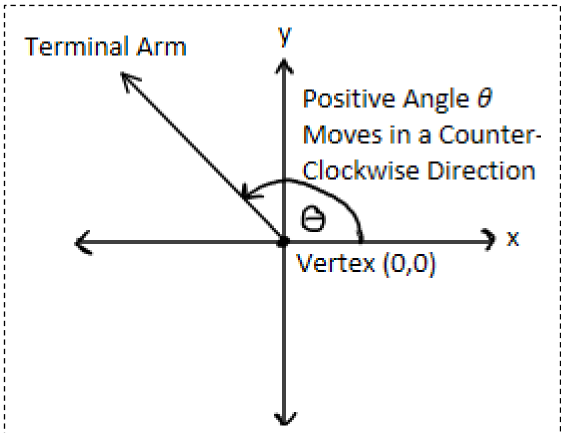
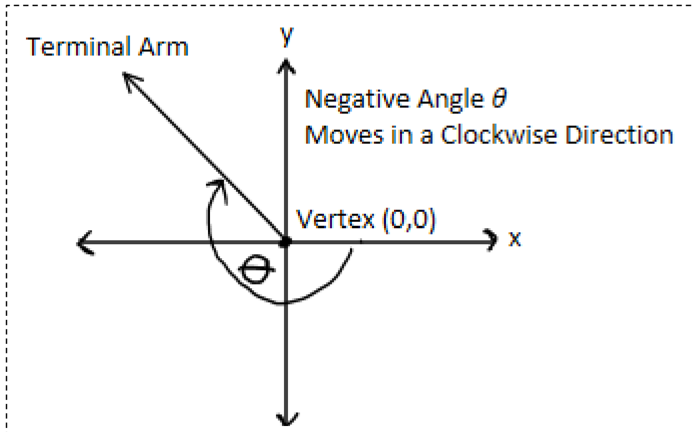
Section 6.1 – Trigonometric Basics

- We will revisit the trigonometry basics here first before we kick it up a notch
- An Angle is determined by rotating an arm in a counter-clockwise position at its endpoint
- The endpoint is called the vertex of the angle
- The arm rotating around is called the terminal arm
  - It can rotate in a clockwise direction (negative angle)
  - Or counter clockwise direction (positive angle)

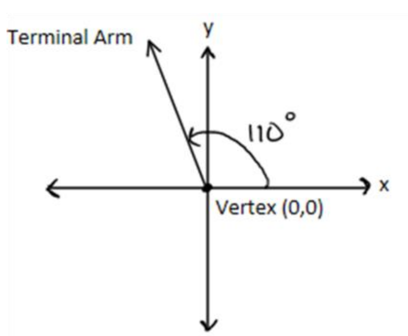


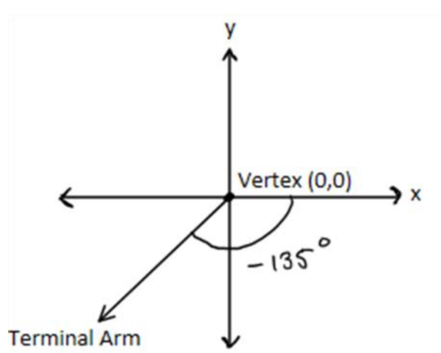


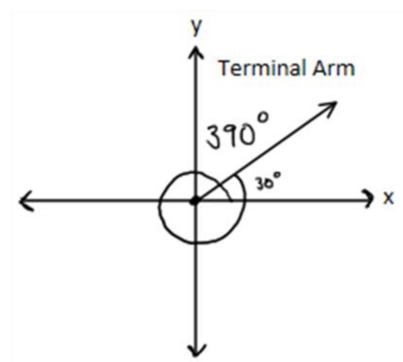
Angles in Standard Position

An angle θ is said to be in **Standard Position** if its **vertex is at the origin** and it **originates from the positive $x - axis$**. **Rotating the terminal arm counter clockwise** about the vertex **forms a positive angle θ**, where **rotating clockwise** about the **vertex forms a negative angle θ**.

