# Section 6.1 – Quadratic Functions

## The Parabola

### Rules for graphing Non-Linear Equations

1. Use positive numbers, negative numbers, and zero whenever possible.
2. If any value is to an even power both positive and negative values must be used.
3. Use values between 0 and 1 when the variable is in the denominator, or is in the exponent

**Example:** A quadratic function in a function that has the most basic form:

$$y = x^2$$

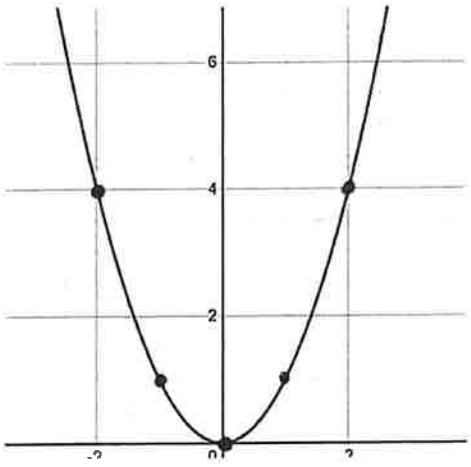
**Graph:** Since $x$ is an even power, positive and negative values of x are used.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 4 | 1 | 0 | 1 | 4 |

$y = x^2$

$y = (-2)^2$

$= 4$




- The following section will be focused on the basics of this quadratic, how to sketch it, how it is manipulated through parameters, and how to solve for the roots (solutions, zero's)

- Quadratic Functions are the backbone of many day-to-day situations.
- Everything that flies through the air has a parabolic arc – A Quadratic Function
- Flow of water through a pipe, the support cables of a suspension bridge: Quadratic function

**Definition of a Quadratic Function**

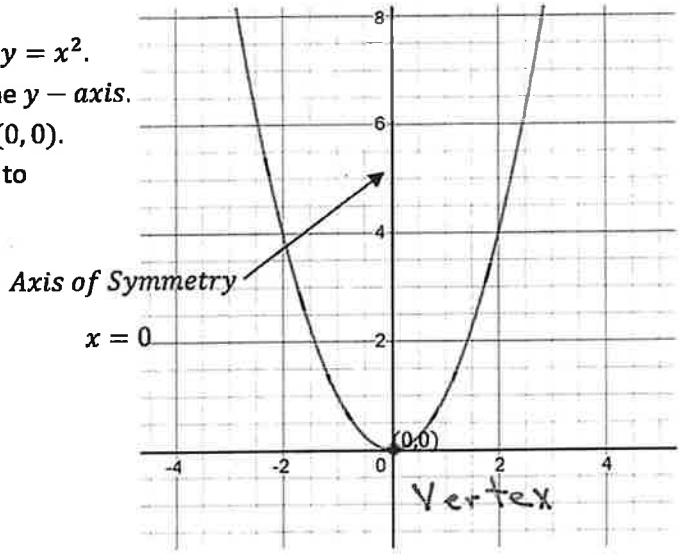
A quadratic function is a function that can be written in the form:

$$f(x) = ax^2 + bx + c$$

Where $a, b, and\ c$ are Real Numbers, and $a \neq c$.
The graph of the quadratic function is called a $Parabola$

- The simplest quadratic function is the function $y = x^2$.
- This is a U-shaped curve that is symmetric to the $y - axis$.
- The lowest point is called the **vertex**, and is at $(0, 0)$.

And the line of symmetry divides the parabola into to identical halves.




**Graphing a Parabola**

**Step 1:** Find the vertex and axis of symmetry

**Step 2:** Find the $y - intercept$ by evaluating $f(0)$, and the $x - intercepts$ by evaluating $f(x) = 0$

**Step 3:** Add additional points if needed

**Step 4:** Sketch graph

## The Graph of $y = ax^2$

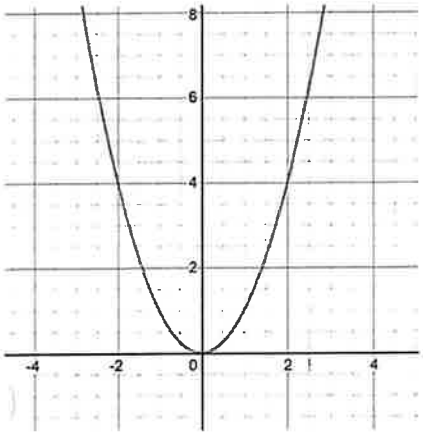
- The graph of $y = ax^2$ is a parabola with vertex at the origin $(0, 0)$
- If $a > 0$: opens upward, vertex is a minimum point.
- If $a < 0$: opens downward, vertex is a maximum point.
- The parabola will be wide if: $-1 < a < 1$, and narrow if $a > 1$ *or* $a < -1$
- The axis of symmetry is $x = 0$

