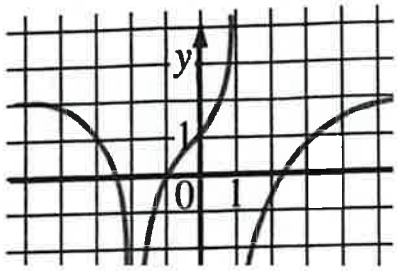
Section 5.2 – Practice Problems

1. State the equations of the horizontal and vertical asymptotes

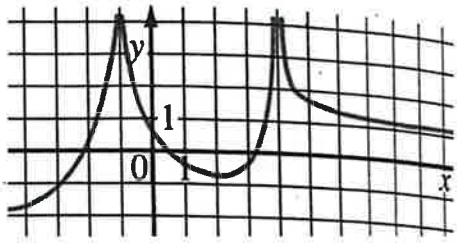
a)



HA: $y = 2$

VA: $x = -2$, $x = 1$

b)



HA: $y = 1$

VA: $x = 4$, $x = -1$

2. Find the limit

a) $\lim_{x \to \infty} \frac{6}{\sqrt{x}}$

0

b) $\lim_{x \to -\infty} 3x^{-5} \rightarrow \lim_{x \to -\infty} \frac{3}{x^5}$

0

c) $\lim_{x \to \infty} \frac{2x+1}{x-3}$

$\frac{2+\frac{1}{x}}{1-\frac{3}{x}} \rightarrow 2$

d) $\lim_{x \to -\infty} \frac{2x+1}{x-3}$

$\frac{2+\frac{1}{x}}{1-\frac{3}{x}} = 2$

e) $\lim_{x \to \infty} \frac{1-x}{3+5x}$

$\frac{\frac{1}{x}-1}{\frac{3}{x}+5} \rightarrow -\frac{1}{5}$

f) $\lim_{x \to -\infty} \frac{x^2-x+1}{x^2+3x-2}$

$\frac{1-\frac{1}{x}+\frac{1}{x^2}}{1+\frac{3}{x}-\frac{2}{x^2}}$

1

g) $\lim_{x \to \infty} \frac{x+3}{x^2-5x+7}$

$\frac{\frac{1}{x}+\frac{3}{x^2}}{\frac{x^2}{x^2}-\frac{5}{x}+\frac{7}{x^2}} \rightarrow 0$

h) $\lim_{x \to \infty} \frac{x^2-1}{(x+3)(2x+4)} \rightarrow \frac{x^2-1}{2x^2+10x+12}$

$\frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{2x^2}{x^2}+\frac{10x}{x^2}+\frac{12}{x^2}} \rightarrow \frac{1-0}{2+0+0} = \frac{1}{2}$

i) $\lim_{x \to -\infty} \frac{3x^3 + x^2 - 5}{x^3 - 4x + 1}$

$\frac{3 + \frac{1}{x} - \frac{5}{x^3}}{1 - \frac{4}{x^2} + \frac{1}{x^3}} \to \boxed{3}$

j) $\lim_{x \to -\infty} \frac{12x^2 - 2x + 1}{3x^4 - 14x^2 + x - 3}$

$\frac{\frac{12}{x^2} - \frac{2}{x} + \frac{1}{x^2}}{3 - \frac{14}{x^2} + \frac{1}{x^3} - \frac{3}{x^4}} \to \boxed{0}$

