Section 5.3 – Practice Problems

1. Write the following logarithmic expression in terms of $\log 2$ *and* $\log 3$

a) $\log 6$

$\log(2\cdot 3)$

$\boxed{\log 2 + \log 3}$

b) $\log 12$

$\log(4\cdot 3)$

$\log 4 + \log 3$

$\log 2^2 + \log 3$

$\boxed{2\log 2 + \log 3}$

c) $\log 72$

$\log(8\cdot 9)$

$\log 8 + \log 9$

$\log 2^3 + \log 3^2$

$\boxed{3\log 2 + 2\log 3}$

d) $\log 3200$

$\log(32\cdot 100)$

$\log 32 + \log 100$

$\log 2^5 + \log 10^2$

$\log 10^2$ is base ten log

$2\log 10 \quad \log 10 = 1$

$\boxed{5\log 2 + 2}$

e) $\log 0.36$

$\log \frac{36}{100} \rightarrow \log 36 - \log 100$

$\log(4\cdot 9) - \log 10^2$

$\log 2^2 + \log 3^2 - 2\log 10$

$\boxed{2\log 2 + 2\log 3 - 2}$

f) $\log_2 216$

$\log_2(2^3\cdot 3^3)$

$\log_2 2^3 + \log_2 3^3$

$3\log_2 2 + 3\log_2 3$

$3 + 3\log_2 3$

change of base

$\boxed{3 + 3\left[\frac{\log 3}{\log 2}\right]}$

216
2 108
2 54
6 9
2 3 3 3

g) $\log 5.4$

$\log \frac{54}{10} \rightarrow \log 54 - \log 10$

$\log(6\cdot 9) - \log 10$

$\log(2\cdot 3\cdot 3^2) - \log 10$

$\log 2 + \log 3^3 - \log 10$

$\boxed{\log 2 + 3\log 3 - 1}$

h) $\log_6 180$

$\frac{\log 180}{\log 6} \rightarrow \frac{\log(18\cdot 10)}{\log(2\cdot 3)} \rightarrow \frac{\log 18 + \log 10}{\log 2 + \log 3}$

$\frac{\log 2\cdot 9 + 1}{\log 2 + \log 3} \rightarrow \frac{\log 2 + \log 3^2 + 1}{\log 2 + \log 3} \rightarrow \boxed{\frac{\log 2 + 2\log 3 + 1}{\log 2 + \log 3}}$

i) $\log_{18} 2160 \rightarrow \frac{\log 2160}{\log 18}$

$\frac{\log(216\cdot 10)}{\log(2\cdot 9)} \rightarrow \frac{\log 216 + \log 10}{\log 2 + \log 9}$

$\frac{\log 2^3\cdot 3^3 + 1}{\log 2 + \log 3^2} \rightarrow \boxed{\frac{3\log + 3\log 3 + 1}{\log 2 + 2\log 3}}$

2160
216 10
2 108
2 54
6 9
2 3 3 3

j) $\log_{12} 0.108$

$\frac{\log \frac{108}{1000}}{\log 12} \rightarrow \frac{\log 2^2\cdot 3^3}{\log 10^3} \rightarrow \frac{2\log 2 + 3\log 3}{3\log 10}$

$\log 3\cdot 4 \quad \log 3 + \log 2^2$

$\frac{2\log 2 + 3\log 3 - 3\log 10}{\log 3 + 2\log 2} \rightarrow \boxed{\frac{2\log 2 + 3\log 3 - 3}{\log 3 + 2\log 2}}$

108
2 54
6 9
2 3 3 3

$\log_b b = 1$

2. Find the exact solution without a calculator

a) $\log_3 81$

$\log_3 3^4 = 4\log_3 3 = \boxed{4}$

b) $\log_2 \frac{1}{32}$

$\rightarrow \log_2 32^{-1}$

$\rightarrow \log_2 2^{-5} = -5\log_2 2$

$\boxed{-5}$

c) $\log_2 \sqrt[4]{8}$

$\log_2 8^{\frac{1}{4}} \rightarrow \log_2 (2^3)^{\frac{1}{4}} \rightarrow \log_2 2^{3/4}$