**Example:** Graph $y = ax^2$

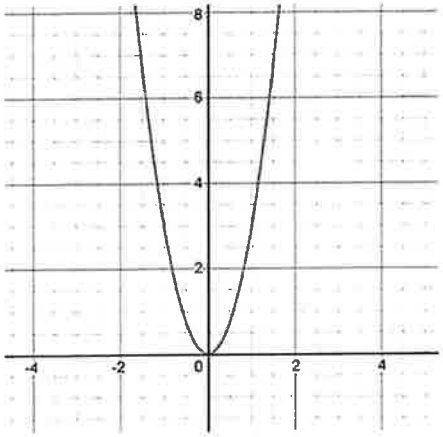
narrower

wider

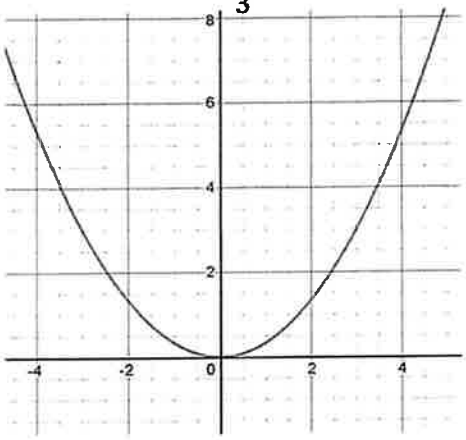
$a = 1$



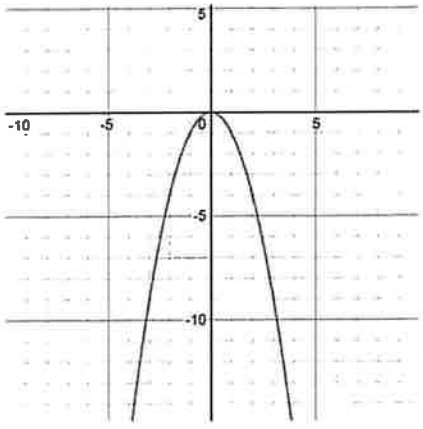
$a = 3$



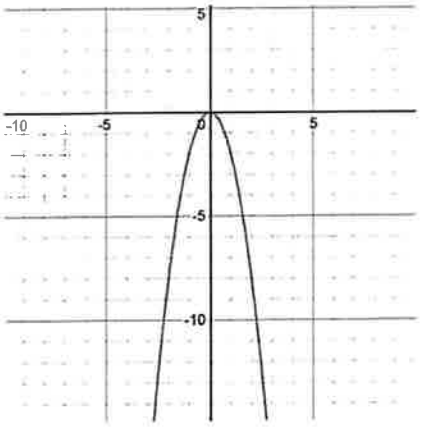
$a = \frac{1}{3}$



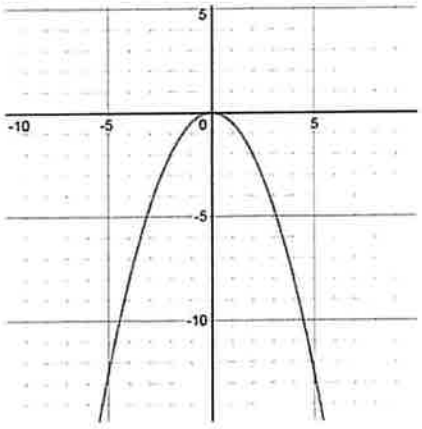
$a = -1$



$a = -2$



$a = -\frac{1}{2}$



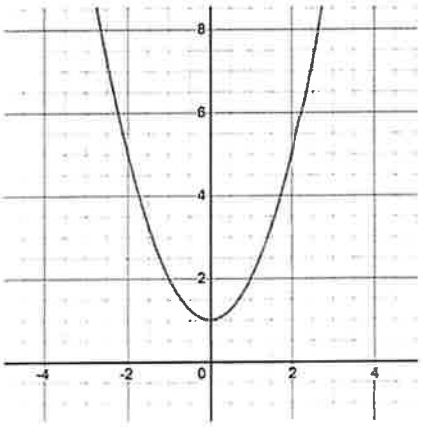
$a < 0$ reflection about $x$-axis

## The Graph of $y = ax^2 + k$

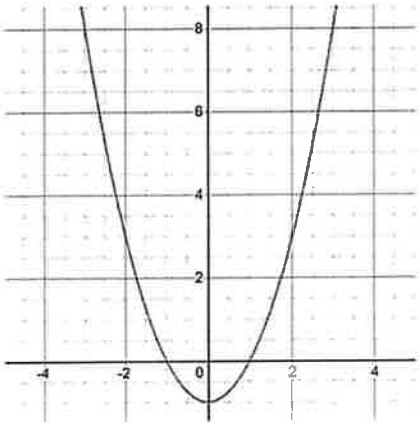
- The graph of $y = ax^2 + k$ shifts vertically up if $k > 0$ and vertically down if $k < 0$
- The vertex of the parabola is $(0, k)$ and the axis of symmetry is $x = 0$

