

Name: KEY

Section 5.3 – Properties of Logarithms

1. Expand the following expression

$$\log \sqrt[3]{x^3(x+5)}$$

$$\log [x^3(x+5)]^{\frac{1}{3}}$$

$$\frac{1}{3} [\log x^3(x+5)]$$

$$\frac{1}{3} [\log x^3 + \log(x+5)]$$

$$\frac{1}{3} [3 \log x + \log(x+5)]$$

$$\boxed{\log x + \frac{1}{3} \log(x+5)}$$

3. Determine an expression for $\log x$ if:

$$x = \frac{\sqrt[3]{a}}{bc^2}$$

$$\log \left(\frac{\sqrt[3]{a}}{bc^2} \right)$$

$$\log (\sqrt[3]{a}) - [\log bc^2]$$

$$\log a^{\frac{1}{3}} - [\log b + \log c^2]$$

$$\boxed{\frac{1}{3} \log a - [\log b + 2 \log c]}$$

2. Condense into one logarithm

$$\log_3(2x-3) - \log_3(2x^2-x-3) + \log_3 3(x+1)$$

$$\log_3(2x-3) + \log_3 3(x+1) - \log_3(2x^2-x-3)$$

$$\log_3(2x-3)(x+1)(3) - \log_3(2x^2-x-3)$$

$$\log_3 \left[\frac{3(2x-3)(x+1)}{2x^2-x-3} \right] \rightarrow \log_3 \left[\frac{3(2x-3)(x+1)}{(2x-3)(x+1)} \right]$$

$$\log_3 \left[\frac{3 \cancel{(2x-3)} \cancel{(x+1)}}{\cancel{(2x-3)} \cancel{(x+1)}} \right]$$

$$\log_3 3 = \boxed{1}$$

4. Solve:

$$\log_3(x+5) - \log_3(x-3) = 2$$

$$\log_3 \frac{(x+5)}{(x-3)} = 2$$

$$3^2 = \frac{(x+5)}{(x-3)} \rightarrow 9 = \frac{(x+5)}{(x-3)}$$

$$9(x-3) = x+5$$

$$9x - 27 = x + 5$$

$$-32 = -8x$$

$$\boxed{x=4}$$

✓ satisfies Domain

5. Solve:

$$2^{(3 \log_8 5)} = x$$

$$2^{(\log_8 5^3)} = x$$

$$\log_2 2^{\log_8 5^3} = \log_2 x$$

$$(\log_8 125)(\log_2 2) = \log_2 x$$

$$\frac{\log 125}{\log 8} \cdot \log_2 2 = \log_2 x$$

$$\frac{\log 5^3}{\log 2^3} \cdot \log_2 2 = \log_2 x$$

$$\log 5 = \log x$$

$$x = 5$$

6. Determine an equivalent expression for:

$$\log_3 \left(\frac{a}{9b^2} \right)$$

$$\log_3 a - [\log_3 9 + \log_3 b^2]$$

$$\log_3 a - [\log_3 3^2 + 2\log_3 b]$$

$$\log_3 a - [2\log_3 3 + 2\log_3 b]$$

$$\log_3 a - [2 + 2\log_3 b]$$

$$\log_3 a - 2\log_3 b - 2 \rightarrow \boxed{\log_3 a - \log_3 b^2 - 2}$$

7. If $\log_2 5 = a$ and $\log_2 3 = b$, determine an expression in terms of a and b for:

$$\log_2 \left(\frac{25}{72} \right)$$

$$\log_2 25 - \log_2 72$$

$$\log_2 5^2 - \log_2 8 \cdot 9$$

$$2\log_2 5 - [\log_2 8 + \log_2 9]$$

$$2\log_2 5 - [\log_2 2^3 + \log_2 3^2]$$

$$2\log_2 5 - [3\log_2 2 + 2\log_2 3]$$

$$2\log_2 5 - [3 + 2\log_2 3]$$

$$2a - [3 + 2b]$$

$$\boxed{2a - 2b - 3}$$