Section 5.4 – Practice Problems

1. Solve for the unknown value. Check for extraneous roots.

a) $\log_5(2x-1)+\log_5(x-2)=1$

$\log_5(2x-1)(x-2)=1$

$\log_5(2x^2-5x+2)=1$

$5^1=2x^2-5x+2 \rightarrow 2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x=-\frac{1}{2}$ ← Reject gives negative log object

$\boxed{x=3}$

b) $\log_2(2-2x)+\log_2(1-x)=5$

if $x=5$ negative

$\log_2(2-2x)(1-x)=5$

$\log_2(2-4x+2x^2)=5$

$2^5=2x^2-4x+2$

$32=2x^2-4x+2 \rightarrow 2x^2-4x-30=0$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=5$ ← reject

$\boxed{x=-3}$

c) $\frac{1}{2}-\log_{16}(x-3)=\log_{16}x$

$\frac{1}{2}=\log_{16}x+\log_{16}(x-3)$

$\frac{1}{2}=\log_{16}x(x-3) \rightarrow 16^{\frac{1}{2}}=x(x-3)$

$4=x^2-3x \rightarrow 0=x^2-3x-4$

$0=(x-4)(x+1)$

$\boxed{x=4}$

$x=-1$ ← reject

d) $\log_2(3x+1)+\log_2(x-1)=\log_2(10x+14)$

$\log_2(3x+1)(x-1)=\log_2(10x+14)$

$\log_2(3x+1)(x-1)-\log_2(10x+14)=0$

$\log_2\left[\frac{(3x+1)(x-1)}{(10x+14)}\right]=0 \rightarrow 2^0=\frac{(3x+1)(x-1)}{(10x+14)}$

$1=\frac{(3x+1)(x-1)}{(10x+14)} \rightarrow 10x+14=(3x+1)(x-1)$

$0=3x^2-3x+x-1-10x-14$

$3x^2-3x-9x-15=0$

$3(x^2-4x-5)=0$

$3(x-5)(x+1)=0$

$\boxed{x=5}$

$x=-1$ ← gives negative log object

e) $\log_4(3x^2-5x-2)-\log_4(x-2)=1$

$\log_4 \frac{(3x^2-5x-2)}{(x-2)}=1$

$\log_4 \frac{(3x+1)\cancel{(x-2)}}{\cancel{(x-2)}}=1$

$\log_4(3x+1)=1 \rightarrow 4^1=3x+1$

$4=3x+1$

$3=3x$

$x=1 \leftarrow$ reject

$\boxed{\text{NO SOLUTION}}$

f) $\log x+\log(29-x)=2$

$\log(x(29-x))=2$

$\log[29x-x^2]=2$

$10^2=29x-x^2$

$x^2-29x+100=0$

$(x-25)(x-4)=0$

$\left.\begin{array}{l} x=25 \\ x=4 \end{array}\right\}$ both work

g) $\log_{25}(x-1)+\log_{25}(x+3)=\log_7\sqrt{7}$

$\log_{25}(x-1)(x+3)=\log_7 7^{\frac{1}{2}}$

$\log_{25}(x^2+2x-3)=\frac{1}{2}$

$25^{\frac{1}{2}}=(x^2+2x-3)$

$5=x^2+2x-3$

$0=x^2+2x-8$

$0=(x+4)(x-2)$

$x=-4 \leftarrow$ reject negative log

$\boxed{x=2}$

h) $2\log(4-x)-\log 3=\log(10-x)$

$\log(4-x)^2-\log 3=\log(10-x)$

$\log(4-x)^2-\log 3-\log(10-x)=0$

$\log(4-x)^2-[\log 3+\log(10-x)]=0$

$\log(4-x)^2-[\log 3(10-x)]=0$

$\log\frac{(4-x)^2}{30-3x}=0 \rightarrow 10^0=\frac{(4-x)^2}{30-3x}$

$1=\frac{(4-x)^2}{30-3x} \rightarrow 30-3x=(4-x)^2$

$30-3x=16-8x+x^2$

$x^2-8x+3x+16-30=0$

$x^2-5x-14=0 \qquad (x-7)(x+2)=0$

$x=7 \leftarrow$ reject negative log

$\boxed{x=-2}$

i) $2\log_2(x+2) - \log_2(3x-2) = 2$

$\log_2 (x+2)^2 - \log_2(3x-2) = 2$

$\log_2 \frac{(x+2)^2}{(3x-2)} = 2 \rightarrow 2^2 = \frac{(x+2)^2}{(3x-2)}$

$4(3x-2) = (x+2)(x+2)$

$12x - 8 = x^2 + 4x + 4$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$\left.\begin{array}{l} x=6 \\ x=2 \end{array}\right\}$ both work