**Example:** Graph of $y = ax^2 + k$

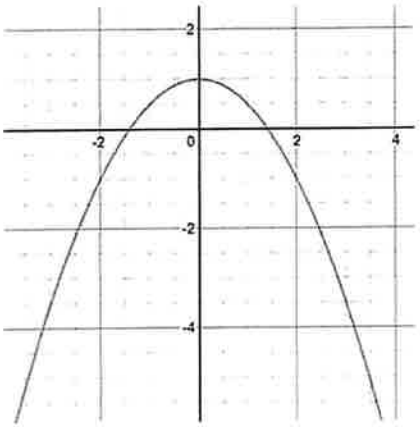
$y = x^2 + 1$



$y = x^2 - 1$



$y = -\frac{1}{2}x^2 + 1$

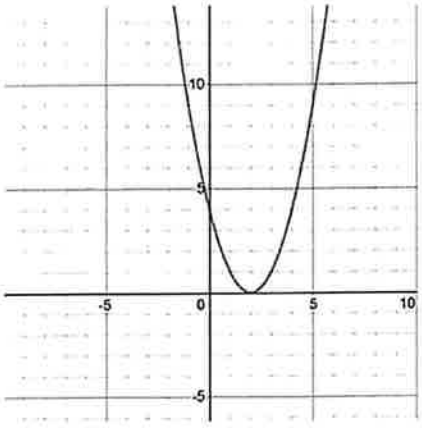


## The Graph of $y = a(x - h)^2$

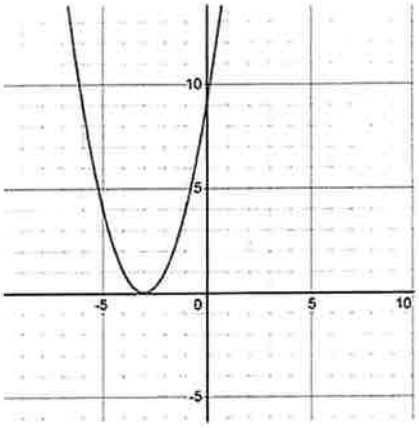
- The graph of shifts horizontally to the right if $h > 0$ and horizontally to the left if $h < 0$
- The vertex of the parabola is $(h, 0)$ and the axis of symmetry is $x = h$

**Example:** Graph of $y = a(x - h)^2$

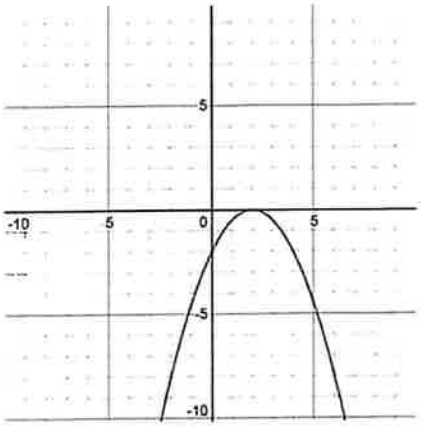
$y = (x - 2)^2$



$y = (x + 3)^2$



$y = -\frac{1}{2}(x - 2)^2$

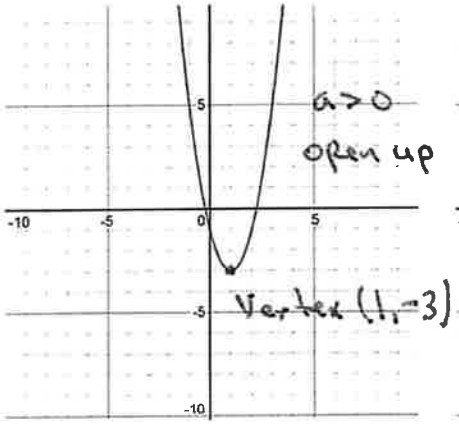


## The Graph of $y = a(x-h)^2 + k$

- This quadratic formula is what we call **Standard** Form
- The vertex of the parabola is $(h, k)$
- The axis of symmetry is $x = h$
- The parabola opens upward if $a > 0$ and downward if $a < 0$
- The parabola will have a minimum value if $a > 0$ and a maximum value if $a < 0$

