## Section 5.6 – Practice Problems

1. Find the equation of the slant asymptote.

a)

$$y = \frac{2x - x^2 - 1}{x}$$

$$\begin{array}{r|l} & -x+2 \\ \hline x & -x^2+2x-1 \\ & -x^2 \\ \hline & 2x-1 \\ & 2x \\ \hline & -1 \end{array}$$

$$f(x) = 2 - x - \frac{1}{x}$$

$$f(x) - (2-x)$$

$$\rightarrow 2 - x - \frac{1}{x} - 2 + x$$

$$= -\frac{1}{x}$$

$$\lim_{x \to \pm\infty} = 0$$

so $\boxed{y = 2-x \text{ is the slant asymptote}}$

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b)

$$y = \frac{x^3 - 1}{x^2}$$

$$\begin{array}{r|l} & x \\ \hline x^2 & x^3+0x^2+0x-1 \\ & x^3 \\ \hline & -1 \end{array}$$

$$f(x) = x - \frac{1}{x^2}$$

$\boxed{\text{slant asymptote: } y = x}$

$$\lim_{x \to \pm\infty} [f(x) - (x)]$$

$$x - \frac{1}{x^2} - x$$

$$= -\frac{1}{x^2} = 0$$

c)

$$y = \frac{3x^2+4x+2}{x+1}$$

$$\begin{array}{rl} & 3x+1 \\ x+1 \big) & 3x^2+4x+2 \\ & 3x^2+3x \\ \hline & x+2 \\ & x+1 \\ \hline & 1 \end{array}$$

$$f(x) = 3x+1 - \frac{1}{(x+1)}$$

$$\boxed{SA: \; y = 3x+1}$$

$$\lim_{x\to\pm\infty} f(x) - (3x+1)$$

$$\lim_{x\to\pm\infty} 3x+1 - \frac{1}{x+1} - 3x - 1$$

$$\lim_{x\to\pm\infty} \frac{-1}{x+1} \to \frac{\frac{-1}{x}}{\frac{x}{x}+\frac{1}{x}} \to \frac{0}{1-0} = 0$$

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d)

$$y = \frac{4x^2}{2x+1}$$

$$\begin{array}{rl} & 2x \quad -1 \\ 2x+1 \big) & 4x^2+0x+0 \\ & 4x^2+2x \\ \hline & -2x+0 \\ & -2x-1 \\ \hline & 1 \end{array}$$

$$f(x) = 2x-1 + \frac{1}{2x+1}$$

$$\lim_{x\to\pm\infty} f(x) - (2x-1)$$

$$\boxed{SA: \; y = 2x-1}$$

$$2x-1 + \frac{1}{2x+1} - 2x + 1$$

$$\lim_{x\to\pm\infty} \frac{1}{2x+1} \to \frac{1}{\infty} = 0$$

e)

$$y = \frac{x^3 + 4x^2 + 5x + 16}{x^2 + 4}$$

$$\begin{array}{r|l} & x + 4 \\ \hline x^2+4 & x^3 + 4x^2 + 5x + 16 \\ & x^3 \qquad\quad 4x \\ \hline & 4x^2 + x + 16 \\ & 4x^2 \qquad + 16 \\ \hline & x \end{array}$$

$$f(x) = x + 4 + \frac{x}{x^2+4}$$

$$\lim_{x \to \pm\infty} (f(x) - (x+4))$$

$$x + 4 + \frac{x}{x^2+4} - x - 4$$

$$\lim_{x \to \pm\infty} \frac{x}{x^2+4} \to \frac{\frac{1}{x}}{1 + \frac{4}{x^2}} \to 0$$

$$\boxed{SA: \ y = x + 4}$$

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f)

$$y = \frac{x + x^2 - x^4}{x^3 - 1}$$

$$\begin{array}{r|l} & -x \\ \hline x^3-1 & -x^4 + x^2 + x + 0 \\ & -x^4 \qquad\quad + x \\ \hline & x^2 \qquad + 0 \end{array}$$