j) $2\log_4 x + \log_4(x-2) - \log_4 2x = 1$

$\log_4 x^2 + \log_4(x-2) - \log_4 2x = 1$

$\log_4 \frac{x^2(x-2)}{2x} = 1 \rightarrow 4^1 = \frac{x^2(x-2)}{2x}$

$8x = x^3 - 2x^2$

$x^3 - 2x^2 - 8x = 0$

$x(x^2 - 2x - 8) = 0 \rightarrow x(x-4)(x+2) = 0$

$x = 0$ ← reject

$\boxed{x = 4}$

$x = -2$ ← reject

2. Express in the terms stated in each question

a) If $\log x = a$ and $\log y = b$, what is $\log\frac{x^3}{y^2}$ in terms of a and b

$\log \frac{x^3}{y^2} \rightarrow \log x^3 - \log y^2$

$3\log x - 2\log y$

$\boxed{3a - 2b}$

b) If $a = \log_2 3$, find $\log_{16} 81$ in terms of a

$\log_{16} 81 \rightarrow \frac{\log 81}{\log 16} \rightarrow \frac{\log 3^4}{\log 2^4} = \frac{\cancel{4}\log 3}{\cancel{4}\log 2}$

$\log_2 3$ (Reverse base change)

$\boxed{\log_2 3 = a}$

c) If $\log 3 = a$ and $\log 25 = b$ determine an expression for $\log\frac{9}{5}$ in terms of a and b

$\log \frac{9}{5} \rightarrow \log 9 - \log 5$

$\log 3^2 - \log 25^{\frac{1}{2}}$

$2\log 3 - \frac{1}{2}\log 25$

$\boxed{2a - \frac{1}{2}b}$

d) If $a = \log 2$ *and* $b = \log 3$, what is $\log\frac{25}{9}$ in terms of a and b

$\log \frac{25}{9} \rightarrow \log 25 - \log 9$

$\log 5^2 - \log 3^2$

$2\log 5 - 2\log 3$

$2\log \frac{10}{2} - 2\log 3$

$2[\log 10 - \log 2] - 2\log 3$

$2[1 - \log 2] - 2\log 3$

$2 - 2\log 2 - 2\log 3$

$\boxed{2 - 2a - 2b}$

e) If $\log A = 2$ *and* $\log B = 3$, what is:

i) $\log A/B^2$

$\log A - \log B^2$

$\log A - 2\log B$

$2 - 2(3)$

$2 - 6 \rightarrow \boxed{-4}$

ii) $(\log AB)^2$

$(\log A + \log B)^2$

$(2+3)^2$

5^2

$\boxed{25}$

f) If $\log 2 = a$ and $\log 3 = b$, what is $\log_5 12$ in terms of a and b

$\log_5 12 \rightarrow \dfrac{\log 12}{\log 5} \rightarrow \dfrac{\log(3\cdot 4)}{\log \frac{10}{2}}$

$\dfrac{\log 3 + \log 4}{\log 10 - \log 2} \rightarrow \dfrac{\log 3 + \log 2^2}{1 - \log 2}$

$\dfrac{2\log 2 + \log 3}{1 - \log 2} \rightarrow \boxed{\dfrac{2a+b}{1-a}}$

g) If $\log AB = 8$ and $\log B = -4$ then what value does A equal

$\log AB \rightarrow \log A + \log B$

$8 = \log A + (-4)$

$\boxed{\log A = 12}$

h) If $\log 3 = x$, $\log 5 = y$, *and* $\log 7 = z$, then find $\log_2 \sqrt[3]{12.6}$ in terms of x, y, z

$\log_2 \sqrt[3]{126} \rightarrow \log_2 \left(\frac{126}{10}\right)^{\frac{1}{3}} \rightarrow \frac{1}{3}\left[\log_2\left(\frac{126}{10}\right)\right]$

$\frac{1}{3}\left[\log_2 126 - \log_2 10\right] \rightarrow \frac{1}{3}\left[\frac{\log 126}{\log 2} - \frac{\log 10}{\log 2}\right]$

