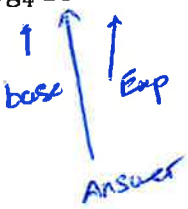
Section 5.2 – Practice Problems

1. Write the following in exponential form

a) $\log_4 16 = 2$



$4^2 = 16$

b) $\log_3 81 = 4$

$3^4 = 81$

c) $\log_6 \frac{1}{36} = -2$

$6^{-2} = \frac{1}{36}$

d) $\log \frac{1}{100} = -2$

no log base means $\log_{10}$

$10^{-2} = \frac{1}{100}$

e) $\log_{32} 8 = \frac{3}{5}$

$32^{3/5} = 8$

Recall $32^{3/5} = \sqrt[5]{32}^3$

$= 2^3$

$= 8$

f) $\log_8 8 = 1$

$8^1 = 8$

g) $\log_5 1 = 0$

$5^0 = 1$

h) $\log 1000 = 3$

$10^3 = 1000$

i) $\log_8 4 = \frac{2}{3}$

$8^{2/3} = 4$

j) $\log_4 \frac{1}{8} = -\frac{3}{2}$

$4^{-\frac{3}{2}} = 8$

Recall $4^{-3/2} = \frac{1}{4^{3/2}} = \frac{1}{\sqrt{4}^3} = \frac{1}{2^3} = \frac{1}{8}$

2. Write the following in Logarithmic Form

a) $2^4 = 16$ (answer ↑, object of log ←, base of log ↑) $\log_2 16 = 4$

b) $8^2 = 64$ $\log_8 64 = 2$

c) $16^{\frac{1}{4}} = 2$ $\log_{16} 2 = \frac{1}{4}$

d) $3^{-2} = \frac{1}{9}$ $\log_3 \frac{1}{9} = -2$

e) $3^0 = 1$ $\log_3 1 = 0$

f) $10^{-2} = 0.01$ $\log 0.01 = -2$

g) $5^1 = 5$ $\log_5 5 = 1$

h) $9^{\frac{3}{2}} = 27$ $\log_9 27 = \frac{3}{2}$

i) $8^{\frac{4}{3}} = 16$ $\log_8 16 = \frac{4}{3}$

j) $\left(\frac{2}{3}\right)^{-4} = \frac{81}{16}$ $\log_{\frac{2}{3}}\left(\frac{81}{16}\right) = -4$

3. Evaluate the log without a calculator. If base the same; exponents must be equal too

a) $f(x) = \log_2 8$

$y = \log_2 8 \rightarrow 2^y = 8$

$2^y = 2^3$ $\boxed{y = 3}$

b) $f(x) = \log_4 16$

$y = \log_4 16 \rightarrow 4^y = 16$

$4^y = 4^2$ $\boxed{y = 2}$

c) $f(x) = \log_8 2$

$y = \log_8 2 \rightarrow 8^y = 2$

$2^{3y} = 2 \quad 3y = 1 \quad \boxed{y = \frac{1}{3}}$

d) $f(x) = \log_{16} 4$

$y = \log_{16} 4 \rightarrow 16^y = 4 \rightarrow 4^{2y} = 4$

$2y = 1 \quad \boxed{y = \frac{1}{2}}$

e) $f(x) = \log_5 1$

$y = \log_5 1 \rightarrow 5^y = 5^0$

$5^y = 1 \quad \boxed{y = 0}$

f) $f(x) = \log_7 7$

$y = \log_7 7 \rightarrow 7^y = 7$

$\boxed{y = 1}$

g) $f(x) = \log_a a$

$y = \log_a a$

$a^y = a \quad \boxed{y = 1}$

h) $f(x) = \log_a a^3$

$y = \log_a a^3$

$a^y = a^3 \quad \boxed{y = 3}$

i) $f(x) = \log_b b^{-4}$

$y = \log_b b^{-4}$

$b^y = b^{-4} \quad \boxed{y = -4}$

j) $f(x) = \log_5 0$

$y = \log_5 0$

$5^y = 0 \rightarrow$ not possible

$y = \varnothing$ undefined means 'empty set'