$$f(x) = -x + \frac{x^2}{x^3-1}$$

$$\lim_{x \to \pm\infty} f(x) - (-x)$$

$$-x + \frac{x^2}{x^3-1} + x$$

$$\boxed{SA: \ y = -x}$$

$$\frac{x^2}{x^3-1} \to \frac{\frac{1}{x}}{1 - \frac{1}{x^3}} \to \boxed{0}$$

2. Find the slant asymptote of the curve. Thus, use it to help graph the curve.

a) VA: $x = 0$

SA: $y = 0$

$$y = \frac{x^2+9}{x}$$

$x \overline{) x^2+9}$ quotient $x$; $x^2$; remainder $9$

no x-ints

no y-ints

$f(-x) = \frac{x^2+9}{-x}$

$f(-x) = -f(x)$ symmetric about origin

$f(x) = x + \frac{9}{x}$

$f(x) - x \rightarrow \frac{9}{x}$

$\lim_{x \to \infty} \frac{9}{x} = 0$

$f'(x) = \frac{x(2x) - (x^2+9)}{x^2} \rightarrow \frac{x^2-9}{x^2}$

crit pts

$x = \pm 3$

ppc $x = 0$

| Interval | $(x^2-9)$ | $x^2$ | $f'(x)$ | $f(x)$ |
|---|---|---|---|---|
| $(-\infty, -3)$ | + | + | + | inc |
| $(-3, 0)$ | - | + | - | dec |
| $(0, 3)$ | - | + | - | dec |
| $(3, \infty)$ | + | + | + | inc |

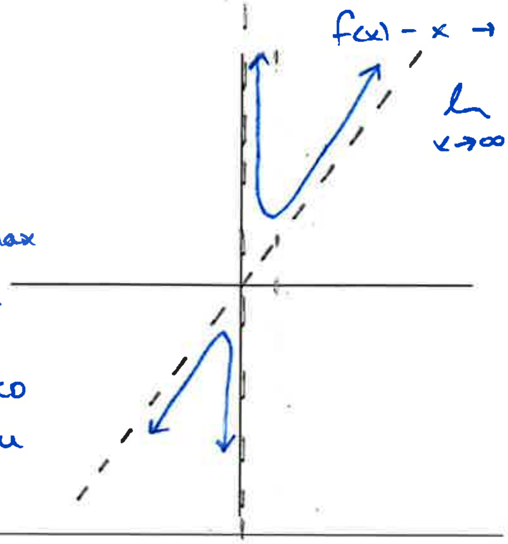
$f(-3) = -6$ localmax

$f(3) = 6$ localmin

$f''(x) = \frac{x^2(2x) - (x^2-9)(2x)}{x^4} \rightarrow \frac{-9}{x^4}$

$x > 0$ $f''(x) < 0$ CD

$x < 0$ $f''(x) > 0$ CU



b)

$x \overline{) x^2-2x-1}$ quotient $x-2$; $x^2-2x$; remainder $-1$

$f(x) = x - 2 - \frac{1}{x}$

$$y = \frac{x^2-2x-1}{x}$$

$\lim_{x \to \pm\infty} f(x) - (x-2)$

$x - 2 - \frac{1}{x} - x + 2 = -\frac{1}{x} = \boxed{0}$

- no y-int
- x-int (Quad equation) $1 \pm \sqrt{2}$
- no symmetry

VA: $x = 0$

SA: $y = x - 2$

HA: None

$f'(x) = \frac{x(2x-2) - (x^2-2x-1)}{x^2} = \frac{2x^2-2x-x^2+2x+1}{x^2}$

$\frac{x^2+1}{x^2}$

if $x > 0$ $f(x) > 0$

$x < 0$ $f(x) > 0$

always increasing

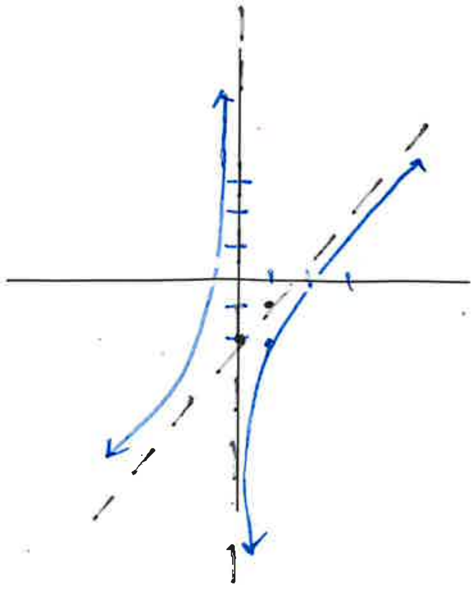
no min/max

$f''(x) = \frac{x^2(2x) - (2x)(x^2+1)}{x^4}$

$\frac{2x^3 - 2x^3 - 2x}{x^4} \rightarrow \frac{-2x}{x^4} = \frac{-2}{x^3}$

if $x < 0$ $f''(x) > 0$ CU

$x > 0$ $f''(x) < 0$ CD



c)