$\frac{3}{4}\log_2 2 = \boxed{3/4}$

d) $\log_5 \sqrt{125}$

$\log_5 125^{\frac{1}{2}} \rightarrow \log_5 5^{3/2} \rightarrow \frac{3}{2}\log_5 5$

$\boxed{\frac{3}{2}}$

e) $\log_9 27^{2.2}$

$2.2\left[\frac{\log 27}{\log 9}\right]$

$2.2\left[\frac{\log 3^3}{\log 3^2}\right] \searrow$

$2.2\left[\frac{3\cancel{\log 3}}{2\cancel{\log 3}}\right] = \frac{6.6}{2} = \boxed{3.3}$

f) $\log_4 \frac{1}{32}$

$\log_4 32^{-1} \rightarrow -1\log_4 32$

$-1\left[\frac{\log 32}{\log 4}\right] \rightarrow -1\left[\frac{\log 2^5}{\log 2^2}\right] \rightarrow -1\left[\frac{5\cancel{\log 2}}{2\cancel{\log 2}}\right] = \boxed{-\frac{5}{2}}$

g) $(\log_4 8)(\log_{16} 32)$

$\left(\frac{\log 8}{\log 4}\right)\left(\frac{\log 32}{\log 16}\right) \rightarrow \left(\frac{\log 2^3}{\log 2^2}\right)\left(\frac{\log 2^5}{\log 2^4}\right)$

$\left(\frac{3\cancel{\log 2}}{2\cancel{\log 2}}\right)\left(\frac{5\cancel{\log 2}}{4\cancel{\log 2}}\right) \rightarrow \boxed{\frac{15}{8}}$

h) $\dfrac{\log_{27} 81}{\log_{25} 125}$

$\rightarrow \frac{\log 81}{\log 27} \div \frac{\log 125}{\log 25} = \frac{\log 3^4}{\log 3^3} \div \frac{\log 5^3}{\log 5^2}$

$\frac{4\log 3}{3\log 3} \div \frac{5\log 5}{2\log 5} \rightarrow \frac{4}{3} \div \frac{5}{2} \rightarrow \frac{4}{3}\cdot\frac{2}{5} = \boxed{\frac{8}{15}}$

i) $\log_4 2 + \log_2 32$

$\frac{\log 2}{\log 4} + \frac{\log 32}{\log 2} \rightarrow \frac{\log 2}{2\log 2} + \frac{5\log 2}{\log 2}$

$\frac{1}{2} + 5 = \boxed{\frac{11}{2} \text{ or } 5\frac{1}{2}}$

j) $\log_9 16 - 2\log_3 2$

$\frac{\log 16}{\log 9} - \frac{2\log 2}{\log 3} \rightarrow \frac{\log 2^4}{\log 3^2} - \frac{2\log 2}{\log 3}$