$\frac{1}{3}\left[\frac{\log(2\cdot 7\cdot 9)}{\log 2} - \frac{\log(2\cdot 5)}{\log 2}\right]$

$\frac{1}{3}\left[\frac{\cancel{\log 2} + \log 7 + \log 9 - \cancel{\log 2} - \log 5}{\log 2}\right]$

$\frac{1}{3}\left[\frac{\log 7 + \log 3^2 - \log 5}{\log 2}\right]$

$\frac{1}{3}\left[\frac{z + 2x - y}{\log \frac{10}{5}}\right] \rightarrow \frac{1}{3}\left[\frac{2x - y + z}{\log 10 - \log 5}\right]$

$\rightarrow \frac{1}{3}\left[\frac{2x-y+z}{1-y}\right] \rightarrow \boxed{\dfrac{2x-y+z}{3(1-y)}}$

i) If $\log_8 3 = a$ and $\log_3 5 = b$, find $\log 5$ in terms of a and b

$a = \log_8 3 \rightarrow a = \dfrac{\log 3}{\log 8}$

$b = \dfrac{\log 5}{\log 3}$

$ab = \dfrac{\log 3}{\log 8} \cdot \dfrac{\log 5}{\log 3}$

$\downarrow$

$ab = \dfrac{\log 5}{\log 8} \rightarrow ab\log 8 = \log 5$

$\updownarrow$

$ab\log 2^3 \rightarrow 3ab\log 2 \rightarrow 3ab\log \frac{10}{5}$

$3ab(\log 10 - \log 5) \rightarrow \underbrace{3ab\log 10}_{1} - 3ab\log 5$

$3ab - 3ab\log 5 = \log 5$

$3ab = \log 5 + 3ab\log 5$

$3ab = \log 5(1 + 3ab)$

$\boxed{\log 5 = \dfrac{3ab}{3ab+1}}$

3. Solve the following

$\log 10 = 1$

a) Solve for B

$$A = \log 3B - \log C$$

$A + \log C = \log 3B$

$A + \log C = \log 3 + \log B$

$A + \log C - \log 3 = \log B$

$A + \log\left(\frac{C}{3}\right) = \log B$

$10^{A + \log\frac{C}{3}} = B$

$B = 10^A \cdot 10^{\log\frac{C}{3}}$ Rule 7

$B = 10^A \cdot \frac{C}{3}$

$\boxed{B = \frac{C10^A}{3}}$

b) Solve for A

$$1 + \log(AB) = \log C$$

$1 + \log A + \log B = \log C$

$\log A = \log C - \log B - 1$

$\log A = \log C - [\log B + \log 10]$

$\log A = \log C - [\log 10B]$

$\log A = \log \frac{C}{10B} \rightarrow \log A = \log \frac{C}{10B}$

$A = \frac{C}{10B}$

c) Solve for A

$$3\log A + \log B = \log C$$

$3\log A = \log C - \log B$

$3\log A = \log \frac{C}{B}$

$\log A^3 = \log \frac{C}{B}$

$A^3 = \frac{C}{B}$

$A = \sqrt[3]{\frac{C}{B}}$

d) Solve for x

$$\log A = \log B - C\log x$$

$\log A - \log B = -C\log x$

$C\log x = \log B - \log A$

$\log x^C = \log \frac{B}{A}$

$x^C = \frac{B}{A}$ $\quad x = \sqrt[C]{\frac{B}{A}}$

Take log of both sides

4. Solve for x but in terms of logarithmic functions

a) $2^{3x} = 5^{x-1}$

$\log 2^{3x} = \log 5^{x-1}$

$3x\log 2 = (x-1)\log 5$

$3x\log 2 = x\log 5 - \log 5$

$3x\log 2 - x\log 5 = -\log 5$

$x(3\log 2 - \log 5) = -\log 5$

$x(\log 2^3 - \log 5) = -\log 5$

$\cancel{-1}x(\log 5 - \log 8) = \cancel{-}\log 5$

$x = \frac{\log 5}{\log 5 - \log 8} \rightarrow \frac{\log 5}{\log \frac{5}{8}} \rightarrow \boxed{\log_{5/8} 5}$

