Section 5.5 – Practice Problems

1. On average, a car depreciates 15% in value each year. How long does it take for a new car worth $40 000 to depreciate to $10 000?

Type of Decay

$A = A_0(x)^{\frac{t}{T}}$

Depreciates 15% mean $x = 0.85$

$A = 10000 \quad A_0 = 40000 \quad T = 1 \quad t = ?$

$10000 = 40000(0.85)^{\frac{t}{1}}$

$\frac{1}{4} = 0.85^t \rightarrow 0.25 = 0.85^t$

$\log_{0.85} 0.25 = t$

$\frac{\log 0.25}{\log 0.85} = t \qquad \boxed{t = 8.53 \text{ yrs}}$

2. If you invest $10 000, how long does it take to become a millionaire ($1 000 000) if you invest in an account at 12% compounded:
a) quarterly b) continuously

$P = 10000$
$A = 1000000$
$r = 0.12$

a) $n = 4$

$A = P(1+\frac{r}{n})^{n \cdot t}$

$1000000 = 10000(1+\frac{0.12}{4})^{4t} \rightarrow 100 = 1.03^{4t}$

$\log_{1.03} 100 = 4t \rightarrow \frac{\log 100}{\log 1.03} = 4t$

$t = \frac{\log 100}{4 \log 1.03} = \boxed{38.95 \text{ yrs}}$

b) $A = Pe^{rt}$

$1000000 = 10000e^{0.12t}$

$100 = e^{0.12t} \rightarrow \ln 100 = \ln e^{0.12t}$

$\ln 100 = \ln e^{0.12t}$

$\ln 100 = 0.12t$

$t = \frac{\ln 100}{0.12}$

$\boxed{t = 38.38 \text{yrs}}$

3. What interest rate is needed if money is to triple in 15 years if the interest is compounded:
a) semi-annually b) continuously

a) $A = P(1+\frac{r}{n})^{n \cdot t} \qquad A = 3P$

$3P = P(1+\frac{r}{2})^{2(15)}$

$3 = (1+\frac{r}{2})^{30}$ take $\sqrt[30]{}$ or $\frac{1}{30}$

$3^{\frac{1}{30}} = 1 + \frac{r}{2} \rightarrow \frac{r}{2} = 3^{\frac{1}{30}} - 1$

$\frac{r}{2} = 3^{\frac{1}{30}} - 1 \rightarrow r = 2(3^{\frac{1}{30}} - 1)$

$\boxed{r = 7.46\%}$

b) $A = Pe^{rt} \qquad 3P = Pe^{r(15)}$

$3 = e^{15r} \qquad \ln 3 = \ln e^{15r}$

$\ln 3 = 15r$

$r = \frac{\ln 3}{15} = \boxed{7.32\%}$

4. It is estimated that 20% of Herlonium decays in 30 hours. What is the half-life of the substance?

Method 1

$A = A_0 x^{\frac{t}{T}} \qquad x = 0.80$

$0.8 = 1\left(\frac{1}{2}\right)^{\frac{30}{T}}$

$0.8 = \left(\frac{1}{2}\right)^{\frac{30}{T}}$

$\log_{\frac{1}{2}} 0.8 = \frac{30}{T}$

$\frac{\log 0.8}{\log 0.5} = \frac{30}{T}$

$T = \frac{30 \log 0.5}{\log 0.8}$

$\boxed{T = 93.2 \text{hrs}}$

$A = A_0 e^{kt}$

$0.8 = 1e^{kt}$

$0.8 = e^{k30}$

$\ln 0.8 = \ln e^{30k}$

$\ln 0.8 = 30k$

$k = \frac{\ln 0.8}{30}$

So now

$\frac{1}{2} = 1e^{\left(\frac{\ln(0.8)}{30}\right)t}$

$\ln 0.5 = \ln e^{\left(\frac{\ln 0.8}{30}\right)t}$

$\ln 0.5 = \left(\frac{\ln 0.8}{30}\right)t$

$\frac{30 \ln 0.5}{\ln 0.8} = t$

$\boxed{t = 93.2 \text{hr}}$

larger pH means less acidic or more alkaline
smaller pH means more acidic

5. The pH scale measures the acidity $(0-7)$ or alkalinity $(7-14)$ of a solution with 7 being neutral water. It is a logarithmic scale in base 10. A pH of 9 is 10 times more alkaline than a pH of 8, and a pH of 5 is 10 times more acidic than a pH of 6.

   a) If lemon juice has a pH of 2.1, how many times more acidic is it than black coffee, with a pH of 4.8?
   b) If tomato juice has a pH of 4.2 and is 75 times more acidic than milk. What is the pH of the milk?

