Section 5.5 – Applications of Inequalities

This Booklet Belongs to: ____________ Block: ______

Strategy for Solving Word Problems

1. Read the problem carefully, more than once. Know what you need to solve for and what is given.
2. Let a variable represent the unknown information (usually x), and represent the rest in terms of it.
3. Make a diagram if possible/necessary
4. Write an equation relating your unknown quantities to what you are given
5. Solve the equation
6. Check your solutions in terms of the original problem to make sure your answer makes sense.

Example 1: The total resistance of two electric circuits is given by: $R^2 - R + 1$, where R is the resistance in ohms. When is the resistance more than 7 $units$?

Solution 1: First find what does our inequality look like?

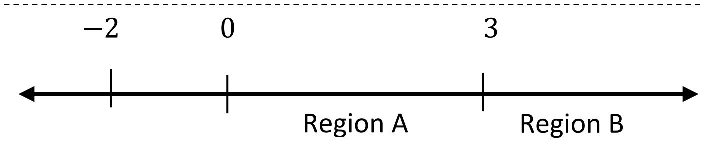
$R^2 - R + 1 > 7$

$R^2 - R - 6 > 0$

$\boldsymbol{(R-3)(R+2) > 0}$

$\boldsymbol{R = 3, R = -2}$

Resistance cannot be negative, so disregard the $-2 \rightarrow 0$ portion of the interval. Our Regions become: $0 < R < 3$ and $R > 3$



Test a Value from Region A: $R^2 - R + 1 > 7$

Selected 1

$(1)^2 - (1) \mp 1 > 7 \quad \rightarrow \quad 1 - 1 + 1 > 7$

$1 > 7$

Since $1 > 7$ is a **FALSE statement**, Region A **DOES NOT belong** to the solution set.

Test a Value from Region B: $R^2 - R + 1 > 7$

Selected 5

$(5)^2 - 5 + 1 > 7 \quad \rightarrow \quad 25 - 5 + 1 > 7$

$21 > 7$

Since $21 > 7$ is a **TRUE statement**, Region B **belongs** to the solution set.

Therefore $\boldsymbol{R > 3}$ ohms.

Example 2: The height in meters of a projectile shot from the top of a building is given by: $h(t) = -16t^2 + 60t + 25$, where t represents the time in seconds the projectile is in the air.

a) Find the time the projectile id in the air before hitting the ground.
b) Find the time interval that the projectile is above $25\ meters$

Solution 2: a) The projectile hits the ground when $h(t) = 0\ \ (0\ on\ the\ y - axis)$

$-16t^2 + 60t + 25 = 0$ Does not Factor Easily, use Quadratic Equation

$$t = \frac{-60 \pm \sqrt{60^2 - 4(-16)(25)}}{2(-16)} = \frac{-60 \pm \sqrt{5200}}{-32} = -0.378, and\ 4.128$$

Omit Negative Time, so $\boldsymbol{t = 4.128\ seconds}$

b)

$$-16t^2 + 60t + 25 > 25$$

$$-16t^2 + 60t > 0$$

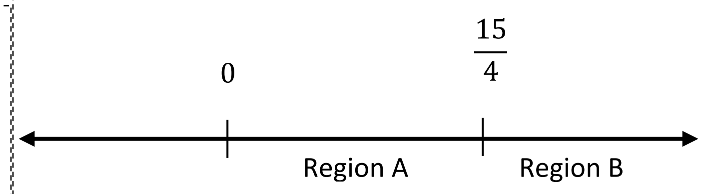
Flip

Divided by a Negative

$$4t^2 - 15t < 0$$

$$t(4t - 15) > 0$$

$$t = 0, \frac{15}{4}$$



Test a Value from Region A: $4t^2 - 15t < 0$

Selected 1

$$4(1)^2 - 15(1) < 0 \quad \rightarrow \quad 4 - 15 < 0$$

$$-11 < 0$$

Since $-11 < 0$ is a **TRUE statement**, Region A **belongs** to the solution set.

Test a Value from Region B: $4t^2 - 15t < 0$

Selected 6

$$4(6)^2 - 15(6) < 0 \quad \rightarrow \quad 144 - 90 < 0$$

$$54 < 0$$

Since $54 < 0$ is a **FALSE statement**, Region B **does NOT belong** to the solution set.

Therefore when: $\boldsymbol{0 < t < \frac{15}{4}}$ **seconds, the ball is above $\boldsymbol{25m}$**

Example 3: The price a stereo will be sold for is given by $s(x) = 200 - 0.1x,\ 0 \le x \le 2000$, where x is the number of stereos produced each day. It costs $18 000 per day to operate the factory and $15 for material to produce each stereo.

a) Find the daily revenue
b) Find the daily cost of producing the stereos
c) Find the interval that produces a profit

Solution 3:

a) Revenue: $R(x) = \textit{the number of stereos product x price per stereo}$

$$= x(200 - 0.1x)$$

$$= \mathbf{200x - 0.1x^2}$$

b) Cost: $C(x) = 18\,000 + 15x$

c) Profit: $P(x) = R(x) - C(x) = (200x - 0.1x^2) - (18\,000 + 15x)$

$$= -0.1x^2 + 185x - 18\,000$$

Use the Quadratic Equation to Solve

$$x = \frac{-185 \pm \sqrt{185^2 - 4(-0.1)(-18\,000)}}{2(-0.1)} = 103.04, 1746.96$$

Since the number of stereos must be a whole number (can’t have a decimal or fraction of a stereo)

$$104 < x < 1746$$

Will produce a profit.

Section 5.5 – Practice Problems

1. A store sells two brands of computers. It stocks twice as many sets of brand X than brand Y. It must carry at least 10 computers of brand Y. There is room for not more than 60 computers. Find a system of inequalities that describe all possibilities.

2. A person has $16 000 to invest in stocks and bonds, with at least $2000 in stocks, and at least three times that amount of bonds. Find a system of inequalities that describes the possibilities of the investment.

3. A rectangular dog run is to be built with $120\ ft$ of fencing. If one side of the dog run uses the side of a barn, for what values will the width have the enclosed area less than or equal to $1600 ft^2$

4. The number, N, of bacteria per m^3, found in unchlorinated water depends on the temperature, T, in degrees Celsius. If the number of bacteria is given by: $N = 60T - T^2$, at what temperature will the number of bacteria exceed $500\ units/m^3$?

5. A window manufacturer projects that profit in dollars from making x windows per week will be: $P(x) = -x^2 + 45x - 450$. How many windows per week must be manufactured to make a profit?

6. The average cost in dollars of producing x units of golf clubs is: $x^2 - 18x + 140$. Determine the number of golf clubs to produce each hour to keep the cost below \$75 per club.

7. The profit for a construction company is: $P(x) = -0.1x^2 + 50x - 5250$, where x is the total number of hours worked by the employees in a week. What total hours worked by the employees will produce a profit for the company?

8. The height in meters of a ball thrown upward from a building is: $h(t) = -4.9t^2 + 29.4t + 24.3$, where t is the time in seconds after releasing the ball. During what time interval will the ball be above $40\ meters$?

Answer Key – Section 5.5

1.	$x \geq 2y; y \geq 10; x + y \leq 60$
2.	$s \geq 2000; b \geq 6000; s + b \leq 16\,000$
3.	$0 < w \leq 20, 40 \leq w < 60$
4.	$10℃ < T < 50℃$
5.	$15 < x < 30\ windows$
6.	$5 < x < 13\ golf\ clubs\ per\ hour$
7.	$150 < x < 350\ hours$
8.	$0.59 < t < 5.41\ seconds$

Extra Work Space