b) $7^{2x-1} = 17^x$

$\log 7^{2x-1} = \log 17^x \rightarrow (2x-1)\log 7 = x\log 17$

$2x\log 7 - \log 7 = x\log 17$

$2x\log 7 - x\log 17 = \log 7$

$x(2\log 7 - \log 17) = \log 7$

$x(\log 49 - \log 17) = \log 7$

$x\log\frac{49}{17} = \log 7$

$x = \frac{\log 7}{\log\frac{49}{17}} = \log_{\frac{49}{17}} 7$

Reverse Change of Base

Same as 4a

c) $3^{x-1} = 9 \cdot 10^x$

$3^{x-1} = 3^2 \cdot 10^x \rightarrow \frac{3^{x-1}}{3^2} = 10^x$

$3^{(x-1)-2} = 10^x \rightarrow 3^{x-3} = 10^x$

$\log 3^{x-3} = \log 10^x$

$(x-3)\log 3 = x$

$x\log 3 - 3\log 3 = x$

$x\log 3 - x = 3\log 3$

$x((\log 3 - 1) = \log 3^3$

$x(\log 3 - \log 10) = \log 27$

$x = \frac{\log 27}{\log\frac{3}{10}} \rightarrow \boxed{\log_{3/10} 27}$

d) $7^{x-1} = 2 \cdot 5^{1-2x}$

$7^{x-1} = 2 \cdot 5^{-2x+1} \rightarrow 7^{x-1} = 2 \cdot 5 \cdot 5^{-2x}$

$7^{x-1} = 10 \cdot 5^{-2x}$

$\log 7^{x-1} = \log(10 \cdot 5^{-2x})$

$(x-1)\log 7 = \log 10 + \log 5^{-2x}$

$x\log 7 - \log 7 = 1 + (-2x)\log 5$

$x\log 7 + 2x\log 5 = 1 + \log 7$

$x(\log 7 + 2\log 5) = \log 10 + \log 7$

$x(\log 7 + \log 25) = \log(10 \cdot 7)$

$x\log 7 \cdot 25 = \log 70 \rightarrow x\log 175 = \log 70$

$x = \frac{\log 70}{\log 175} = \boxed{\log_{175} 70}$

5. Solve for the unknown

a) $\log_2(\log_8 x) = -1$

$2^{-1} = \log_8 x$

$\frac{1}{2} = \log_8 x$

$8^{\frac{1}{2}} = x$

$x = \sqrt{8}$

$\boxed{x = 2\sqrt{2}}$

b) $\log_2(\log_x(\log_3 27)) = -1$

$2^{-1} = \log_x(\log_3 27)$

$\frac{1}{2} = \log_x(\log_3 27)$

$\frac{1}{2} = \log_x(\log_3 3^3)$

$\frac{1}{2} = \log_x(3\log_3 3) \rightarrow \frac{1}{2} = \log_x 3$

$x^{\frac{1}{2}} = 3$

$x = 3^2$

$\boxed{x = 9}$

c) $\log_{\frac{1}{2}}(\log_4(\log_2 x)) = 1$

$\frac{1}{2}^1 = \log_4(\log_2 x)$

$2^{-1} = \log_4(\log_2 x)$

$4^{\frac{1}{2}} = \log_2 x$

$2 = \log_2 x$

$2^2 = x$

$\boxed{x = 4}$

d) $\log x = \frac{2}{3}\log 27 + 2\log 2 - \log 3$

$\log x = \log 27^{2/3} + \log 2^2 - \log 3$

$\log x = \log \sqrt[3]{27}^2 + \log 4 - \log 3$

$\log x = \log 9 + \log 4 - \log 3$

$\log x = \log \frac{9 \cdot 4}{3}$

$\log x = \log \frac{36}{3}$

$\log x = \log 12$

$\boxed{x = 12}$

e) $\log x = \log 2 + 3\ \log_{\sqrt{10}} y - \log 2z$

$\log x = \log 2 + \log_{\sqrt{10}} y^3 - \log 2z$

$\log x = \log 2 + \dfrac{\log y^3}{\log \sqrt{10}} - \log 2z$

$\log x = \log 2 + \dfrac{\log y^3}{\frac{1}{2}\log 10} - \log 2z$

$\log x = \log 2 + 2\log y^3 - \log 2z$

$\log x = \log 2 + \log y^6 - \log 2z$

$\log x = \log \dfrac{2y^6}{2z}$ $\quad \log x = \log \dfrac{y^6}{z}$ $\quad \boxed{x = \dfrac{y^6}{z}}$

f) $2\log x = -\log a + \log b + 4\log\frac{1}{c}$

$\log x^2 = \log a^{-1} + \log b + \log\left(\frac{1}{c}\right)^4$

$\log x^2 = \log a^{-1} + \log b + \log c^{-4}$

$\log x^2 = \log a^{-1} b c^{-4}$

$\log x^2 = \log \dfrac{b}{ac^4}$

$x^2 = \dfrac{b}{ac^4} \rightarrow x = \sqrt{\dfrac{b}{ac^4}}$

$x = \dfrac{\sqrt{b}}{c^2\sqrt{a}} = \dfrac{\sqrt{b}\sqrt{a}}{c^2\sqrt{a}\sqrt{a}} = \boxed{\dfrac{\sqrt{ab}}{ac^2}}$

