Section 5.2 – Solving Systems of Non-Linear Equations Algebraically

This booklet belongs to:______________________Block:_____

- When solving system of non-linear equations, we can similar strategies as linear
- The most straightforward way of doing this is to use the concept of equality

$$If \quad a = b \quad and \quad a = c \quad then \quad b = c$$

- We can use this concept to simplify the systems of equations by:
  - Writing both equations in **terms of one variable**
  - Setting them **equal to one another**
  - Solving for the **given variable**
  - **Substituting** back into the equation to solve for the remaining variable
  - Check our solution

Example 1: Solve the system: $y = x^2 - 3x - 4$ and $2x - y = 4$

Solution 1: Since one is already in terms of y, rearrange the other to also be in terms of y

$$2x - y = 4 \quad \rightarrow \quad y = 2x - 4$$

- Now since they are both equal to y we can set them equal to each other and solve for x

$y = x^2 - 3x - 4$ and $y = 2x - 4$ $\quad (Since\ they\ both\ equal\ y, set\ them\ equal\ to\ each\ other)$

Solve for x first:

$2x - 4 = x^2 - 3x - 4 \quad \rightarrow \quad x^2 - 5x = 0$

$x^2 - 5x = 0 \quad \rightarrow \quad x(x - 5) = 0$

$x = 0$ or $x = 5$

Now solve for y

$y = 2x - 4$	$y = 2x - 4$
$y = 2(0) - 4$	$y = 2(5) - 4$
$y = -4$	$y = 6$

Check:

$y = x^2 - 3x - 4$	$y = x^2 - 3x - 4$
$-4 = (0)^2 - 3(0) - 4$	$6 = (5)^2 - 3(5) - 4$
$-4 = -4$	$6 = 6$

System has Solutions:

$\mathbf{(0, -4)}$ ***and*** $\mathbf{(5, 6)}$

Example 2: Solve the system: $y = -\frac{1}{2}x^2 + 2x - 3$ and $y = x - 2$

Solution 2: Since they are both already in terms of y, set them equal to each other and solve for x

Solve for x first:

$x - 2 = -\frac{1}{2}x^2 + 2x - 3 \quad \rightarrow \quad \frac{1}{2}x^2 - x + 1 = 0 \qquad (Does\ not\ Factor)$

$x^2 - 2x + 2 = 0 \quad (Multiply\ by\ the\ LCM)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} \qquad Use\ the\ Quadratic\ Equation$

$$x = \frac{2 \pm \sqrt{-4}}{2} = \emptyset$$

The line and parabola **do not intersect**. There are **No Real Solutions**.

Example 3: Solve the system: $y = x^2 - 3x - 4$ and $2x - y = 3$

Solution 3: Since one is already in terms of y, rearrange the other to also be in terms of y

$$2x - y = 3 \quad \rightarrow \quad y = 2x - 3$$

Now since they are both equal to y we can set them equal to each other and solve for x

$y = x^2 - 3x - 4$ and $y = 2x - 3 \qquad Since\ they\ both\ equal\ y, set\ them\ equal\ to\ each\ other$

Solve for x first:

$2x - 3 = x^2 - 3x - 4 \quad \rightarrow \quad x^2 - 5x - 1 = 0 \qquad (Does\ not\ factor)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2(1)} \qquad Use\ the\ Quadratic\ Equation$

$$x = \frac{5 \pm \sqrt{29}}{2}$$

Now solve for y

$y = 2x - 3$	$y = 2x - 3$
$y = 2\left(\frac{5 + \sqrt{29}}{2}\right) - 3$	$y = 2\left(\frac{5 - \sqrt{29}}{2}\right) - 3$
$y = 5 + \sqrt{29} - 3$	$y = 5 - \sqrt{29} - 3$
$y = 2 + \sqrt{29}$	$y = 2 - \sqrt{29}$

System has Solutions:

$\left(\frac{5 + \sqrt{29}}{2}, 2 + \sqrt{29}\right)$ ***and*** $\left(\frac{5 - \sqrt{29}}{2}, 2 - \sqrt{29}\right)$

Example 4: Solve the system: $y = x^2 - x - 3$ and $y = 2x^2 - x + 7$

Solution 4: Since they are both already in terms of y, set them equal to each other and solve for x

Solve for x first:

$2x^2 - x + 7 = x^2 - x - 3$

$x^2 = -10 \qquad (Does\ not\ Factor)$

$x = \emptyset$

The line and parabola **do not intersect**. There are **No Real Solutions**.