Examples:







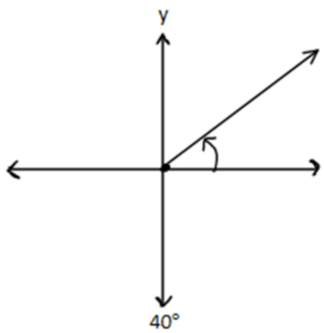
- Counter Clockwise
- Positive Direction
- Past the 90° Mark
- An extra 20° into Q2
- Making 110°

- Clockwise
- Negative Direction
- Past the $-90°$ Mark
- An extra $-45°$ into Q3
- Making $-135°$

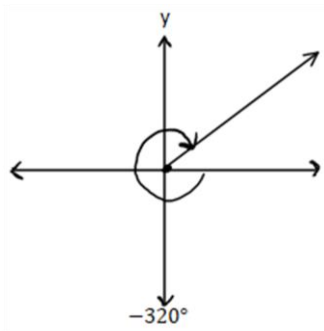
- Counter Clockwise
- Positive Direction
- Past the 360° Mark
- An extra 30° into Q1
- Making 390°

Co-Terminal Angles

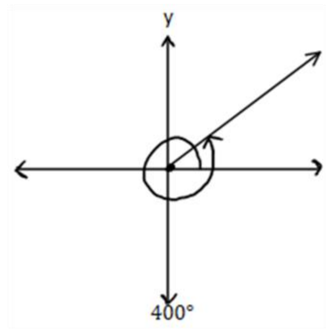
- Angles in Standard Position that have the same terminal side are called C0-Terminal angles
- Think about adding or subtracting an entire rotation $360°$, from the given angle
- There are an infinite number of Co-Terminal angles, we can add or subtract multiples of $360°$



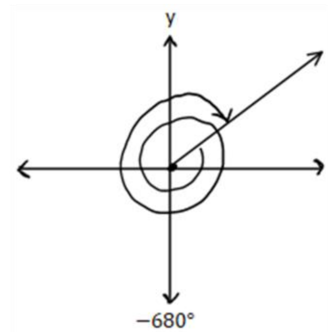
Rotation of $40°$



Rotation of $40° - 360°$



Rotation of $40° + 360°$



Rotation of $40° - 2(360°)$

<u>Example 1:</u> If $\theta = 120°$, in Standard Position, find two positive and two negative angles that are co-terminal with θ

<u>Solution 1:</u> There are an infinite number of solutions, so add or subtract $360°$ to your hearts content

$120° + 360° = 480°$ $\quad$ $120° + 2(360°) = 840°$ $\quad$ $120° + 8(360°) = 3000°$

$120° - 360° = -240°$ $\quad$ $120° - 2(360°) = -600°$ $\quad$ $120° - 8(360°) = -2760°$

<u>Example 2:</u> Find the smallest positive co-terminal angle for

a) $2692°$
b) $-1940°$

<u>Solution 2:</u>

a) Divide 2692 by 360 to see now many full rotations have occurred

$\frac{2692}{360} = 7.47777..$ $\quad$ $7\ full\ rotations:$ $\quad$ $7 \cdot 360 = 2520°$

$2692° - 2520° = 172°$

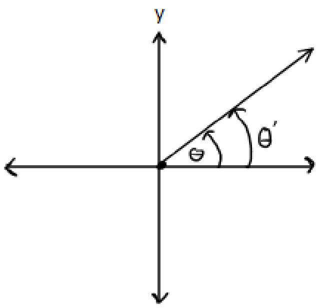
b) Divide 1940 by 360 to see now many full rotations in the negative direction

$\frac{-1940}{360} = -5.3888\ ...$ $\quad$ *Since we went negative we will need 6 rotations to get back to the positive.* $\quad$ $6 \cdot 360 = 2160°$

$-1940° + 2160° = \mathbf{220°}$

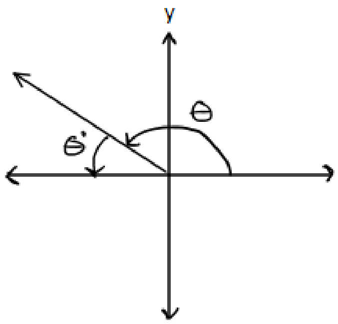
Reference Angles

- For and angle $\boldsymbol{\theta}$ **in Standard Position**, the **reference angle** is the **positive acute angle** $\boldsymbol{\theta'}$ that is **formed** with the **terminal side of** $\boldsymbol{\theta}$ **and the** $\boldsymbol{x-axis}$**.**
- Read that again…
- A reference angle is between $0°\ and\ 90°$: $0° \leq \theta' \leq 90°$
- This will start to make a lot more sense in applications in Section 7.2