6. Determine an equation for the following in terms of logarithmic functions

Take log of both sides

a) $x = \dfrac{a^2}{b^3 \cdot c^{\frac{1}{2}}}$

$\log x = \log\left[\dfrac{a^2}{b^3c^{\frac{1}{2}}}\right]$

$\log x = \log a^2 - \log b^3c^{\frac{1}{2}}$

$\log x = 2\log a - [\log b^3 + \log c^{\frac{1}{2}}]$

$\log x = 2\log a - \log b^3 - \log c^{\frac{1}{2}}$

$\boxed{\log x = 2\log a - 3\log b - \frac{1}{2}\log c}$

b) $x = \dfrac{a^{-2}b^3}{c^{-\frac{1}{2}}}$

$\log x = \log\left[\dfrac{a^{-2}b^3}{c^{-\frac{1}{2}}}\right]$

$\log x = \log a^{-2}b^3 - \log c^{-\frac{1}{2}}$

$\log x = \log a^{-2} + \log b^3 - \log c^{-\frac{1}{2}}$

$\boxed{\log x = -2\log a + 3\log b + \frac{1}{2}\log c}$

c) $x = \dfrac{\sqrt[3]{a^2} \cdot b^{-\frac{2}{5}}}{c^{\frac{1}{2}}}$

$\log x = \log\left[\dfrac{\sqrt[3]{a^2}\, b^{-2/5}}{c^{1/2}}\right]$

$\log x = \log a^{2/3} b^{-2/5} - \log c^{1/2}$

$\log x = \log a^{2/3} + \log b^{-2/5} - \log c^{1/2}$

$\boxed{\log x = \frac{2}{3}\log a - \frac{2}{5}\log b - \frac{1}{2}\log c}$

d) $x = \dfrac{\sqrt{a^5} \cdot b^{-\frac{1}{3}}}{c^3 \cdot d^{-\frac{2}{3}}}$

$\log x = \log\left[\dfrac{\sqrt{a^5}\, b^{-\frac{1}{3}}}{c^3 d^{-2/3}}\right]$

$\log x = \log \sqrt{a^5}\, b^{-\frac{1}{3}} - \log c^3 d^{-2/3}$

$\log x = \log a^{5/2} b^{-\frac{1}{3}} - [\log c^3 + \log d^{-2/3}]$

$\log x = \log a^{5/2} + \log b^{-1/3} - \log c^3 - \log d^{-2/3}$

$\log x = \frac{5}{2}\log a - \frac{1}{3}\log b - 3\log c + \frac{2}{3}\log d$

$\boxed{\log x = \frac{5}{2}\log a + \frac{2}{3}\log d - \frac{1}{3}\log b - 3\log c}$

7. Solve of the unknown value x

a) $\log_2 16^{2x+1} = 8$

$\log_2 16^{2x+1} = 8$

$\underbrace{\log_2 2^{4(2x+1)}}_{\text{Rule 7}} = 8$

$4(2x+1) = 8$

$8x + 4 = 8$

$8x = 4$

$\boxed{x = \frac{1}{2}}$

b) $\log_{16} x + \log_4 x + \log_2 x = 7$

$\dfrac{\log x}{\log 16} + \dfrac{\log x}{\log 4} + \dfrac{\log x}{\log 2} = 7$

$\dfrac{\log x}{\log 2^4} + \dfrac{\log x}{\log 2^2} + \dfrac{\log x}{\log 2} = 7$

$\dfrac{\log x}{4\log 2} + \dfrac{\log x}{2\log 2} + \dfrac{\log x}{\log 2} = 7$

$\dfrac{\log x}{4\log 2} + \dfrac{2\log x}{4\log 2} + \dfrac{4\log x}{4\log 2} = 7$