Example 5: Solve the system: $x^2 - 4x + y + 1 = 0$ and $2x^2 - 2x - y + 2 = 0$

Solution 5: Even though they are both equal to zero, set them both equal to y to eliminate one variable

$x^2 - 4x + y + 1 = 0 \quad \rightarrow \quad y = -x^2 + 4x - 1$ and $2x^2 - 2x - y + 2 = 0 \quad \rightarrow \quad y = 2x^2 - 2x + 2$

Now since they are both equal to y we can set them equal to each other and solve for x

$y = -x^2 + 4x - 1$ and $y = 2x^2 - 2x + 2$ $(Since\ they\ both\ equal\ y, set\ them\ equal\ to\ each\ other)$

Solve for x first:

$-x^2 + 4x - 1 \ = 2x^2 - 2x + 2 \quad \rightarrow \quad 3x^2 - 6x + 3 = 0$

$3(x^2 - 2x + 1) = 0 \quad \rightarrow \quad 3(x - 1)(x - 1) = 0$

$x = 1$

Now solve for y

$(1)^2 - 4(1) + y + 1 = 0$

$1 - 4 + y + 1 = 0$

$y = 2$

Check:

$2(1)^2 - 2(1) - 2 + 2 = 0$

$2 - 2 - 2 + 2 = 0$

System has Solutions:

$(\mathbf{1}, \mathbf{2})$

Section 5.2 – Practice Problems

Find all the real solutions of the system of equations

1. $2x^2 - y = 1 \quad and \quad y = 5x + 2$

2. $x^2 - y = 3 \quad and \quad y = 3x + 7$

3. $x^2 = 2y \quad and \quad y = x - \frac{1}{2}$

4. $x^2 + y = 4 \quad and \quad 1 = 2x + y$

5. $3x^2 - 10y = 5 \quad and \quad x - y = -2$

6. $2x^2 - 3y = 2 \quad and \quad x - 2y = -2$

7. $x^2 + 2y = -2 \quad and \quad -2x + y = 1$

8. $x + y = 2 \quad and \quad y = 1 - x^2$

9. $y = x^2 - x \quad and \quad y = 2x$

10. $y = x^2 - 6x \quad and \quad y = x - 12$

11. $y = x^2 + 8x - 10 \quad and \quad y = 3x + 4$

12. $x^2 = y \quad and \quad 1 = 2x - y$

13. $x^2 + y = 9 \quad and \quad 16 = 3x + 2y$

14. $x^2 - y = 10 \quad and \quad 2x - 3y = -10$

15. $y = x^2 \quad and \quad x + y = 3$

16. $y + 2x^2 - 2 = 0 \quad and \quad 3y - x - 3 = 0$

17. $y - x^2 = 0 \quad and \quad x^2 - 2x + y = 6$

18. $2x^2 + y = 9 \quad and \quad y - x^2 - 5x = 1$

19. Find all the points of intersection of the parabola $y = x^2 - 4x + 2$ and the $x - axis$

20. Find all the points of intersection of the parabola $y = 75x^2 - 33x + 157$ and the $y - axis$

Answer Key – Section 5.2

1. $\left(-\frac{1}{2},-\frac{1}{2}\right),(3,17)$	11. $(-7,-17),(2,10)$
2. $(5,22),(-2,1)$	12. $(1,1)$
3. $\left(1,\frac{1}{2}\right)$	13. $(2,5),\left(-\frac{1}{2},\frac{35}{4}\right)$
4. $(3,-5),(-1,3)$	14. $(4,6),\left(-\frac{10}{3},\frac{10}{9}\right)$
5. $(5,7),\left(-\frac{5}{3},\frac{1}{3}\right)$	15. $\left(\frac{-1+\sqrt{13}}{2},\frac{7-\sqrt{13}}{2}\right),\left(\frac{-1-\sqrt{13}}{2},\frac{7+\sqrt{13}}{2}\right)$
6. $(2,2),\left(-\frac{5}{4},\frac{3}{8}\right)$	16. $\left(\frac{-1+\sqrt{73}}{12},\frac{35+\sqrt{73}}{36}\right),\left(\frac{-1-\sqrt{73}}{12},\frac{35-\sqrt{73}}{36}\right)$
7. $(-2,-3)$	17. $\left(\frac{1+\sqrt{13}}{2},\frac{7+\sqrt{13}}{2}\right),\left(\frac{1-\sqrt{13}}{2},\frac{7-\sqrt{13}}{2}\right)$
8. $No\ Solution$	18. $\left(-\frac{8}{3},-\frac{47}{9}\right),(1,7)$
9. $(0,0),(3,6)$	19. $\left(2+\sqrt{2},0\right),\left(2-\sqrt{2},0\right)$
10. $(3,-9),(4,-8)$	20. $(0,157)$

Extra Work Space