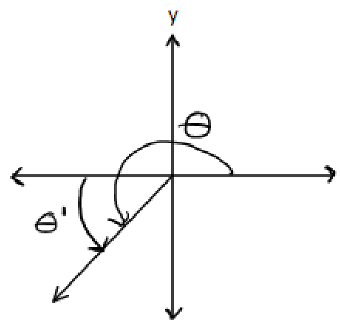
θ in Quadrant 1

$\theta' = \theta$



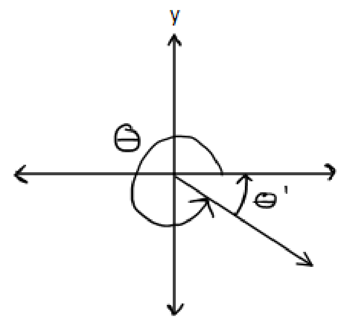
θ in Quadrant 2

$\theta' = 180° - \theta$



θ in Quadrant 3

$\theta' = \theta - 180°$



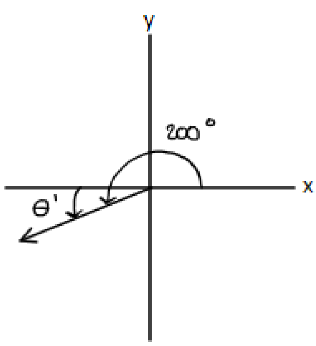
θ in Quadrant 4

$\theta' = 360° - \theta$

<u>Example 3:</u> Find the reference angle for: a) $200°$ b) $300°$ c) $-200°$

<u>Solution 3:</u>

a)

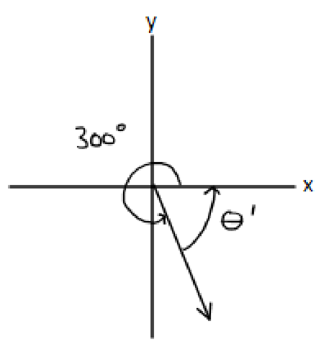


$\theta' = \theta - 180°$

$\theta' = 200° - 180°$

$\boldsymbol{\theta' = 20°}$

b)

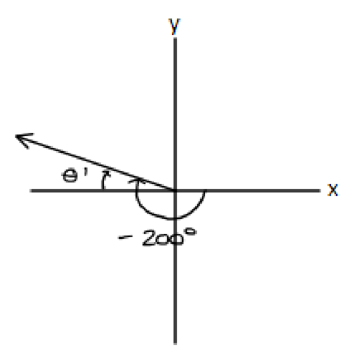


$\theta' = 360° - \theta$

$\theta' = 360° - 300°$

$\boldsymbol{\theta' = 60°}$

c)



The Co-Terminal Angle of $-200°\ is\ 160°$

$\theta' = 180° - \theta$

$\theta' = 180° - 160°$

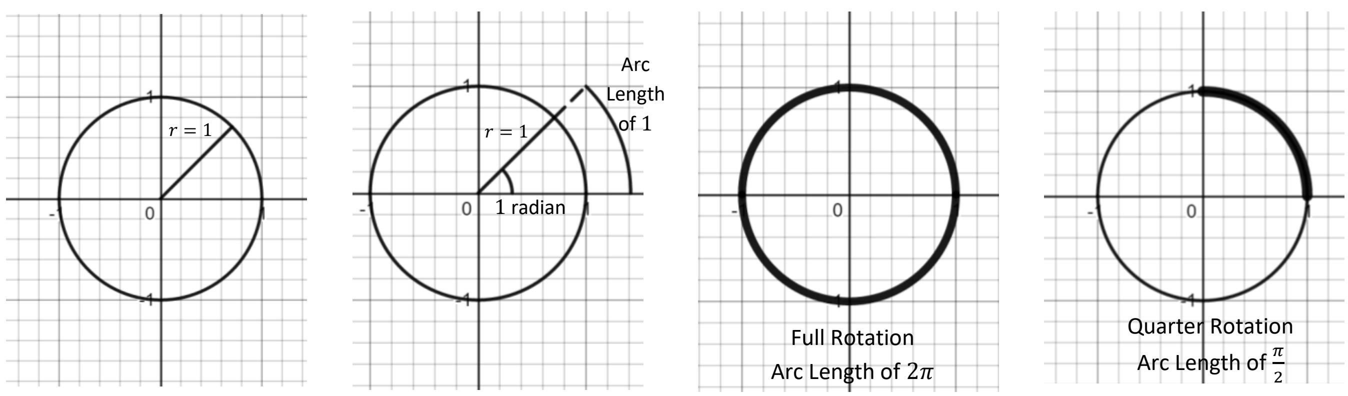
$\boldsymbol{\theta' = 20°}$

Radians and Converting from Degrees

- Radians, the new thing! Radians are another way to measure rotation.
- A radian is not a degree value, but has applications in science and engineering, because it helps measure the arc length of a circle (we will see this shortly)
- In order to understand radians, we need to consider a circle with radius of 1.
- This is called the: **Unit Circle**