$\dfrac{7\log x}{4\log 2} = 7 \rightarrow 7\log x = 28\log 2$

$\log x = 4\log 2$

$\log x = \log 16$

$\boxed{x = 16}$

Let $\log x = y$

$\log 2 = z$

c) $\log_9 x + 3\log_3 x = 7$

$$\frac{\log x}{\log 9} + \frac{3\log x}{\log 3} = 7$$

$$\frac{\log x}{2\log 3} + \frac{3\log x}{\log 3} = 7$$

$$\frac{\log x}{2\log 3} + \frac{6\log x}{2\log 3} = 7 \rightarrow \frac{7\log x}{2\log 3} = 7$$

$$7\log x = 7(2\log 3)$$

$$\log x = 2\log 3 \rightarrow \log x = \log 9$$

$$\boxed{x = 9}$$

d) $2\log_4 x - 3\log_x 4 = 5$

$$\frac{2\log x}{\log 4} - \frac{3\log 4}{\log x} = 5$$

$$\frac{2\log x}{2\log 2} - \frac{3\log 4}{\log x} = 5$$

$3\log 2^2$

$6\log 2$

$$\frac{\log x}{\log 2} - \frac{3\log 4}{\log x} = 5$$

$$\frac{\log x \log x - 6\log 2 \log 2}{\log x \log 2} = 5$$

$$(\log x)^2 - 6(\log 2)^2 = 5(\log x \log 2)$$

$$y^2 - 5yz - 6z^2 = (y-6z)(y+1z) = 0$$

$$(\log x - 6\log 2)(\log x + \log 2) = 0$$

$$(\log x - \log 64)(\log x + \log 2)$$

$\log x = \log 64$

or

$\log x = -\log 2$

$$\boxed{x = 64}$$

$$\boxed{x = 2^{-1} = \frac{1}{2}}$$

e) $(\log_4 a)(\log_a 2a)(\log_{2a} x) = \log_a a^3$

$$\frac{\log a}{\log 4} \cdot \frac{\log 2a}{\log a} \cdot \frac{\log x}{\log 2a} = \frac{\log a^3}{\log a}$$

$$\frac{\cancel{\log a}}{2\log 2} \cdot \frac{\cancel{\log 2a}}{\cancel{\log a}} \cdot \frac{\log x}{\cancel{\log 2a}} = \frac{\log a^3}{\log a}$$

$$\frac{\log x}{2\log 2} = \frac{3\cancel{\log a}}{\cancel{\log a}}$$

$$\frac{\log x}{2\log 2} = 3$$

$$\log x = 6\log 2$$

$$\log x = \log 2^6$$

$$\log x = 64$$

$$\boxed{x = 64}$$

f) $\sqrt{\log x} = \log\sqrt{x}$

$$\sqrt{\log x} = \log x^{\frac{1}{2}}$$

$$\sqrt{\log x} - \frac{1}{2}\log x = 0$$

Recall $\sqrt{\log x} \cdot \sqrt{\log x} = \log x$

Factor $\sqrt{\log x}$

$$\sqrt{\log x}\left(1 - \frac{1}{2}\sqrt{\log x}\right) = 0$$

Either:

$\sqrt{\log x} = 0 \rightarrow \log x = 0$

$10^0 = x$ $\boxed{x = 1}$

or:

$1 - \frac{1}{2}\sqrt{\log x} = 0$

$-\frac{1}{2}\sqrt{\log x} = -1 \rightarrow \sqrt{\log x} = 2$

$\log x = 4$

$x = 10^4$

$$\boxed{x = 10\,000}$$

8. Find the error in the statements written below

a)

$$2 > 1$$

$$\frac{2}{4} > \frac{1}{4}$$

$$\frac{1}{2} > \frac{1}{4}$$

$$\log\frac{1}{2} > \log\frac{1}{4}$$

$$\log\frac{1}{2} > \log\left(\frac{1}{2}\right)^2$$

From here to here

$$\log\frac{1}{2} > 2\log\frac{1}{2}$$

$$1 > 2$$

Divided by $\log \frac{1}{2}$

but $\log \frac{1}{2}$ is negative so the inequality should have flipped

b)

$$3 > 2$$

$$3\log\frac{1}{2} > 2\log\frac{1}{2}$$

multiplied by $\log \frac{1}{2}$ which is negative so inequality should have flipped

$$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$$

$$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2$$

$$\frac{1}{8} > \frac{1}{4}$$

$$1 > 2$$

See Website for Detailed Answer Key

Extra Work Space