4. Find the unknow information without a calculator If exponents equal, bases are equal

a) $\log_x 27 = 3$

$x^3 = 27$

$x^3 = 3^3 \quad \boxed{x = 3}$

b) $\log_4 x = -3$

$4^{-3} = x \rightarrow x = \frac{1}{4^3} \rightarrow \boxed{x = \frac{1}{64}}$

c) $\log 1000 = x$

$10^x = 1000$

$10^x = 10^3 \quad \boxed{x = 3}$

d) $\log_x 8 = 1$

$x^1 = 8$

$x^1 = 8^1 \quad \boxed{x = 8}$

e) $\log_7 x = -2$

$7^{-2} = x$

$\frac{1}{7^2} = x$

$x = \frac{1}{49}$

f) $\log_9 27 = x$

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = \frac{3}{2}$

g) $\log_x 32 = 2$

can't be negative

$x^2 = 32$

$x = \sqrt{32}$

$x = \sqrt{16 \cdot 2}$

$x = 4\sqrt{2}$

h) $\log_4 x = 0$

$4^0 = x$

$1 = x$

i) $\log_{32} 8 = x$

$32^x = 8$

$(2^5)^x = 2^3$

$2^{5x} = 2^3$

$5x = 3$

$x = 3/5$

j) $\log_x 625 = 4$

$x^4 = 625$

$x^4 = 5^4$

$x = 5$

625
5 ^ 125
5 ^ 25
5 ^ 5

k) $\log_4 x = \frac{3}{2}$

$4^{3/2} = x$

$\sqrt{4}^3 = x$

$2^3 = x$

$x = 8$

l) $\log_4 0.25 = x$

$4^x = 0.25$

$4^x = \frac{1}{4}$

$4^x = 4^{-1}$

$x = -1$

m) $\log_{\sqrt{2}} x = 8$

Recall $\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt{2}^8 = x$

$(2^{\frac{1}{2}})^8 = x$

$2^4 = x$

$x = 16$

n) $\log_{\sqrt{3}} x = 4$

$\sqrt{3}^4 = x$

$(3^{\frac{1}{2}})^4 = x$

$3^2 = x$

$x = 9$

o) $\log_x \sqrt{3} = \frac{1}{2}$

$x^{\frac{1}{2}} = \sqrt{3}$

$x^{\frac{1}{2}} = 3^{\frac{1}{2}}$

$x = 3$

p) $\log_{3x} 36 = 2$

$(3x)^2 = 36$

$9x^2 = 36$

$x^2 = 4$

$x = 2$

q) $\log_{\sqrt{2}} 16 = x$

$\sqrt{2}^x = 16$

$2^{\frac{1}{2}x} = 16$

$2^{\frac{1}{2}x} = 2^4$

$\frac{1}{2}x = 4$

$x = 8$

r) $\log_{\sqrt{3}} 9 = x$

$\sqrt{3}^x = 9$

$3^{\frac{1}{2}x} = 3^2$

$\frac{1}{2}x = 2$

$x = 4$

s) $\log_7(x^2 + 24) = 2$

$7^2 = (x^2 + 24)$

$49 = x^2 + 24$

$0 = x^2 - 25$

$0 = (x+5)(x-5)$

$x = \pm 5$

t) $\log(x-2)^2 = -2$

$10^{-2} = (x-2)^2 \rightarrow \left(\frac{1}{10}\right)^2 = (x-2)^2$

$\pm\frac{1}{10} = x - 2$

$x = 2 \pm \frac{1}{10}$

5. What is the Domain of the following functions.

Since $y = \log_a x \rightarrow a^y = x \leftarrow x > 0$ so

a) $f(x) = \log_3(x-1)$

Base of log cannot be negative, zero, or 1

$x - 1 > 0$

$x > 1$

b) $f(x) = -\log_2 x + 3$

$x > 0$

c) $f(x) = \log_{(2-x)} 5$

$2 - x > 0 \rightarrow 2 > x$

and $2 - x \neq 1$

$x \neq 1$

d) $f(x) = \log_3(-x)$

$x < 0$

since $-x > 0$

$x < 0$

e) $f(x) = \log_{x+1}(x-2)$

$x + 1 > 0$

$x > -1$

and

$x + 1 \neq 1$

$x \neq 0$

$x - 2 > 0$

$x > 2$

to satisfy all only need $x > 2$

f) $f(x) = \log_{x-2}(x+1)$

$x - 2 \neq 1$

$x \neq 3$

$x + 1 > 0$

$x > -1$

$x - 2 > 0$

$x > 2$

satisfy all

$x > 2; x \neq 3$

reflected in y and shift right 1 unit (1,0) → (−1,0) → (0,0)