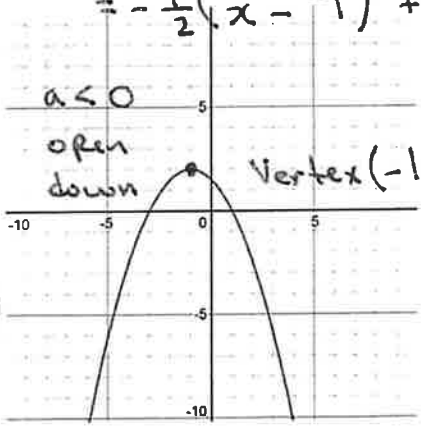
**Example:** Graph of $y = a(x-h)^2 + k$

$y = 2(x-1)^2 - 3$



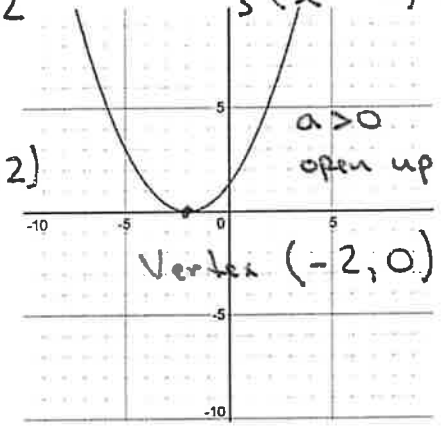

$y = -\frac{1}{2}(x+1)^2 + 2$

$= -\frac{1}{2}(x-(-1))^2 + 2$




$y = \frac{1}{3}(x+2)^2$

$= \frac{1}{3}(x-(-2))^2 + 0$




## Domain and Range

**Domain and Range from a Set of Ordered Pairs (x, y)**

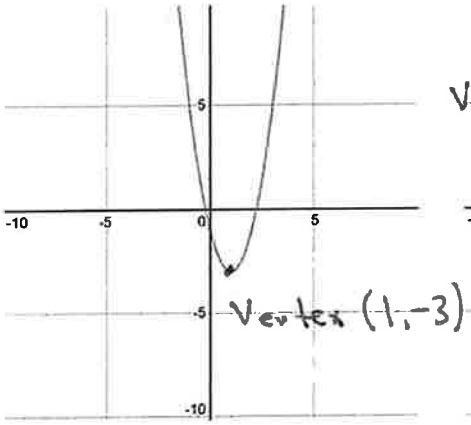
The set of $x - values$, in the ordered pairs, is the **DOMAIN**
The set of $y - values$, in the ordered pairs, is the **RANGE**

**Example:** Determine the Domain and Range of the Ordered Pairs

| | Domain | Range |
|---|---|---|
| A: $\{(1,2), (-3,5), (4,-2)\}$ | $\{-3, 1, 4\}$ | $\{-2, 2, 5\}$ |
| B: $\{(-3,4), (1,0), (0,2), (3,2)\}$ | $\{-3, 0, 1, 3\}$ | $\{0, 2, 4\}$ |
| C: $\{(-2,1), (1,0), (3,3), (1,4)\}$ | $\{-2, 1, 3\}$ | $\{0, 1, 3, 4\}$ |
| D: $\{(-3,-1), (-3,3), (2,3), (4,0)\}$ | $\{-3, 2, 4\}$ | $\{-1, 0, 3\}$ |

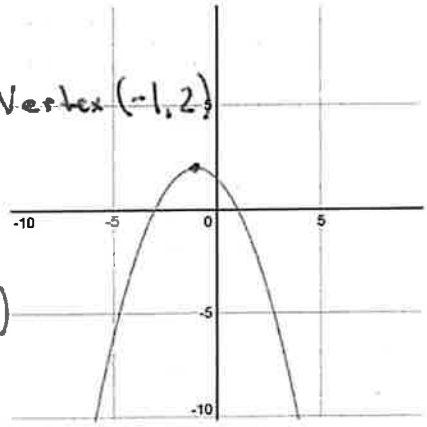
**Example:** Determine the domain and range of the following parabolas:

$y = 2(x-1)^2 - 3$ $\quad$ $y = -\frac{1}{2}(x+1)^2 + 2$ $\quad$ $y = \frac{1}{3}(x+2)^2$



