Section 5.5 – Practice Problems

Discuss and sketch the curve in the following questions. Include:
A: Domain
B: Intercepts
C: Symmetry
D: Asymptotes
E: Intervals of Increase and Decrease
F: Local Maximums and Minimums (if any)
G: Concavity and Points of Inflection

$3x^5 - 10x^3 + 45x$

$\underbrace{x(3x^4 - 10x^2 + 45)}_{\text{no more factors}}$

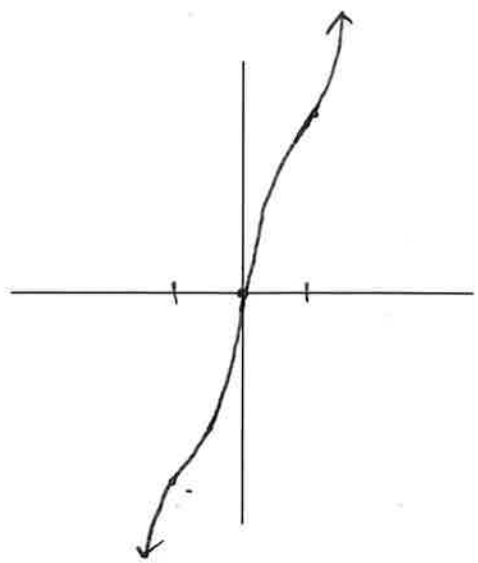
1. $3x^5 - 10x^3 + 45x$

A: D: $\mathbb{R}$ all Real #'s

B: y-int $(0,0)$
x-int $(0,0)$

C: $f(-x) = 3(-x)^5 - 10(-x)^3 + 45(-x)$
$= -3x^5 + 10x^3 - 45x$
$-(3x^5 - 10x^3 + 45x)$

$f(-x) = -f(x)$ odd symmetry (symmetric about the origin)



D: VA: none

HA: $\lim_{x\to\infty} \infty$

$\lim_{x\to-\infty} -\infty$

E: $15x^4 - 30x^2 + 45$
$15(x^4 - 2x^2 + 3)$
$15(x^4 - 2x^2 + 1 + 2)$
$15((x^2-1)^2 + 2)$
↑
always positive, always increases

F: No max/min

G: $f''(x) = 60x^3 - 60x$
$= 60x(x^2 - 1)$
crit pts
$x = 0$
$x = \pm 1$

Interval	$60x$	(x^2-1)	$f''(x)$	$f(x)$
$(-\infty, -1)$	−	+	−	CD
$(-1, 0)$	−	−	+	CU
$(0, 1)$	+	−	−	CD
$(1, \infty)$	+	+	+	CU

Inflection
$f(-1) = -38$
$f(0) = 0$
$f(1) = 38$

2. $y = (x^2 - 1)^3$

A: All Real #'s

B: y-int: $(0,-1)$

x-int: $(1,0)$ $(-1,0)$

C: $f(-x) = ((-x)^2 - 1)^3$

$(x^2-1)^3$

symmetry $f(-x) = f(x)$

even symmetry

symmetric about y-axis

D: VA: none

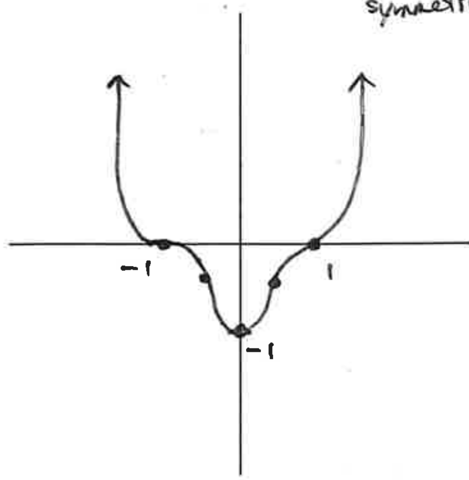
HA: $\lim\limits_{x \to \infty} \infty$

$\lim\limits_{x \to -\infty} \infty$

E: $y' = 3(x^2-1)^2(2x)$

$= 6x(x^2-1)^2$

always positive



F: $f(0) = -1$ local min

Decreases $x < 0$

increases $x > 0$

G: $y'' = 6x(2(x^2-1)(2x)) + 6(x^2-1)^2$

$= 24x^2(x^2-1) + 6(x^2-1)^2$

$= 6(x^2-1)(4x^2 + (x^2-1))$

$= 6(x^2-1)(5x^2-1)$

crit pts

$x = \pm 1$ $x = \pm \frac{1}{\sqrt{5}}$

Interval	(x^2-1)	$(5x^2-1)$	$f''(x)$	$f(x)$
$(-\infty, -1)$	+	+	+	CU
$(-1, -\frac{1}{\sqrt{5}})$	−	+	−	CD
$(-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}})$	−	−	+	CU
$(\frac{1}{\sqrt{5}}, 1)$	−	+	−	CD
$(1, \infty)$	+	+	+	CU

Inflection Pts.