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shift 1 unit right (1,0) → (2,0)

6. Match the Equation to the Graph

a) $f(x) = \log_3(x-1)$	E
b) $f(x) = \log_3(1-x)$	B
c) $f(x) = \log_3 x + 2$	C
d) $f(x) = -\log_3 x$	F
e) $f(x) = -\log_3(-x)$	A
f) $f(x) = -\log_3(x+2)$	D

−(x−1)

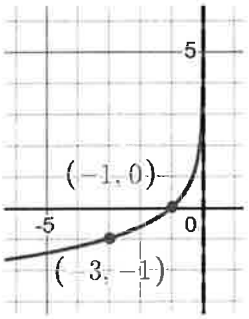
(1,0)→(1,2) up 2

Reflect in x-axis

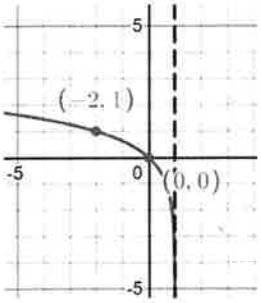
Double reflection

Reflect y-values shift left 2 units

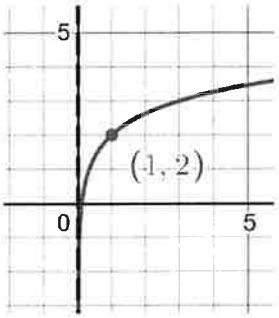
A



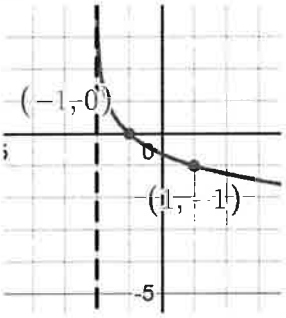
B



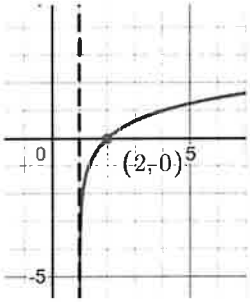
C



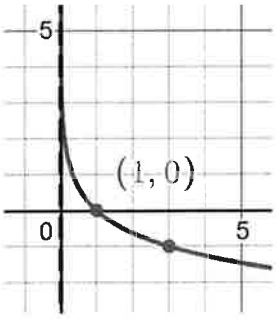
D



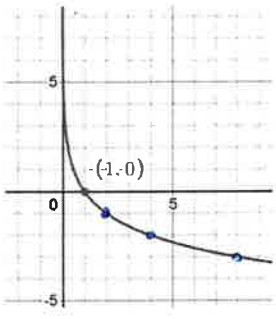
E



F

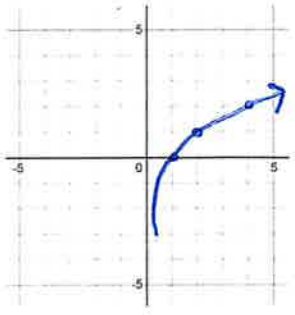


7. If $y = \log_b a$ is shown by the graph below left, what is the shape of:



a) $y = -\log_b a$

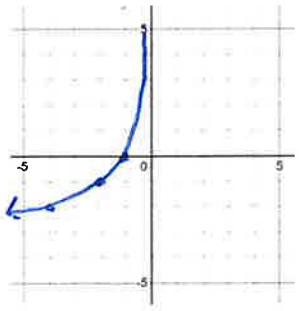
Reflection of y-values



original: $y = \log_b a \rightarrow b^y = a$

b) $y = \log_b(-a)$

reflection of x-values

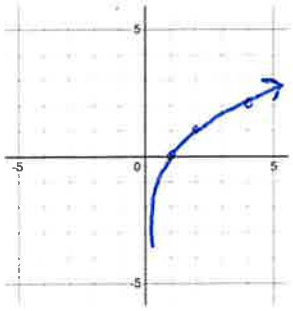


c) $y = \log_{\frac{1}{b}} a$

$\left(\frac{1}{b}\right)^y = a$

$b^{-y} = a$

reflection of exponential in x-axis



8. If point (a, b) is on the graph of $y = 5^x$, what point satisfies $y = \log_5 x$?

$y = 5^x \rightarrow \log_5 y = x$

So $y = \log_5 x$ is the inverse of $x = \log_5 y$

so $(a,b) \rightarrow (b,a)$

9. If point on the graph of $y = \log_2 x$ is $(1,0)$, what point must be on the graph of $y = -\log_2 x$?

↑ reflection of y-values

$(1,0) \rightarrow (1,0)$

10. If (c, d) is on the graph of $y = \log_b a$ what point must be on the graph of $y = \log_{\frac{1}{b}} a$?