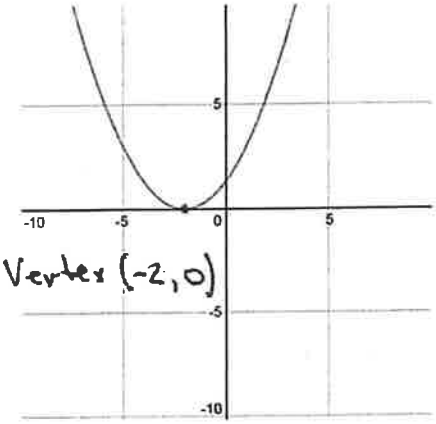

Domain: all real numbers

Range: $y \geq -3$




Domain: all real numbers

Range: $y \leq 2$




Domain: all real numbers

Range: $y \geq 0$

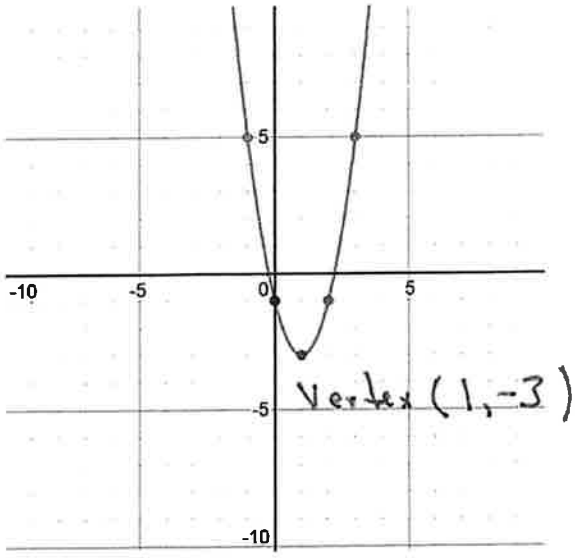
## Graphing Quadratic Equations

$y = a(x-h)^2 + k$ Standard Form

**Example 1:** Graph the following quadratic equation, state the vertex, max/min values, axis of symmetry, domain and range. Plot at least 4 points other than the vertex. $y = 2(x-1)^2 - 3$

| $y = 2(x-1)^2 - 3$ |
|---|
| $y = 2(-1-1)^2 - 3 = 5$ |
| $y = 2(0-1)^2 - 3 = -1$ |
| $y = 2(2-1)^2 - 3 = -1$ |
| $y = 2(3-1)^2 - 3 = 5$ |

| $x$ | $y$ |
|---|---|
| $-1$ | 5 |
| 0 | -1 |
| 2 | -1 |
| 3 | 5 |




Vertex: $(1,-3)$

Max/Min: Min value $y = -3$

Axis of Symmetry: $x = 1$

Domain: all real numbers

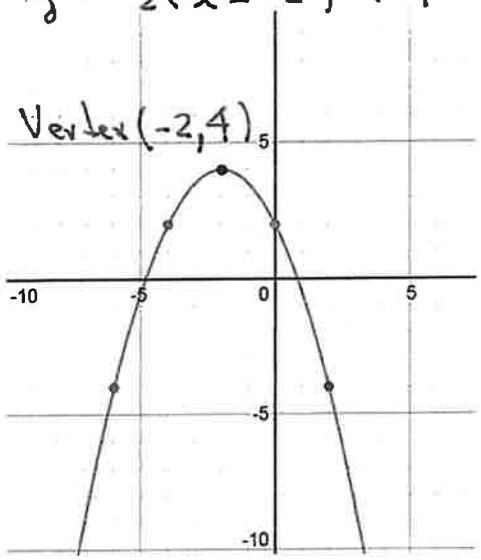
Range: $y \geq -3$

**Example 2:** Graph the following quadratic equation, state the vertex, max/min values, axis of symmetry, domain and range. Plot at least 4 points other than the vertex. $y = -\frac{1}{2}(x+2)^2 + 4$

| $y = -\frac{1}{2}(x+2)^2 + 4$ |
|---|
| $y = -\frac{1}{2}(-6+2)^2 + 4 = -4$ |
| $y = -\frac{1}{2}(-4+2)^2 + 4 = 2$ |
| $y = -\frac{1}{2}(0+2)^2 + 4 = 2$ |
| $y = -\frac{1}{2}(2+2)^2 + 4 = -4$ |

| $x$ | $y$ |
|---|---|
| $-6$ | $-4$ |
| $-4$ | $2$ |
| $0$ | $2$ |
| $2$ | $-4$ |

$y = -\frac{1}{2}(x-^{-}2)^2 + 4$




Vertex: $(-2, 4)$

Max/Min: Max $y = 4$

Axis of Symmetry: $x = -2$

Domain: all real numbers

Range: $y \leq 4$

PQ's # 1 - 14

## Finding the Equation of a Parabola from a Graph

Finding the equation of a parabola from a graph requires two things:

1. The vertex
2. The value that determines the shape and direction of the parabola

**Example 1:** Determine an equal for the parabola.

Vertex: $(-2, 5)$ $y = a(x-h)^2 + k$

So, $y = a(x+2)^2 + 5$

- Now, solve for $a$, by plugging in any point on the line, other than the vertex
- We can use: $(0,3), (-4,3), or\ (2,-3)$
- I will use $(0,3)$

