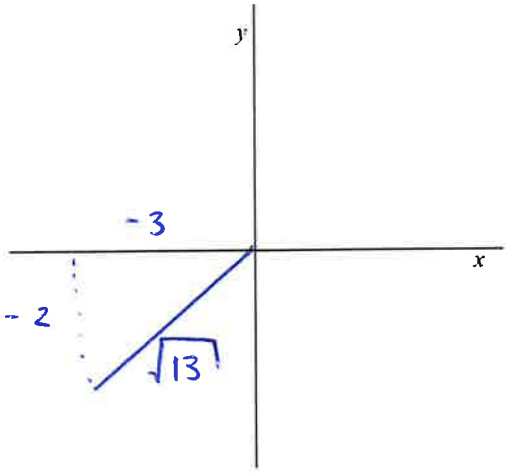
$x^2 + y^2 = r^2$

$(-3)^2 + (-2)^2 = r^2$

$9 + 4 = r^2$

$13 = r^2$

$\sqrt{13} = r$



$\sin\theta = \frac{-2}{\sqrt{13}}$	$\cos\theta = \frac{-3}{\sqrt{13}}$	$\tan\theta = \frac{2}{3}$
$\csc\theta = \frac{\sqrt{13}}{-2}$	$\sec\theta = \frac{\sqrt{13}}{-3}$	$\cot\theta = \frac{3}{2}$

c) $\sqrt{5}x + 2y = 0, \quad y \leq 0$

$2y = -\sqrt{5}x$

$y = -\frac{\sqrt{5}}{2}x$

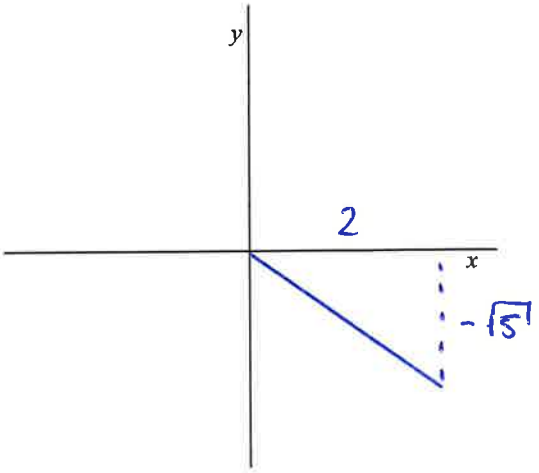
$x^2 + y^2 = r^2$

$2^2 + (-\sqrt{5})^2 = r^2$

$4 + 5 = r^2$

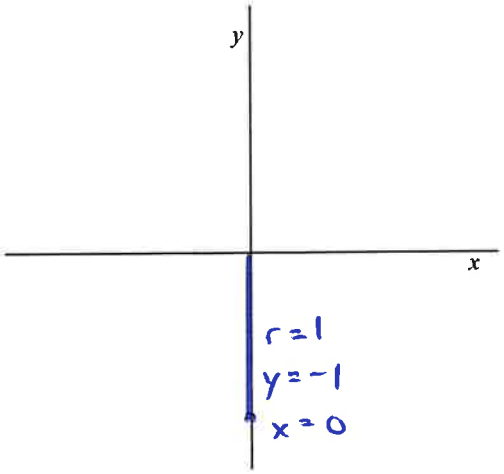
$9 = r^2$

$r = 3$



$\sin\theta = -\frac{\sqrt{5}}{3}$	$\cos\theta = \frac{2}{3}$	$\tan\theta = -\frac{\sqrt{5}}{2}$
$\csc\theta = -\frac{3}{\sqrt{5}}$	$\sec\theta = \frac{3}{2}$	$\cot\theta = -\frac{2}{\sqrt{5}}$

d) $x = 0 \quad y \leq 0$



$\sin\theta = -1$	$\cos\theta = 0$	$\tan\theta =$ undefined
$\csc\theta = -1$	$\sec\theta =$ undefined	$\cot\theta = 0$

8. Determine the coordinates of the point at the given distance from the origin in the stated quadrant, if θ is its position angle.

a) Distance of 10, $Q2$, $\sin\theta = \frac{3}{5}$

$r = 10$

$5^2 - 3^2 = x^2$

$25 - 9 = x^2$

$16 = x^2$

$4 = x$

$Q2$ so $\boxed{-4}$

we have $(-4,3)$ but $r = 5$

$\boxed{(-8,6)}$ so $r = 10$

b) Distance of 3, $Q3$, $\tan\theta = \frac{1}{1}$ in Q3 $(-1,-1)$

$1^2 + 1^2 = r^2$

$2 = r^2$

$r = \sqrt{2}$

$(-1,-1) \rightarrow r = \sqrt{2}$

$\sqrt{2} \cdot z = 3$

$z = \frac{3}{\sqrt{2}}$

$\left(\frac{-3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right) \Rightarrow r = 3$