> **Definition of a Radian Measure**
>
> An angle of 1radian, is a standard position angle, rotated counter clockwise, of arc length of 1 on a unit circle.

Example of Unit Circle and Radians



Since the **Circumference of a Circle** is given by: $\boldsymbol{C = 2\pi r}$. A circle with Radius 1 (the **Unit Circle**), has: $\boldsymbol{C = 2\pi}$.

It takes $\boldsymbol{360°}$ **for one full rotation**, so we can infer the relationship that: $\boldsymbol{360° = 2\pi}$ and $\boldsymbol{180° = \pi}$

From this relationship we can formulate these two conversion ratios:

Degree to Radians

$$\frac{\pi}{180°}$$

Multiply the given degree by the ratio above

Radians to Degrees

$$\frac{180°}{\pi}$$

Multiply the given radian measure by the ratio above

Example 4: Convert the following Degrees/Radians into their Radian/Degree counterpart

a) $315°$ | b) $35°$ | c) $\frac{5\pi}{6}$ | d) 2.5

Solution 4: Remember that the ratio you use should position the given units in the numerator and those same units in the denominator of the ratio being used, these shows cancelling of units. Simplify the degrees as a fraction if you can!

a) $\mathbf{315°}$

$$315° \cdot \frac{\pi}{180°}$$

$$\frac{315°(\pi)}{180°}$$

$$\boldsymbol{\frac{7\pi}{4}}$$

b) $\mathbf{45°}$

$$45° \cdot \frac{\pi}{180°}$$

$$\frac{45°(\pi)}{180°}$$

$$\boldsymbol{\frac{\pi}{4} \text{ or } 0.79}$$

c) $\boldsymbol{\frac{5\pi}{6}}$

$$\frac{5\pi}{6} \cdot \frac{180°}{\pi}$$

$$\frac{5\pi(180°)}{6\pi}$$

$$\frac{900°}{6} = \mathbf{150°}$$

d) $\mathbf{2.5}$

$$2.5 \cdot \frac{180°}{\pi}$$

$$\frac{2.5(180°)}{\pi}$$

$$\frac{450°}{\pi} = \mathbf{143.2°}$$

It is understood that the absence of units after the conversion implies radians

Arc Length

- The length of an arc of a circle is proportionate to the angle θ in radians, and the radius
- The equation for determining the Arc Length is thus:

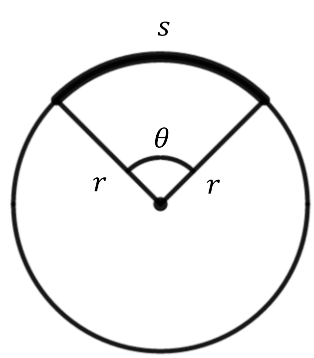
$$\boldsymbol{s = r\theta}$$

where,

$s = arc\ length$

$r = radius$

$\theta = central\ angle\ in\ radians$



<u>Example 5:</u> Determine the Arc Length of a Circle with a radius of $6cm$ and a central angle of $45°$

<u>Solution 5:</u> Use our Arc Length Formula, but… my angle needs to be **converted to Radians**

$$45° \cdot \frac{\pi}{180°}$$

$$\frac{45°(\pi)}{180°}$$

$$\boldsymbol{\frac{\pi}{4}}$$

$$\boldsymbol{s = r\theta}$$

$$s = 6(\frac{\pi}{4})$$

$$s = \frac{6\pi}{4} = \frac{3\pi}{2} = \boldsymbol{4.71cm}$$

<u>Example 6:</u> Find the distance travelled by the tip of a second hand on a clock with a radius of $13cm$, as it moves from $0\ seconds$ to $30\ seconds$.