$y = \log_b a \rightarrow b^y = a$

$y = \log_{\frac{1}{b}} a \rightarrow \left(\frac{1}{b}\right)^y = a$

$b^{-y} = a$

Reflection of y-values

$(c,d) \rightarrow (c,-d)$

11. Without a calculator, between what two integers do we find:

a) $\log 1253$

$\log 1253 = x$ $\quad 10^x = 1253$

$10^3 = 1000$ $\quad 10^4 = 10000$ between 3 and 4

$3 < \log 1253 < 4$

b) $\log 0.025$

$\log 0.025 = x$

$10^x = 0.025$

$10^{-1} = 0.100$ } in between

$10^{-2} = 0.010$ }

$-2 < \log 0.025 < -1$

12. Graph $y = \log(2 - x)$, label asymptotes and axis crossings

↑

$2 - x > 0$

$2 > x$ Domain and asymptote

x-int

$0 = \log(2-x)$

$10^0 = (2-x)$

$1 = 2 - x$

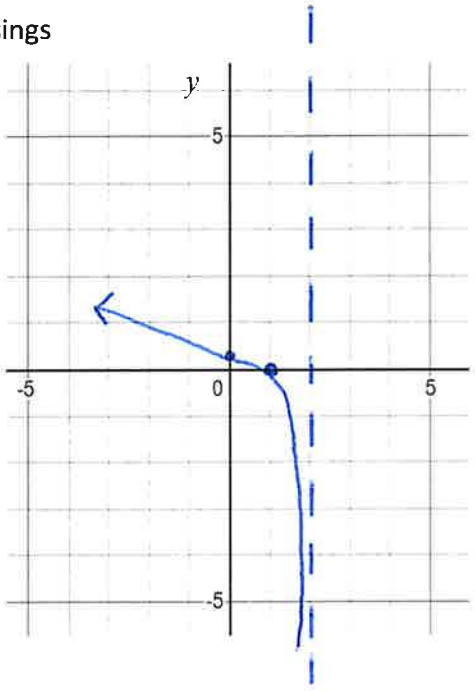
$x = 1$

y-int

$y = \log(2-0)$

$y = \log 2$

$y = 0.301$



13. Determine the inverse of the following functions:

a) $y = 8^{x-2}$ → $x = 8^{y-2}$

$\log_8 x = y - 2$

$y = \log_8 x + 2$ → $f^{-1}(x) = \log_8 x + 2$

b) $f(x) = 5^{4x-1} + 6$ → $y = 5^{4x-1} + 6$

$x = 5^{4y-1} + 6$

$(x-6) = 5^{4y-1}$

$\log_5(x-6) = 4y - 1$ → $\log_5(x-6) + 1 = 4y$

$\frac{1}{4}[\log_5(x-6) + 1] = y$

$f^{-1}(x) = \frac{1}{4}[\log_5(x-6) + 1]$

c) $y + 1 = \log_3(x - 2)$

$y = \log_3(x-2) - 1$ → $x = \log_3(y-2) - 1$

$x + 1 = \log_3(y-2)$

$3^{x+1} = y - 2$ $\quad y = 3^{x+1} + 2$

$f^{-1}(x) = 3^{x+1} + 2$

d) $f(x) = 2 + \log(5x - 3)$

$y = 2 + \log(5x-3)$

$x = 2 + \log(5y-3)$ → $x - 2 = \log(5y-3)$

$10^{x-2} = 5y - 3$

$10^{x-2} + 3 = 5y$ → $\frac{10^{x-2} + 3}{5} = y$

$f^{-1}(x) = \frac{10^{x-2} + 3}{5}$

See Website for Detailed Answer Key

Extra Work Space