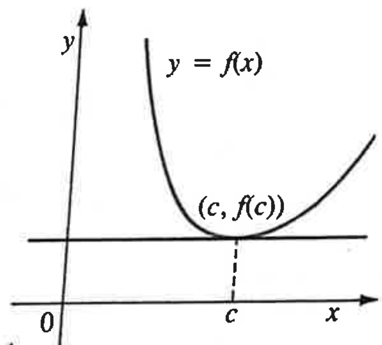
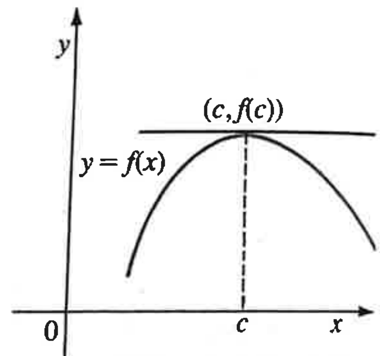
5.4 The Second Derivative Test

We can use the second derivative test in locating local maximum and minimums as well. We have to assume that $f''(x)$ exists and is continuous throughout the entire domain of f. The graphs below show the relationship of the first and second derivative.

$f'(x) = 0$ $f''(x) > 0$	$f'(x) = 0$ $f''(x) < 0$
Local Minimum	Local Maximum
 $f'(c) = 0, f''(c) > 0$	 $f'(c) = 0, f''(c) < 0$

Second Derivative Test
If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

Ex.1 Find the local maximum and minimum values of $f(x) = x^3 - 12x + 5$.

crit numbers first

$f'(x) = 3x^2 - 12$

$= 3(x-4)$

$= 3(x+2)(x-2)$

$x = -2$

$x = 2$

Second Derivative

$f''(x) = 6x$

$f'(2) = 0$ $f''(2) = 12$ $12 > 0$

so local minimum

$f'(-2) = 0$ $f''(-2) = -12$ $-12 < 0$

so local maximum

$= x^3(x-8)$ roots $(0,0)$ and $(8,0)$

Ex. 2 Find the maximum and minimum values of $y = x^4 - 8x^3$. Use these, together with concavity and points of inflection, to sketch the curve.

$y' = 4x^3 - 24x^2$

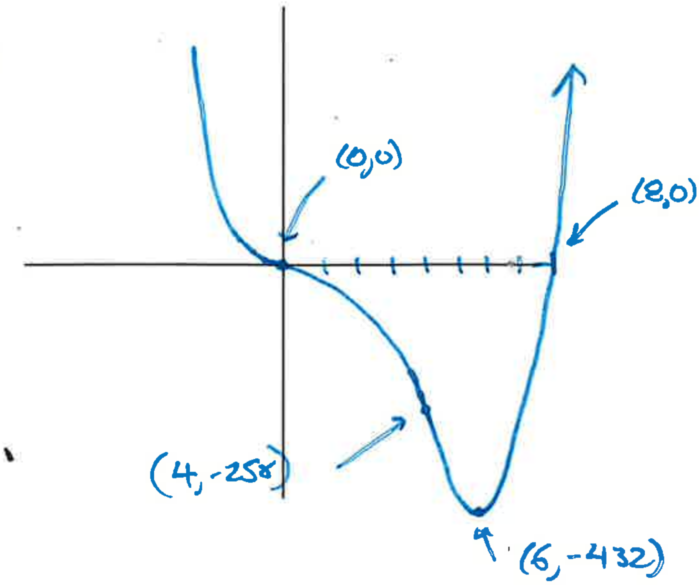
$= 4x^2(x-6)$ ← $f'(0) = 0$ no local max or min

$y'' = 4x^2(1) + (x-6)(8x)$

$= 4x^2 + 8x^2 - 48x$

$= 12x^2 - 48x$

$= 12x(x-4)$ → 2nd Deriv Test



crit numbers: $x = 0$

$x = 6$

$f''(0) = 0$ ← no info given

$f''(6) = 144$ ← local min

crit numbers here

$x = 0$

$x = 4$

Interval	$12x$	$(x-4)$	$f''(x)$	$f(x)$
$(-\infty, 0)$	$-$	$-$	$+$	cu
$(0, 4)$	$+$	$-$	$-$	cd
$(4, \infty)$	$+$	$+$	$+$	cu

Inflection Points

$(0,0)$

$(4,-256)$

$f(6) = -432$

Homework Questions

Practice Problems: #1-3