Section 6.2 – General Form of a Quadratic Functions

- Generally quadratic equations come in **General Form** $f(x) = ax^2 + bx + c$ $f(x) = 2x^2 - 4x - 2$
- The good news is that we can change them to **Standard Form** $f(x) = a(x-h)^2 + k$
- We use a technique called **completing the square,** follow the guidelines below $f(x) = 2(x-1)^2 - 4$

How the equation changes	Steps
$f(x) = ax^2 + bx + c$	• Given Standard Form
$y = ax^2 + bx + c$	• Replace $f(x)$ with y to simplify
$y - c = a(x^2 + \frac{b}{a}x)$	• Add $-c$ to both sides • Factor a out of the right side
$y - c + a\left(\frac{b}{2a}\right)^2 = a(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2)$	• Add $\left(\frac{b}{2a}\right)^2$ to the right side • Add $a * \left(\frac{b}{2a}\right)^2$ to the left to balance
$y - c + \frac{b}{4a}^2 = a(x + \frac{b}{2a})^2$	• Simplify to a perfect square
$y = a(x + \frac{b}{2a})^2 + c - \frac{b}{4a}^2$	• Add c and subtract $\frac{b}{4a}^2$ from both sides
$f(x) = a(x - -\frac{b}{2a})^2 + c - \frac{b}{4a}^2$	• Write in the form $f(x) = a(x-h)^2 + k$

The Vertex Formula

The Graph $f(x) = ax^2 + bx + c$, $a \neq 0$ is a parabola with **vertex** (h, k) and **axis of symmetry of** $x = h$,

where $h = -\frac{b}{2a}$ **and** $k = c - \frac{b^2}{4a}$ Vertex $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

If $a > 0$, the parabola has a **minimum value and opens upward**

If $a < 0$, the parabola has a **maximum value and opens downward**

Example: Determine the vertex directly from the equation $f(x) = 2x^2 - 4x - 3$

Solution: $f(x) = 2x^2 - 4x - 3$ has $a = 2$, $b = -4$, *and* $c = -3$

$$Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) \Rightarrow \left(-\frac{-4}{2(2)}, -3 - \frac{(-4)^2}{4(2)}\right) \Rightarrow \left(-\frac{-4}{4}, -3 - \frac{16}{8}\right)$$

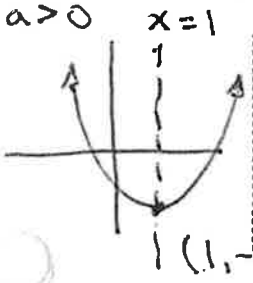
Therefore the vertex is: $(1, -5)$

$$\Rightarrow (-(-1), -3 - 2)$$

$$\Rightarrow (1, -5)$$

Example: Determine the vertex and axis of symmetry for $f(x) = 2x^2 - 4x - 1$

Solution: $f(x) = 2x^2 - 4x - 1$ has $a = 2$, $b = -4$, *and* $c = -1$



$$Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) \Rightarrow \left(-\frac{-4}{2(2)}, -1 - \frac{(-4)^2}{4(2)}\right) \Rightarrow \left(-\frac{-4}{4}, -1 - \frac{16}{8}\right)$$

Therefore the axis of symmetry is: $x = 1$

$$\Rightarrow (-(-1), -1 - 2)$$

$$\Rightarrow (1, -3)$$

Example: Given the following quadratic $f(x) = -2x^2 + 8x - 3$, determine the vertex, axis of symmetry, max/min, domain and range

Solution: $f(x) = -2x^2 + 8x - 3$ has $a = -2$, $b = 8$, *and* $c = -3$

$$Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) \Rightarrow \left(-\frac{8}{2(-2)}, -3 - \frac{(8)^2}{4(-2)}\right) \Rightarrow (2, 5)$$