<u>Solution 6:</u> From $0 - 30 seconds$ is:

$$\frac{30}{60} = \frac{1}{2}\ of\ a\ revolution \quad \rightarrow \quad \frac{1}{2}\ of\ a\ revolution = \frac{1}{2}(2\pi) = \pi\ radians$$

$$s = r\theta \quad \rightarrow \quad s = 13cm(\pi) = \boldsymbol{40.8cm}$$

<u>Example 7:</u> What is the degree measure of a central angle subtended by an arc of length $24cm$ in a circle with a radius of $5cm$?

<u>Solution 7:</u> In this case we need find our angle (it will be in radians) and then **convert to degrees**

$$\boldsymbol{s = r\theta} \quad \rightarrow \quad \boldsymbol{\frac{s}{r} = \theta} \quad \rightarrow \quad \frac{24cm}{5cm} = 4.8\ radians$$

$$4.8\ radians \cdot \frac{180°}{\pi} = \frac{864°}{\pi} = \boldsymbol{275}°$$

Section 6.1 – Practice Problems

1. Determine the Quadrant the standard position angle is in, or if it is not in a specific quadrant, say so.

a) $150°$	b) $-150°$
c) $314°$	d) $-314°$
e) $612°$	f) $-537°$
g) $1100°$	h) $6325°$
i) $810°$	j) $-900°$

2. Find the degree measure of the given rotation (One full rotation is $360°$)

a) $\frac{1}{8}$ rotation	b) $\frac{1}{5}$ rotation
c) $\frac{5}{6}$ rotation	d) $\frac{9}{8}$ rotation
e) $\frac{7}{5}$ rotation	f) $\frac{7}{6}$ rotation

3. Find the radian measure of the given rotation (One full rotation is 2π)

a) $\frac{1}{6}$ rotation	b) $\frac{3}{4}$ rotation
c) $\frac{2}{3}$ rotation	d) $2\frac{1}{4}$ rotation
e) $\frac{13}{12}$ rotation	f) $\frac{11}{8}$ rotation

4. Determine one positive and one negative coterminal angle/

a) $150°$	b) $-150°$
c) $314°$	d) $-314°$
e) $612°$	f) $-537°$
g) $1100°$	h) $6325°$

i) $810°$	j) $-900°$

5. Convert from degrees to radians. Express answer in terms of π.

a) $45°$	b) $90°$
c) $150°$	d) $240°$
e) $300°$	f) $360°$
g) $405°$	h) $420°$
i) $450°$	j) $630°$

6. Convert from degrees to radians. Express answer in to three decimals.

a) $70°$	b) $37.5°$

c) $130°$	d) $\frac{90°}{\pi}$
e) $400°$	f) $527°$
g) $-248°$	h) $718°$
i) $1025°$	j) $-1349°$

7. Convert from radians to degrees.

a) $\frac{\pi}{3}$	b) $\frac{5\pi}{6}$
c) $\frac{3\pi}{4}$	d) $\frac{11\pi}{6}$
e) $\frac{17\pi}{6}$	f) $\frac{21\pi}{4}$

g) $\frac{11\pi}{3}$

h) $\frac{20\pi}{3}$

i) $\frac{31\pi}{6}$

j) $\frac{23\pi}{4}$

8. Convert from radians to degrees. Provide answers to 1 decimal place.

a) 3

b) -4

c) 2.7

d) -1.2

e) 8.2

f) -12.8

9. Find the radius of a circle if an arc of 3 subtends angle of $30°$ on the circle.

10. Find the arc length of a sector of a circle radius $15cm$ if the sector angle is $130°$.

11. Find the angle in degrees if an arc length of $5cm$ has a radius of $6cm$.

12. As the time changes from $2:00pm$ to $2:30pm$ on an analog clock.
a) Determine the change in radian measure of the minute hand.

b) Determine the change in radian measure of the hour hand.

13. A horse on a merry-go-round is $4m$ from the center. How many meters does Kate travel ibn the horse if the merry-go-round makes 15 revolutions before stopping?

14. A flywheel makes 12 revolutions per minute(rpm). How many seconds does it take for the flywheel to turn through 216°?

15. The earth rotates about an axis through its poles, making one revolution per day. The radius of Earth is approximately $6400km$. What distance is traversed by a point on Earth's surface at the equator during an $8-hour$ interval as a result of Earth's rotation about its axis?

16. What distance does a bird fly when flying due South from 40^o north latitude to 20^o north latitude?

See Website for Detailed Answer Key

Extra Work Space