$y = a(x+2)^2 + 5$ sub in $(0,3)$

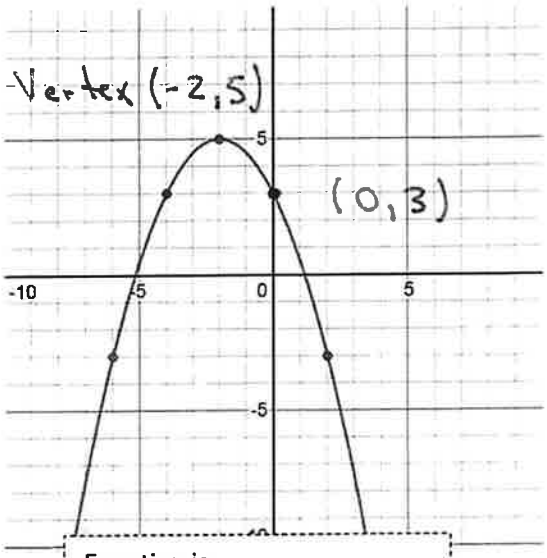
$3 = a(0+2)^2 + 5$

$3 = 4a + 5$

$-2 = 4a$

$a = \frac{-2}{4} = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}(x+2)^2 + 5$




Equation is:

$$y = -\frac{1}{2}(x+2)^2 + 5$$

**Example 2:** Determine an equal for the parabola.

Vertex: $(1,-4)$ $y = a(x-h)^2 + k$

So, $y = a(x-1)^2 - 4$

- Now, solve for $a$, by plugging in any point on the line, other than the vertex
- We can use: $(4,2)\ or\ (-2,2)$
- I will use $(-2.2)$

$y = a(x-1)^2 - 4$ sub in $(-2,2)$

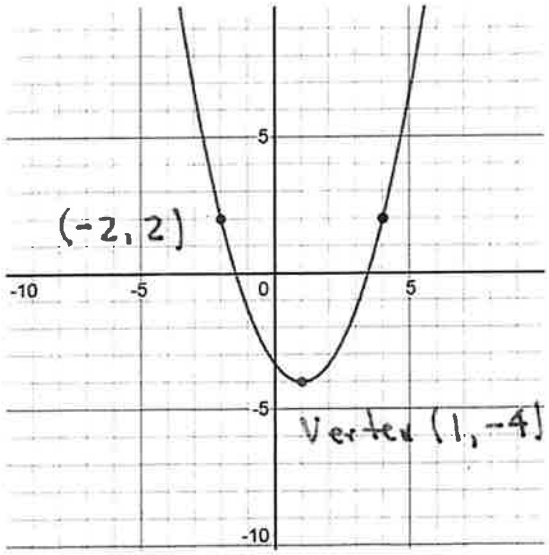
$2 = a(-2-1)^2 - 4$

$2 = 9a - 4$

$6 = 9a$

$a = \frac{6}{9} = \frac{2}{3}$

$\therefore y = \frac{2}{3}(x-1)^2 - 4$




Equation is:

$$y = \frac{2}{3}(x-1)^2 - 4$$

**Practice Questions #17-22**

## Section 6.1 – Practice Questions

**Determine whether the graph of each quadratic opens upwards or downwards, why?**

1. $y = \frac{1}{3}x^2 + 2$

2. $y = -2x^2 + 3$

3. $y = -3(x-1)^2 + 2$

4. $y = 2 - 3x^2$

5. $y + 2x - x^2 = 0$

6. $x^2 + 2x + y = 0$

**Graph the quadratic function. Plot at least 4 points other than vertex.**

7. $y = x^2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

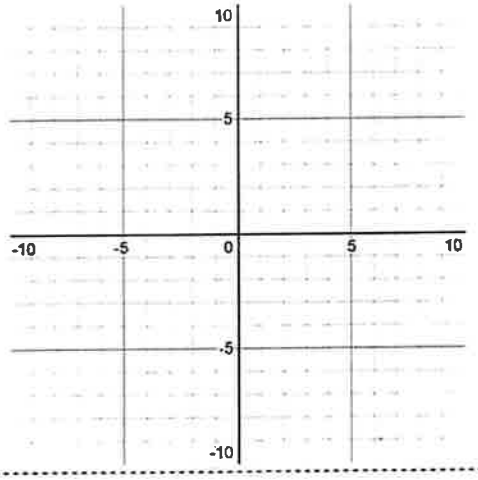
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____




8. $y = -x^2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

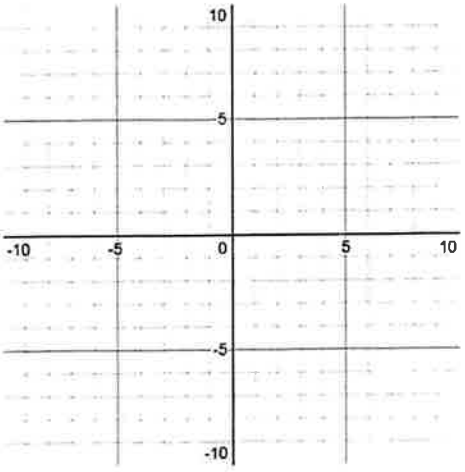
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____