$\frac{4\log 2}{2\log 3} - \frac{2\log 2}{\log 3}$ common denominator

$\frac{4\log 2 - 4\log 2}{2\log 3} = \boxed{0}$

3. Use the properties of logarithms to expand the following (if possible)

a) $\log 100x^2y^3$

$\log 100 + \log x^2 + \log y^3$

$\log 10^2 + \log x^2 + \log y^3$

$2\log 10 + 2\log x + 3\log y$

$\boxed{2 + 2\log x + 3\log y}$

b) $\log \dfrac{x^3}{1000y^2}$

$\log x^3 - [\log 1000y^2]$

$\log x^3 - [\log 1000 + \log y^2]$

$3\log x - [\log 10^3 + 2\log y]$

$3\log x - [3\log 10 + 2\log y]$

$\boxed{3\log x - 3 - 2\log y}$

c) $\log(x^2+y^3)^4$

$4\log(x^2+y^3)$

can't go any further than this

d) $\log^4(x^2+y^3)$

$\downarrow$

$[\log(x^2+y^3)]^4$

can't go any further

$\log_5 25$

$2\log_5 5$

e) $\log_5 \frac{25x^2y^3}{z}$

$\log_5(25x^2y^3) - \log_5 z$

$\log_5 25 + \log_5 x^2 + \log_5 y^3 - \log_5 z$

$2 + 2\log_5 x + 3\log_5 y - \log_5 z$

f) $\log\sqrt{x^2(x+2)}$ $\rightarrow \log(x^2(x+2))^{\frac{1}{2}}$

$\frac{1}{2}\log x^2(x+2) \rightarrow \frac{1}{2}[\log x^2 + \log(x+2)]$

$\frac{1}{2}[2\log x + \log(x+2)]$

$\boxed{\log x + \frac{1}{2}\log(x+2)}$

g) $4\log_2(2x)^{12}$ $\rightarrow 48\log_2(2x)$

$48[\log_2 2 + \log_2 x]$

$48[1 + \log_2 x] \rightarrow \boxed{48 + 48\log_2 x}$

h) $\log_a \sqrt{\frac{x^2y+1}{a^3}}$ $\rightarrow \log_a\left(\frac{x^2y+1}{a^3}\right)^{\frac{1}{2}}$

$\frac{1}{2}[\log_a(x^2y+1) - \log_a a^3]$

$\frac{1}{2}[\log_a(x^2y+1) - 3\log_a a]$

$\frac{1}{2}[\log_a(x^2y+1) - 3] \rightarrow \boxed{\frac{1}{2}\log_a(x^2y+1) - \frac{3}{2}}$

i) $\log\frac{(x^3+y)^3}{x^3}$

$\log(x^3+y)^3 - \log x^3$

$\boxed{3\log(x^3+y) - 3\log x}$

j) $\log\sqrt[3]{\frac{xy^3}{z^6}}$ $\quad \log\left(\frac{xy^3}{z^6}\right)^{\frac{1}{3}}$

$\frac{1}{3}\log\left(\frac{xy^3}{z^6}\right) \rightarrow \frac{1}{3}[(\log x + \log y^3) - \log z^6]$

$\frac{1}{3}[(\log x + 3\log y) - 6\log z]$

$\boxed{\frac{1}{3}\log x + \log y - 2\log z}$

4. Condense into one single logarithmic quantity

a) $\log_5 x - \log_5 25$

$\boxed{\log_5\left(\frac{x}{25}\right)}$

b) $\log_3 x - 2\log_3 27$

$\log_3\left(\frac{x}{27^2}\right) \rightarrow \boxed{\log_3\left(\frac{x}{729}\right)}$

c) $\log\sqrt{x} - \log x^{\frac{3}{2}}$

$\log x^{\frac{1}{2}} - \log x^{\frac{3}{2}}$

$\log\left(\frac{x^{\frac{1}{2}}}{x^{3/2}}\right) \rightarrow \log\left(x^{\frac{1}{2}} \cdot x^{-3/2}\right)$

$\log x^{-2/2} \rightarrow \boxed{\log\left(\frac{1}{x}\right)}$

d) $\log(x^2-1) - \log(x+1) - \log x$

$\log(x^2-1) - [\log(x+1) + \log x]$

$\log(x^2-1) - [\log(x+1)x]$

Diff of Squares

$\log\left[\frac{(x^2-1)}{(x+1)x}\right] \rightarrow \log\left[\frac{(x+1)(x-1)}{(x+1)x}\right]$

$\boxed{\log\left[\frac{x-1}{x}\right]}$

e) $\log(3x^2-5x-2) - \log(x^2-4) - \log(3x+1)$

$\log(3x^2-5x-2) - [\log(x^2-4) + \log(3x+1)]$

$\log(3x^2-5x-2) - [\log(x^2-4)(3x+1)]$

$\log\left[\frac{3x^2-5x-2}{(x^2-4)(3x+1)}\right] \rightarrow \log\left[\frac{\cancel{(3x+1)}\cancel{(x-2)}}{(x+2)\cancel{(x-2)}\cancel{(3x+1)}}\right] \rightarrow \log\left[\frac{1}{(x+2)}\right]$