c) Distance of 8, $Q1$, $\sec\theta = \frac{2}{1}$ $\frac{r}{x}$

$2^2 - 1^2 = y^2$

$4 - 1 = y^2$

$3 = y^2$

$y = \sqrt{3}$

Q1 both positive

$(1, \sqrt{3}) \rightarrow r = 2$

$(4, 4\sqrt{3}) \quad r = 8$

} increase factor of 4

d) Distance of 8, $Q2$, $\csc\theta = \frac{13}{5}$ $\frac{r}{y}$

Q2 y is pos, x is neg

$13^2 - 5^2 = x^2$

$169 - 25 = x^2$

$144 = x^2$

$x = -12$

$(-12,5) \rightarrow r = 13$

$13 \cdot z = 8$

$z = \frac{8}{13}$

$\left(\frac{-96}{13}, \frac{40}{13}\right) \rightarrow r = 8$

9. Let B be an acute angle where $\sin B = a$. Find $\csc B$ and $\cos(90° - B)$ in terms of a.

$\sin B = a$

$\cos(90 - B) = \sin B = a$

$\csc B = \frac{1}{\sin B} = \frac{1}{a}$

10. Let P be an acute angle where $\cos P = b$. Find $\sec P$ and $\sin(\frac{\pi}{2} - P)$ in terms of b

$\cos P = b$

$\sec P = \frac{1}{\cos P} = \frac{1}{b}$

$\sin\left(\frac{\pi}{2} - P\right) = \cos P = b$

11. The terminal side of angle θ in Standard Position, goes through the intersection point of the given curves. Find the intersection point, then find $\sin\theta$ *and* $\cos\theta$

a) $2x - y = 10$ $\quad\sin\theta = -4/5$

$3x + y = 5$ $\quad\cos\theta = 3/5$

$2x - y = 10$

$3x + y = 5$

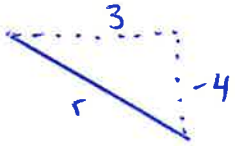
$5x = 15$

$x = 3$

$2(3) - y = 10$

$6 - y = 10$

$y = -4$



$r^2 = 3^2 + (-4)^2$

$r^2 = 9 + 16$

$r^2 = 25 \qquad r = 5$

b) $y = x^2 + 4x$ $\quad\sin\theta = 0$

$y = -4x - 16$ $\quad\cos\theta = -1$

Set equal to each other

$x^2 + 4x = -4x - 16$

$x^2 + 8x + 16 = 0$

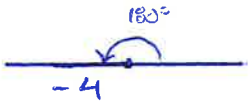
$(x+4)^2 = 0$

$x = -4$

$y = (-4)^2 + 4(-4)$

$y = 16 - 16$

$y = 0$



$r = 4$

$y = 0$

$x = -4$

12. Find all angles of θ, $0 \le \theta < 360°$, where $\sin\theta = \cos\theta$

$\sin\theta = \cos\theta$ when $x = 1, -1$ or $y = 1, -1$

but all angles through 360°

so $\theta = 45°$

45°
135°
225°
315°

] these two only

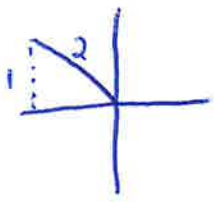
13. If $1 + \sin\theta = 3\sin\theta$, *where* $\tan\theta < 0$. Find $\cos\theta$.

$1 + \sin\theta = 3\sin\theta$

$1 = 2\sin\theta$

$\sin\theta = \frac{1}{2}$ Q2

Q2 or Q4



$2^2 - 1^2 = x^2$

$4 - 1 = x^2$

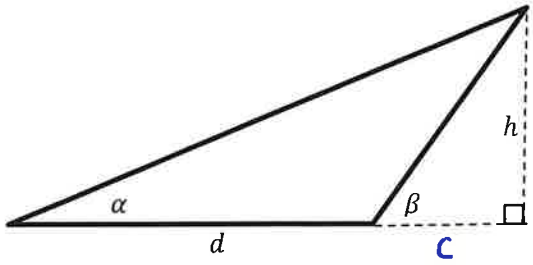
$3 = x^2$

$\sqrt{3} = x$

$\cos\theta = -\frac{\sqrt{3}}{2}$

14. Show that:

$$h = \frac{d}{\cot\alpha - \cot\beta}$$



$\tan\beta = \frac{h}{c}$ so $\cot\beta = \frac{c}{h}$

$\tan\alpha = \frac{h}{d+c}$ so $\cot\alpha = \frac{d+c}{h}$

from $\cot\beta = \frac{c}{h} \rightarrow c = h\cot\beta$

$$\cot\alpha = \frac{d + h\cot\beta}{h}$$

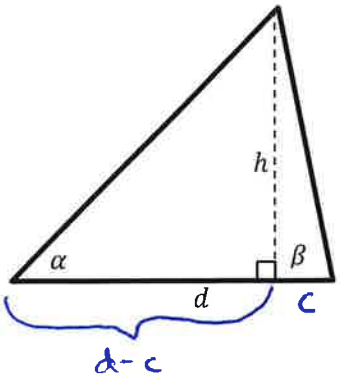
$$h\cot\alpha - h\cot\beta = d$$

$$h(\cot\alpha - \cot\beta) = d$$

$$\boxed{h = \frac{d}{\cot\alpha - \cot\beta}}$$

15. Show that:

$$h = \frac{d}{\cot\alpha + \cot\beta}$$



$\tan\alpha = \frac{h}{d-c} \rightarrow \cot\alpha = \frac{d-c}{h}$

$\tan\beta = \frac{h}{c} \rightarrow \cot\beta = \frac{c}{h}$

$h\cot\beta = c$

$$\cot\alpha = \frac{d - h\cot\beta}{h}$$

$$h\cot\alpha = d - h\cot\beta$$

$$h\cot\alpha + h\cot\beta = d$$

$$h(\cot\alpha + \cot\beta) = d$$

$$\boxed{h = \frac{d}{\cot\alpha + \cot\beta}}$$

See Website for Detailed Answer Key

Extra Work Space