9. $y = x^2 - 2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

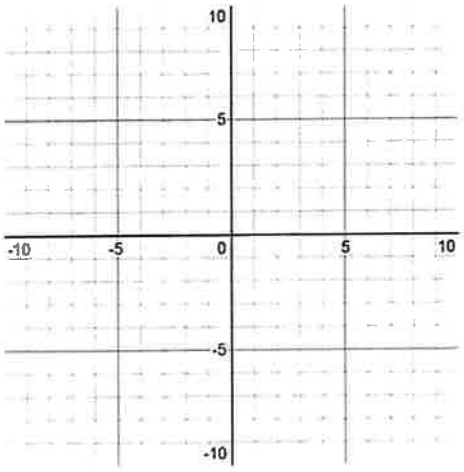
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____




10. $y = (x - 2)^2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

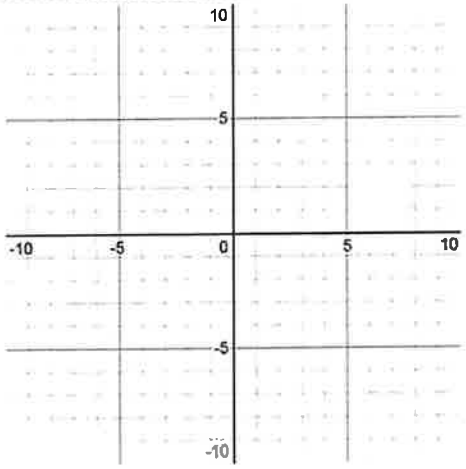
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____




11. $y = -(x + 1)^2 + 2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

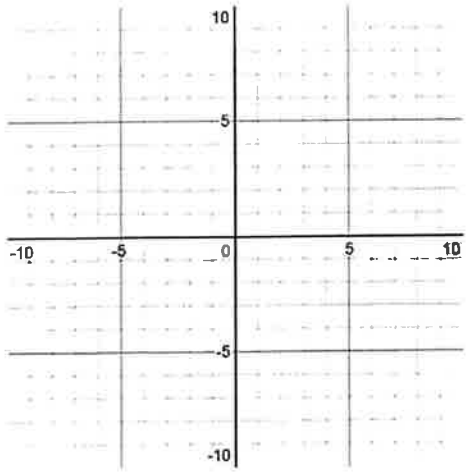
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____



12. $y = \frac{1}{2}(x+2)^2 - 2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

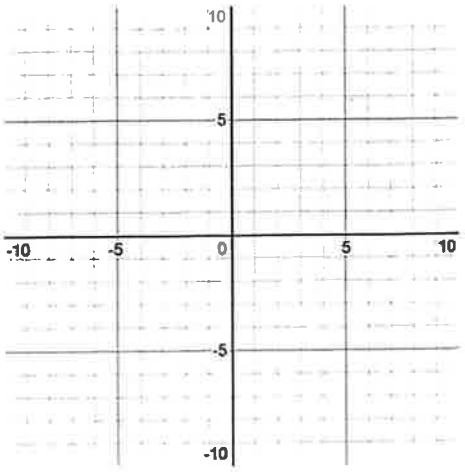
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____




13. $y = -\frac{1}{2}(x-2)^2 + 3$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

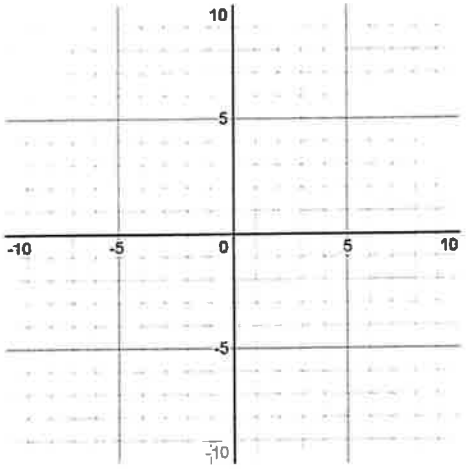
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____




14. $y = -2(x-1)^2 - 2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

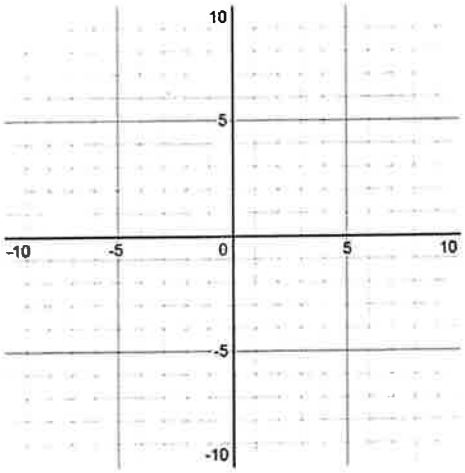
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____