$\log 1 - \log(x+2) \longrightarrow 0 - \log(x+2) \rightarrow \boxed{-\log(x+2)}$

f) $\log_3(2x-3) - \log_3(2x^2 - x - 3) + \log_3 3(x+1)$

$$\log_3(2x-3) - [\log_3(2x^2-x-3) - \log_3 3(x+1)]$$

$$\log_3 \frac{(2x-3)}{\frac{(2x^2-x-3)}{3(x+1)}} \rightarrow \log_3 \frac{(2x-3)(3)(x+1)}{2x^2-x-3} \rightarrow \log_3 \frac{3\cancel{(2x-3)}\cancel{(x+1)}}{\cancel{(2x-3)}\cancel{(x+1)}}$$

$$\log_3 3 = \boxed{1}$$

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g) $2[\log(x^2-1) - \log(x+1) - \log(x-1)]$

$$2[\log(x^2-1) - (\log(x+1) + \log(x-1))]$$

$$2[\log(x^2-1) - [\log(x+1)(x-1)]]$$

$$2\left[\log\frac{(x^2-1)}{(x+1)(x-1)}\right] \rightarrow 2\left[\log\frac{(x+1)(x-1)}{(x+1)(x-1)}\right]$$

$$2[\log 1] = 2(0) = \boxed{0}$$

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h) $\frac{3}{2}\log 4x^4 - \frac{1}{2}\log y^6$

$$\log(4x^4)^{3/2} - \log(y^6)^{\frac{1}{2}}$$

$$\log(4^{3/2}x^6) - \log y^3$$

$$\log(8x^6) - \log y^3$$

$$\boxed{\log\left(\frac{8x^6}{y^3}\right)}$$

i) $\frac{1}{4}[\log(x^2-4)-\log(x-2)]-\log x$

$\frac{1}{4}\left[\log\frac{(x^2-4)}{x-2}\right]-\log x$

$\frac{1}{4}\left[\log\frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}\right]-\log x$

$\frac{1}{4}[\log(x+2)]-\log x$

$\frac{1}{4}\log(x+2)-\log x \rightarrow \log(x+2)^{\frac{1}{4}}-\log x$

$\boxed{\log\frac{\sqrt[4]{x+2}}{x}}$

j) $\log(x^2-4)-[\log(x-2)+\log(x+2)]$

$\log(x^2-4)-[\log(x-2)(x+2)]$

$\log(x^2-4)-[\log(x^2-4)]$

$\log\frac{(x^2-4)}{(x^2-4)}=\log 1=\boxed{0}$

5. Simplify

a) $\log_b x^{\log_x a}$

$\log_x a\cdot\log_b x$

many ways to do this one.

$\frac{\log a}{\cancel{\log x}}\cdot\frac{\cancel{\log x}}{\log b}$

$\frac{\log a}{\log b}=\boxed{\log_b a}$

Reverse Change of Base

b) $x^{\log_x 20-\log_x 4}$

$x^{\log_x\left(\frac{20}{4}\right)}$

$x^{\log_x 5}$

Rule #7

$\boxed{5}$

c) $(\log_2 10)(\log 48 - \log 3)$

$$\left(\frac{\log 10}{\log 2}\right)\left(\log \frac{48}{3}\right)$$

$$\left(\frac{\log 10}{\log 2}\right)\left(\log 16\right)$$

change of base but already base 10

$$\left(\frac{\cancel{\log 10}}{\log 2}\right)\left(\frac{\log 16}{\cancel{\log 10}}\right)$$

$$\frac{\log 2^4}{\log 2} \rightarrow \frac{4\cancel{\log 2}}{\cancel{\log 2}} = \boxed{4}$$

d) $\dfrac{\log x^3 + \log x^5}{\log x^6 - \log x^3}$

$$\frac{3\log x + 5\log x}{6\log x - 3\log x} \rightarrow \frac{8\cancel{\log x}}{3\cancel{\log x}}$$

$$\boxed{\frac{8}{3}}$$

e) $\left(\frac{a}{b}\right)^{\log 0.5} \cdot \left(\frac{a}{b}\right)^{\log 0.2}$

common base add exponents

$$\left(\frac{a}{b}\right)^{\log 0.5 + \log 0.2}$$

$$\left(\frac{a}{b}\right)^{\log (0.5 \cdot 0.2)}$$

$$\left(\frac{a}{b}\right)^{\log 0.1} \rightarrow \left(\frac{a}{b}\right)^{\log 10^{-1}}$$

$$\left(\frac{a}{b}\right)^{-1} = \boxed{\frac{b}{a}}$$

f) $4^{-2\log_4 3}$

$$\left(4^{\log_4 3}\right)^{-2}$$

cancel $\rightarrow$ Rule #7

$$3^{-2} = \boxed{\frac{1}{9}}$$

g) $10\log_4 x - 12\log_8 x$

$$10\left[\frac{\log x}{\log 4}\right] - 12\left[\frac{\log x}{\log 8}\right]$$

$$10\left[\frac{\log x}{2\log 2}\right] - 12\left[\frac{\log x}{\log 2^3}\right]$$