VA: $x = \pm 1$

HA: none

SA: $y = x$

x-int: (0,0)

y-int: (0,0)

$$y = \frac{x^3}{x^2 - 1}$$

$$\begin{array}{r} x \\ x^2-1 \overline{) \; x^3 \quad} \\ \underline{x^3 - x} \\ x \end{array}$$

$f(x) = x - \frac{x}{x^2-1}$

$\lim_{x \to \pm\infty} f(x) - x = \frac{-x}{x^2-1}$

$= 0$

$f'(x) = \frac{(x^2-1)(3x^2) - x^3(2x)}{(x^2-1)^2} \rightarrow \frac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2} = \frac{x^4 - 3x^2}{(x^2-1)^2}$

$= \frac{x^2(x^2-3)}{(x^2-1)^2}$ only this matter

if $x > |3|$ $f(x) > 0$ inc

$-3 < x < 3$ $f(x) < 0$ dec

$f''(x)$ → insert work here ☺

$f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}$

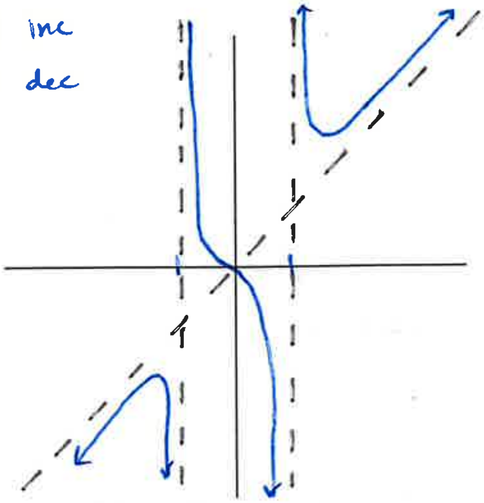
interval points
$x = 0, -1, 1$

$f''(-2) = -$ CD

$f''(-\frac{1}{2}) = +$ cu

$f''(\frac{1}{2}) = -$ CD

$f''(2) = +$ cu



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d)

$(x-1)^3 = x^3 - 3x^2 + 3x - 1$

$$y = \frac{(x-1)^3}{x^2}$$

$$\begin{array}{r} x - 3 \\ x^2 \overline{) \; x^3 - 3x^2 + 3x - 1} \\ \underline{x^3 - 3x^2 \qquad\qquad} \\ 0 \qquad\qquad \\ 3x - 1 \end{array}$$

$f(x) = x - 3 + \frac{3x-1}{x^2}$

$f(x) - (x-3) = \frac{3x-1}{x^2}$

$\lim_{x \to \pm\infty} \frac{3x-1}{x^2} = 0$

A: Domain: $(-\infty, 0) \cup (0, \infty)$

B: y-int: None

x-int: (1,0)

C: No symmetry

D: VA: $x = 0$

HA: None

SA: $x - 3$

$f'(x) = \frac{x^2(3(x-1)^2) - (x-1)^3(2x)}{x^4} \rightarrow \frac{3x^2(x-1)^2 - 2x(x-1)^3}{x^4}$

$= \frac{(x-1)^2(3x^2 - 2x(x-1))}{x^4} = \frac{(x-1)^2(3x - 2x + 2)}{x^4}$

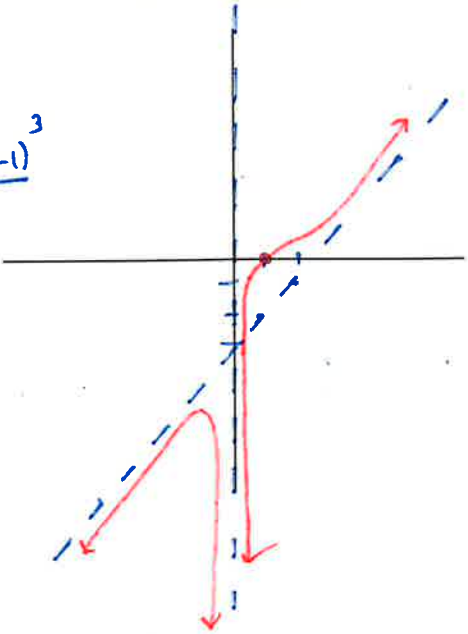
$= \frac{(x-1)^2(x+2)}{x^4}$

$(-\infty, -2)$ $f'(x) > 0$ inc

$(-2, 0)$ $f'(x) < 0$ dec

$(0, \infty)$ $f'(x) > 0$ inc

$f(-2) = -27/4$ local max



$f''(x) = \frac{6(x-1)}{x^4}$

$f''(0) < 0$ CD

$f''(2) > 0$ cu

crit pt is $x = 1$