15. $y - 4 = -\frac{2}{3}(x+2)^2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

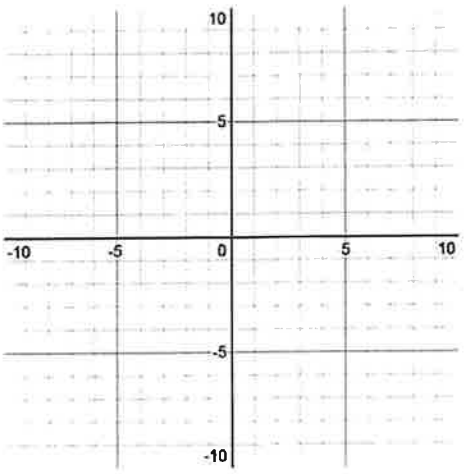
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

Domain: _____

Range: _____




16. $y + 3 = \frac{3}{4}(x-4)^2$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |

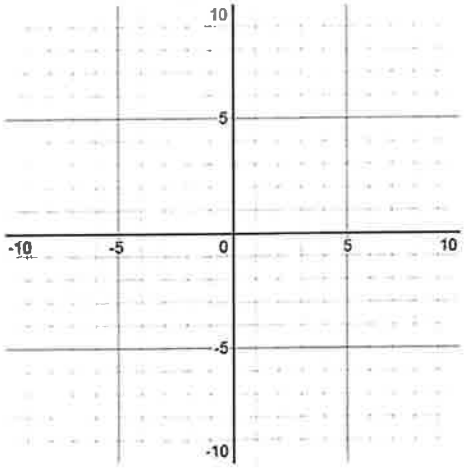
Vertex: _____

Max/Min: _____

Axis of Symmetry: _____

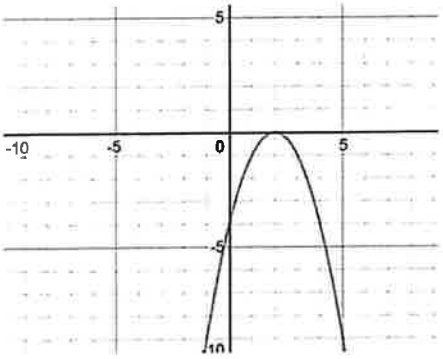
Domain: _____

Range: _____

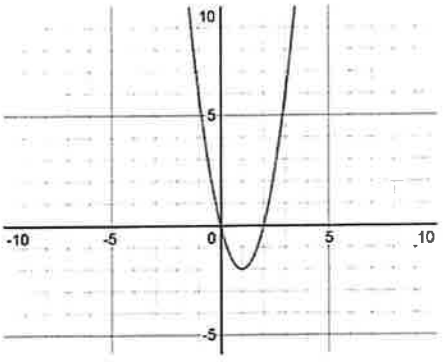



Determine an equation for the Parabola.

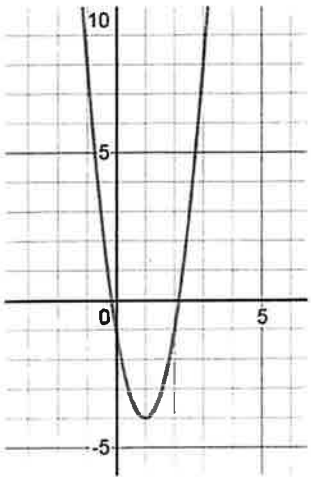
17.



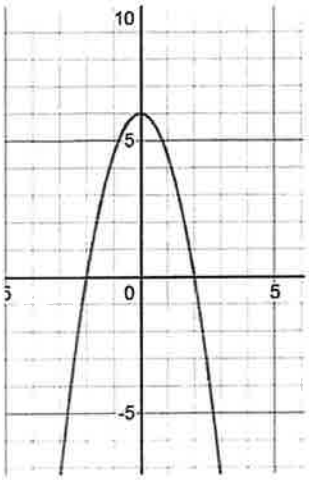

18.



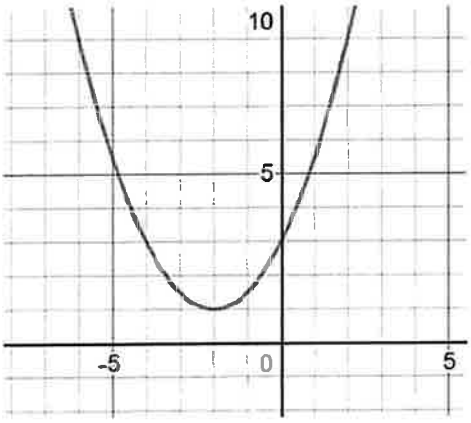
19.




20.




21.




22.