$$\frac{10}{2}\left[\frac{\log x}{\log 2}\right] - \frac{12}{3}\left[\frac{\log x}{\log 2}\right]$$

$$5\left[\frac{\log x}{\log 2}\right] - 4\left[\frac{\log x}{\log 2}\right]$$

$\log 4 = \log 2^2 = 2\log 2$

$$\frac{\log x}{\log 2} = \boxed{\log_2 x}$$

Reverse change of base

h) $\log\pi + \log\frac{\sqrt{2}}{\pi} + \frac{1}{2}\log\frac{3}{2} - \log\frac{\sqrt{3}}{10}$

$$\log\left(\cancel{\pi}\cdot\frac{\sqrt{2}}{\cancel{\pi}}\right) + \log\left(\frac{3}{2}\right)^{\frac{1}{2}} - \log\frac{\sqrt{3}}{10}$$

$$\log\sqrt{2} + \left[\log\frac{\sqrt{\frac{3}{2}}}{\frac{\sqrt{3}}{10}}\right]$$

$$\log\sqrt{2} + \left[\log\frac{\cancel{\sqrt{3}}}{\sqrt{2}}\cdot\frac{10}{\cancel{\sqrt{3}}}\right]$$

$$\log\sqrt{2} + \log\frac{10}{\sqrt{2}}$$

$$\log\left(\sqrt{2}\cdot\frac{10}{\sqrt{2}}\right) \rightarrow \log 10 = \boxed{1}$$

i) $\log(1-x^3) - \log(1+x+x^2) - \log(1-x)$

$$\log(1-x^3) - [\log(1+x+x^2) + \log(1-x)]$$

$$\log(1-x^3) - [\log(1+x+x^2)(1-x)]$$

$$\log(1-x^3) - [\log(1-x^3)]$$

$$\log\frac{(1-x^3)}{(1-x^3)}$$

$$\log 1 = \boxed{0}$$

$(1+x+x^2)(1-x)$

$1 - x + x - x^2 + x^2 - x^3$

$1 - x^3$

j) $\dfrac{\log_a x}{\log_{ab} x} - \dfrac{\log_a x}{\log_b x}$

$$\frac{\frac{\log x}{\log a}}{\frac{\log x}{\log ab}} - \frac{\frac{\log x}{\log a}}{\frac{\log x}{\log b}} \rightarrow \frac{\cancel{\log x}}{\log a} \cdot \frac{\log ab}{\cancel{\log x}} - \frac{\cancel{\log x}}{\log a} \cdot \frac{\log b}{\cancel{\log x}}$$

$$\frac{\log ab}{\log a} - \frac{\log b}{\log a} \rightarrow \frac{\log ab - \log b}{\log a}$$

$$\frac{\log \frac{ab}{b}}{\log a} \rightarrow \frac{\log a}{\log a} = \boxed{1}$$

k) $\dfrac{1}{\log_a x} + \dfrac{1}{\log_b x}$

$$\frac{1}{\frac{\log x}{\log a}} + \frac{1}{\frac{\log x}{\log b}} \rightarrow \frac{\log a}{\log x} + \frac{\log b}{\log x} \rightarrow \frac{\log a + \log b}{\log x}$$

$$\frac{\log ab}{\log x} \rightarrow \boxed{\log_x ab}$$

l) $(\log_5 9)(\log_3 7)(\log_7 5)$

$$\frac{\log 9}{\log 5} \cdot \frac{\log 7}{\log 3} \cdot \frac{\log 5}{\log 7} \rightarrow \frac{\log 3^2}{\log 5} \cdot \frac{\log 7}{\log 3} \cdot \frac{\log 5}{\log 7}$$

$$\frac{2\cancel{\log 3}}{\cancel{\log 5}} \cdot \frac{\cancel{\log 7}}{\cancel{\log 3}} \cdot \frac{\cancel{\log 5}}{\cancel{\log 7}}$$

$$\boxed{2}$$

See Website for Detailed Answer Key

Extra Work Space