Therefore the vertex is: $(2, 5)$

Plotting 4 other Points: $f(x) = -2x^2 + 8x - 3$

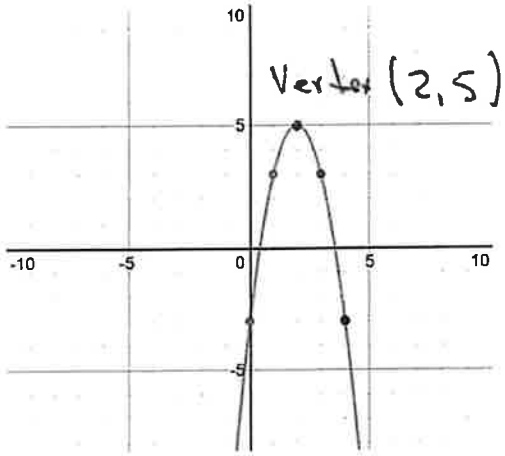
$f(0) = -2(0)^2 + 8(0) - 3 = -3$

$f(1) = -2(1)^2 + 8(1) - 3 = 3$

$f(3) = -2(3)^2 + 8(3) - 3 = 3$

$f(4) = -2(4)^2 + 8(4) - 3 = -3$

x	$f(x)$
0	-3
1	3
3	3
4	-3



Vertex: $(2, 5)$ **Axis of Symmetry:** $x = 2$

Max/Min: Max @ $y = 5$

Domain: all real numbers **Range:** $y \geq 5$

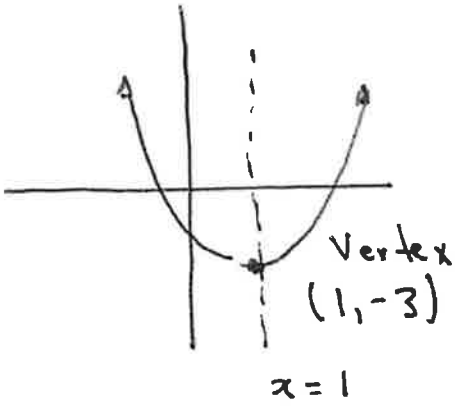
Example: Given that $f(x)$ is a quadratic function with minimum $f(1) = -3$, find the vertex, axis of symmetry, domain and range

Solution: $f(1) = -3$ means the point $(1, -3)$, so the **vertex is** $(1, -3)$

The **Axis of Symmetry is:** $x = 1$

Domain: all real numbers

Range: $y \geq -3$



Example: Determine a quadratic function with vertex $(2, 1)$ and $y - intercept: -3$

Solution:

The **Standard Form** of a Quadratic Function is: $y = a(x - h)^2 + k$, so $y = a(x-2)^2 + 1$

The $y - int$ means that the **graph crosses the y-axis**, at $x = 0$, so it **crosses at:** $(0, -3)$

So now we can **input** and **solve for** a:

$y = a(x-2)^2 + 1$

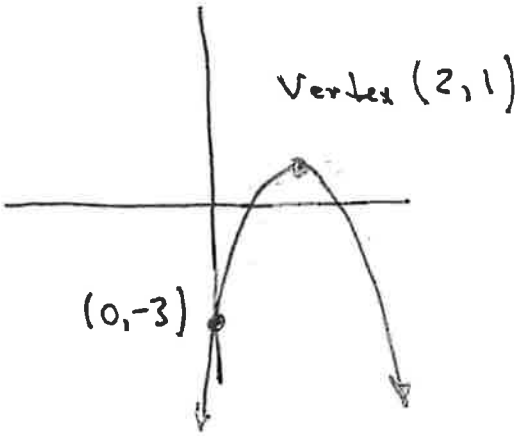
$-3 = a(0-2)^2 + 1$

$-3 = 4a + 1$

$-4 = 4a$ thus

$-\frac{4}{4} = a$ $y = -1(x-2)^2 + 1$

$a = -1$



Practice Questions #7-16

Section 6.2 – Practice Questions

Complete the square, and state if the vertex is a minimum and maximum

1. $y = x^2 - 2x + 4$

2. $f(x) = -2x^2 - 8x + 3$

3. $y = 3x^2 - 6x + 1$

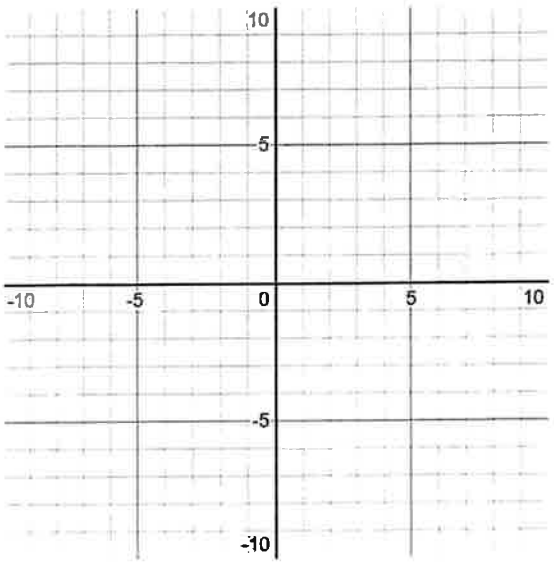
4. $g(x) = -x^2 + 3x - 4$

5. $y = -\frac{1}{3}x^2 + 2x - 4$

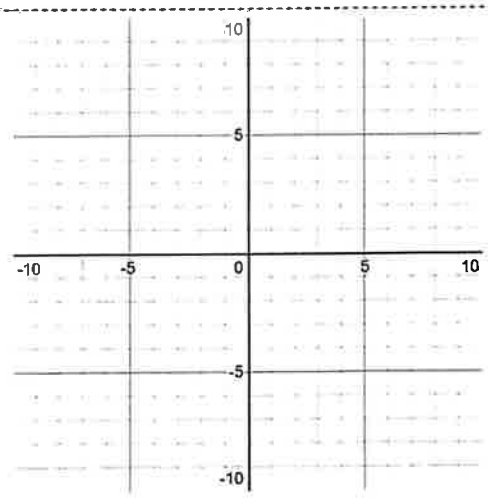
6. $i(x) = 3x^2 - 5x + 2$

Graph the following quadratic functions. Plot at least four points other than the vertex.