a) $4.8 - 2.1 = 2.7$ $\quad 10^{2.7} = 501$

$\boxed{\text{juice 501 times more acidic}}$

b) $10^x = 75$

$\log 75 = x \quad x = 1.9 \quad$ so, $4.2 + 1.9 = \boxed{6.1}$

6. If Vancouver has a population of 400 000 and is growing at a rate of 2% annually, and Surry has a population of 300 000 and is growing at a rate of 3% annually, in how many years will Surrey catch up to Vancouver in population?

pop growth 2% means 102% or 1.02

need A = A

$A = A_0 \times ^{t/T}$

$400\,000(1.02)^{\frac{t}{1}} = 300\,000(1.03)^{\frac{t}{1}}$

$1.02^t = \frac{3}{4}(1.03)^t$

$\log 1.02^t = \log \frac{3}{4}(1.03)^t$

$t \log 1.02 = \log 3/4 + t\log(1.03)$

$t\log 1.02 - t\log 1.03 = \log 3/4$

$t(\log 1.02 - \log 1.03) = \log 3/4$

$t = \dfrac{\log 3/4}{\log \frac{1.02}{1.03}}$

$\boxed{t = 29.5\text{ yrs}}$

7. Find the time needed for money to triple at 8% compounded:
   a) Daily b) continuously

a) $A = P(1 + r/n)^{n \cdot t}$ $\quad A = 3P$

$3P = P(1 + \frac{r}{n})^{n \cdot t} \rightarrow 3 = (1 + \frac{0.08}{365})^{365t}$

$\log 3 = \log\left(1 + \frac{0.08}{365}\right)^{365t}$

$\log 3 = 365t\left[\log\left(1 + \frac{0.08}{365}\right)\right]$

$t = \dfrac{\log 3}{365\log\left(1 + \frac{0.08}{365}\right)} = \boxed{13.73\text{ yrs}}$

b) $A = Pe^{rt} \rightarrow 3 = 1e^{rt} \rightarrow 3 = e^{0.08t}$

$\ln 3 = \ln e^{0.08t} \rightarrow \ln 3 = 0.08t$

$t = \dfrac{\ln 3}{0.08}$ $\quad \boxed{t = 13.73\text{ yrs}}$

8. The amount of a chemical in grams that will dissolve in a solution is given by $C = 8e^{0.3t}$ where t is the temperature in Celsius of the solution. Find t when $C = 100\ grams$

$C = 8e^{0.3t} \rightarrow 100 = 8e^{0.3t}$

$12.5 = e^{0.3t}$

$\ln 12.5 = \ln e^{0.3t}$

$\ln 12.5 = 0.3t$

$t = \dfrac{\ln 12.5}{0.3}$

$t = 8.42$

$\boxed{t = 8.42°C}$

9. The population if Toronto is given by $P(t) = 4\,000\,000e^{0.012t}, where\ t = 0$ corresponds to year 2000. What year will the population reach 6 400 000?

$P(t) = 4\,000\,000\,e^{0.012t}$

$6\,400\,000 = 4000000\,e^{0.012t}$

$1.6 = e^{0.012t}$

$\ln 1.6 = \ln e^{0.012t}$

$\ln 1.6 = 0.012t$

$t = \dfrac{\ln 1.6}{0.012} = 39.2$ yrs

$\boxed{\text{so by 2039}}$ population will reach 6400 000

10. A biologist studying bacteria determines that a certain culture grows exponentially, such that the colony doubles every 4 days. If the initial colony had 1200 bacteria, how long did it take to have 100 000 bacteria?

Method 1: $A = A_0 x^{\frac{t}{T}}$

$T = 4$
$A = 100000$
$A_0 = 1200$
$x = 2$
$t = ?$

$100\,000 = 1200(2)^{\frac{t}{4}}$

$\dfrac{250}{3} = 2^{t/4}$

$\log_2 \dfrac{250}{3} = \dfrac{t}{4}$

$4\left(\dfrac{\log \frac{250}{3}}{\log 2}\right) = t$ $\boxed{t = 25.5 \text{ days}}$

Method 2: use $A = A_0 e^{kt}$ and try it out

11. The half-life of Lukatium is 5570 years. If 500 milligrams are present today, how much is present after 2500 years?

$A = A_0 e^{kt}$ k first

$\frac{1}{2} = 1e^{k5570} \rightarrow \ln \frac{1}{2} = \ln e^{5570k}$

$\ln \frac{1}{2} = 5570k$

$k = \dfrac{\ln \frac{1}{2}}{5570}$

So, $A = 500e^{\left(\frac{\ln \frac{1}{2}}{5570}\right)(2500)}$ $\boxed{A = 366.3 \text{ mg}}$

$A = A_0 x^{\frac{t}{T}}$

$t = 2500$
$T = 5570$
$x = \frac{1}{2}$
$A_0 = 500$

$A = 500\left(\frac{1}{2}\right)^{\frac{2500}{5570}}$

$\boxed{A = 366.3 \text{ mg}}$

See Website for Detailed Answer Key

Extra Work Space