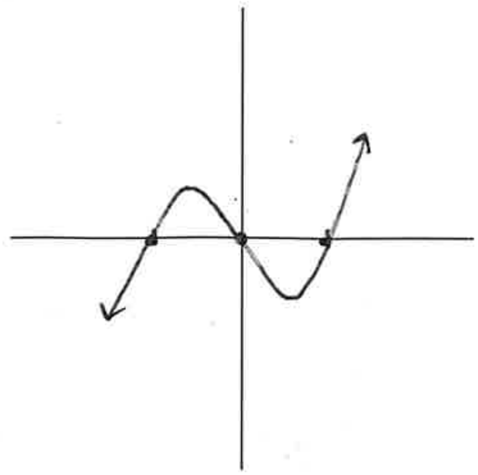
$(-1, 0)$

$(-\frac{1}{\sqrt{5}}, -\frac{64}{125})$

$(\frac{1}{\sqrt{5}}, -\frac{64}{125})$

$(1, 0)$

3. $y = x - \sqrt[3]{x}$ $\longrightarrow$ $x - x^{\frac{1}{3}}$ $\rightarrow$ $x^{\frac{1}{3}}(x^{2/3} - 1)$



A: All Real #'s

B: y-int: (0,0)
x-int: (0,0) (1,0) (-1,0)

C: $f(-x) = -x - \sqrt[3]{-x}$
$= -x + \sqrt[3]{x}$ $f(-x) = -f(x)$
$= -(x - \sqrt[3]{x})$ symmetric about origin

D: No vertical asymptotes
$\lim_{x \to \infty} = \infty$ HA none
$\lim_{x \to -\infty} = -\infty$

E: $f'(x) = 1 - \frac{1}{3}x^{-2/3}$ ppc at 0 $\rightarrow$ $\frac{3x^{2/3} - 1}{3x^{2/3}}$ $3x^{2/3} - 1 = 0$

$\uparrow$ always positive $x^{2/3} = \frac{1}{3}$

when $x < 0$ $f'(x) > 0$ increasing $x = \pm \frac{1}{3}^{3/2}$
$x > 0$ $f'(x) < 0$ decreasing $x = 0$ is a ppc

Interval		
$(-\infty, -\frac{1}{3}^{3/2})$	+	increasing
$(-\frac{1}{3}^{3/2}, 0)$	-	decreasing
$(0, \frac{1}{3}^{3/2})$	-	decreasing
$(\frac{1}{3}^{3/2}, \infty)$	+	increasing

local max $f(-\frac{1}{3^{3/2}}) = \frac{2\sqrt{3}}{9}$

local min $f(\frac{1}{3^{3/2}}) = -\frac{2\sqrt{3}}{9}$

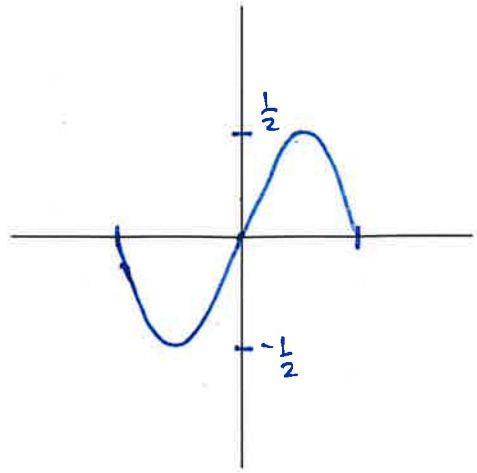
G: $f''(x) = \frac{2}{9}x^{-5/3}$ $x = 0$ ppc $\leftarrow$ inflection pt $f(0) = 0$
$x > 0$ $f''(x) = CU$
$x < 0$ $f''(x) = CD$

4. $y = x\sqrt{1-x^2}$ A: D: $\{x \mid x \in \mathbb{R};\ -1 \le x \le 1\}$

B: y-int: $(0,0)$
x-int: $(0,0)$

D: no asymptotes

C: $f(-x) = -x\sqrt{1-x^2}$
$f(-x) = -f(x)$
so symmetric about the origin



E: $f'(x) = x \dfrac{1(-2x)}{2\sqrt{(1-x^2)}} + \sqrt{1-x^2}$

$f'(x) = \dfrac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}$

$= \dfrac{-x^2 + 1 - x^2}{\sqrt{1-x^2}} = \dfrac{1-2x^2}{\sqrt{1-x^2}}$

$1-2x^2 = 0 \rightarrow x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

$x > \frac{1}{\sqrt{2}}$ $f'(x) < 0$ dec
$x < \frac{1}{\sqrt{2}}$ $f'(x) > 0$ inc
$x > -\frac{1}{\sqrt{2}}$ $f'(x) > 0$ inc
$x < -\frac{1}{\sqrt{2}}$ $f'(x) < 0$ dec

inc: $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ dec: $\left(-1, -\frac{1}{\sqrt{2}}\right)$ $\left(\frac{1}{\sqrt{2}}, 1\right)$

F: local min $f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$
local max $f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

G: $f''(x)$ $\sqrt{1-x^2}(-4x) - (1-2x^2)\left(\dfrac{1(-2x)}{2\sqrt{1-x^2}}\right)$ / denom^2

$-4x\sqrt{1-x^2} + \dfrac{2x(1-2x^2)}{2\sqrt{1-x^2}}$ / denom^2

$= \dfrac{\dfrac{-4x(1-x^2) + x(1-2x^2)}{\sqrt{1-x^2}}}{\sqrt{1-x^2}^{\,2}}$

$x=0$ $x = \pm\sqrt{3/2}$ (outside domain)

$\dfrac{-4x + 4x^3 + x - 2x^3}{\sqrt{1-x^2}^{\,3}} = \dfrac{2x^3 - 3x}{\sqrt{1-x^2}^{\,3}} = \dfrac{x(2x^2-3)}{\sqrt{1-x^2}^{\,3}}$

Interval	$f''(x)$	
$(-1,0)$	+	CU
$(0,1)$	−	CD

Inflection at $(0,0)$