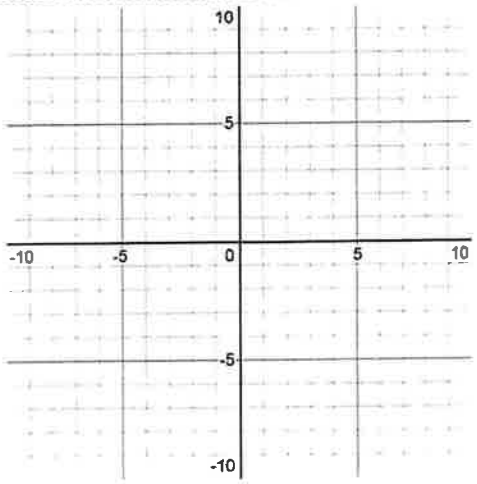
7. $y = x^2 - 4x$



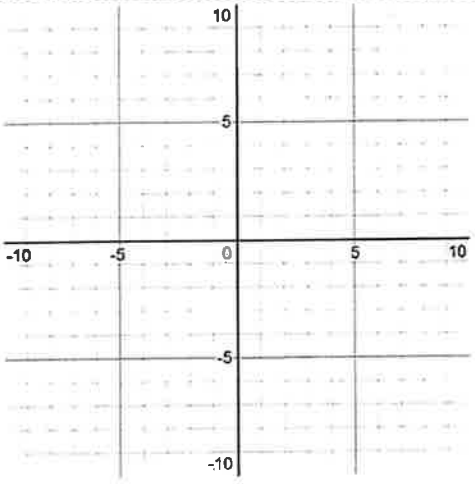
8. $y = 2x^2 + 6x$



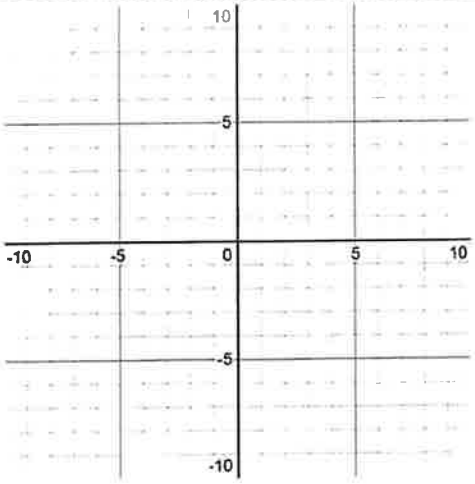
9. $y = -x^2 + 2x + 3$



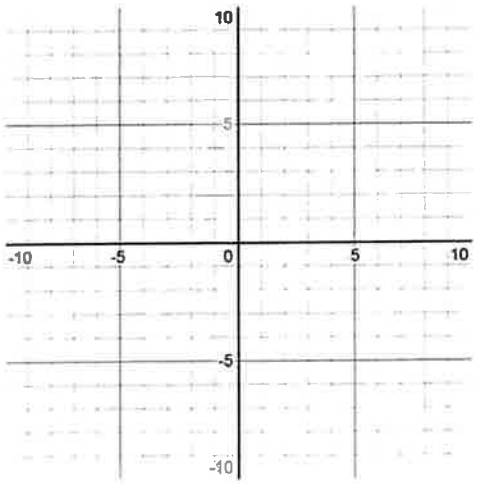
10. $y = x^2 - 6x + 8$



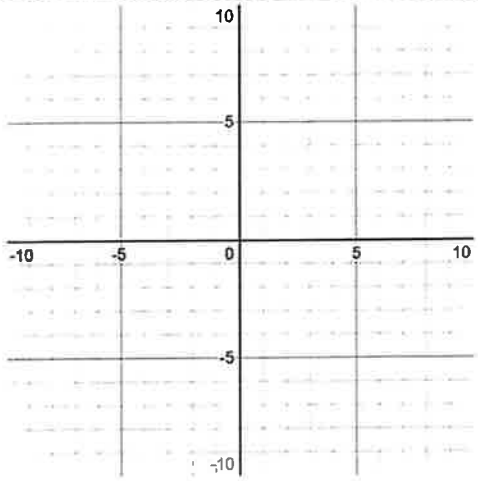
11. $y = -x^2 + 6x - 9$



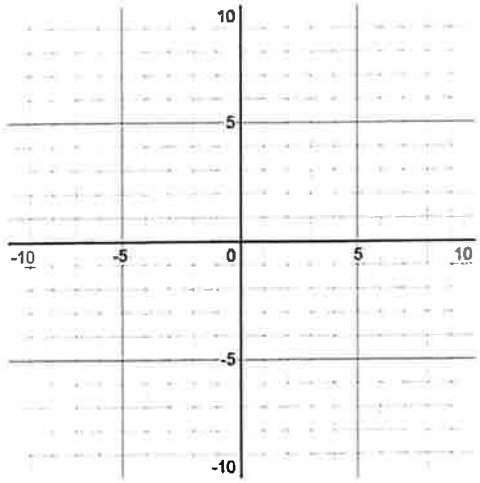
12. $y = -\frac{1}{2}x^2 + 4x - 6$



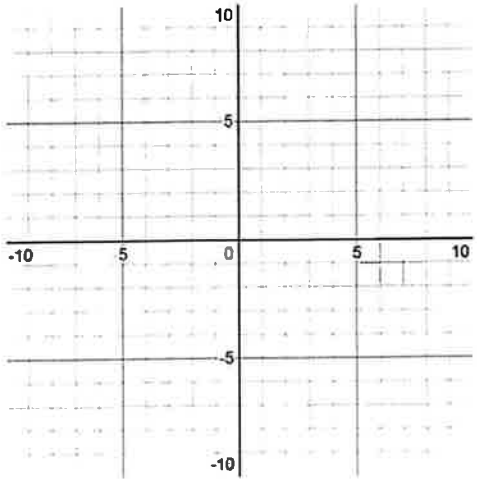
13. $y = x^2 + 2x + 3$



14. $y = -x^2 - 6x - 4$



15. $y = 2x^2 - 4x + 1$



16. $y = -3x^2 - 12x - 8$