3. Find the horizontal asymptotes of each curve

a) $y = \frac{2x - 3}{5 - 4x}$

$\lim_{x \to \pm\infty} \frac{2 - \frac{3}{x}}{\frac{5}{x} - 4} \to -\frac{2}{4} = \boxed{-\frac{1}{2}}$

b) $y = \frac{x}{x^2 + 1}$

$\lim_{x \to \pm\infty} \frac{\frac{1}{x}}{1 + \frac{1}{x^2}} \to \frac{0}{1+0} = \boxed{0}$

c) $y = \frac{x^3 + 1}{x^3 - 1}$

$\lim_{x \to \pm\infty} \frac{\frac{x^3}{x^3} + \frac{1}{x^3}}{\frac{x^3}{x^3} - \frac{1}{x^3}} \to \frac{1+0}{1-0} = \boxed{1}$

d) $y = 1 - \frac{x}{x^2 - 2}$

$\lim_{x \to \pm\infty} \frac{x^2 - 2 - x}{x^2 - 2} \to \boxed{1}$

4. Find the horizontal and vertical asymptotes. Use them, together with intercepts to sketch the graph.

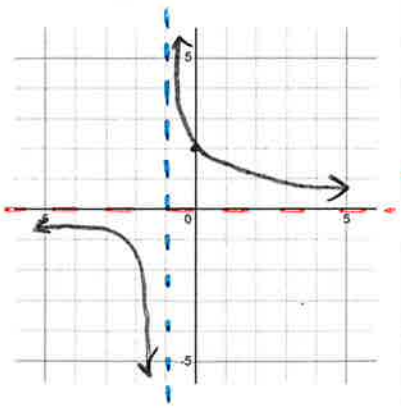
a) $y = \frac{2}{x + 1}$

VA: $x = -1$

HA: $y = 0$

no x-int

y-int: $(0, 2)$



b) $y = \frac{x}{x + 1}$

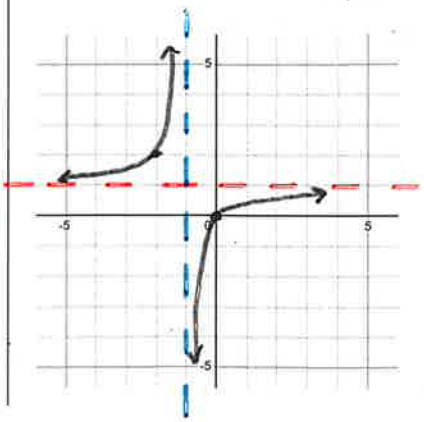
VA: $x = -1$

HA: $y = 1$

$\frac{1}{1+0} = 1$

y-int: $(0, 0)$

x-int: $(0, 0)$



c)

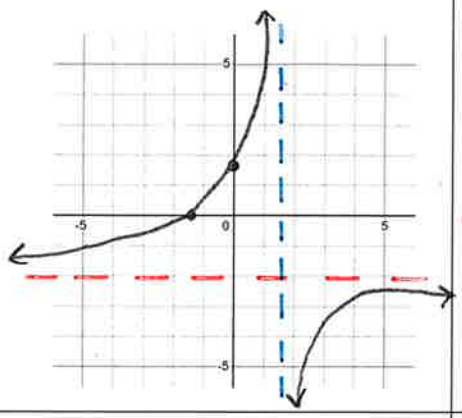
$$y = \frac{4x+5}{3-2x}$$

VA: $x = \frac{3}{2}$

HA: $y = -2$

y-int: $(0, 5/3)$

x-int: $(-5/4, 0)$



d)

$$y = \frac{1}{x^2-1}$$

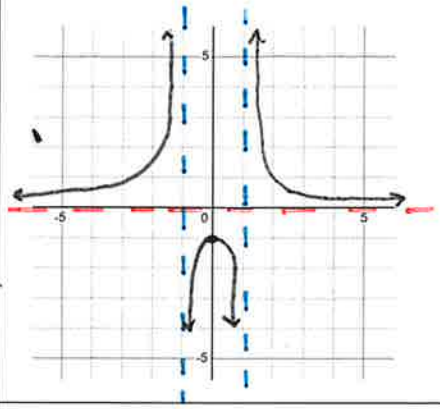
VA: $x = \pm 1$

HA: $y = 0$

no x-ints

y-int: $(0,-1)$

$$\frac{\frac{1}{x^2}}{1-\frac{1}{x^2}} = 0$$



e)

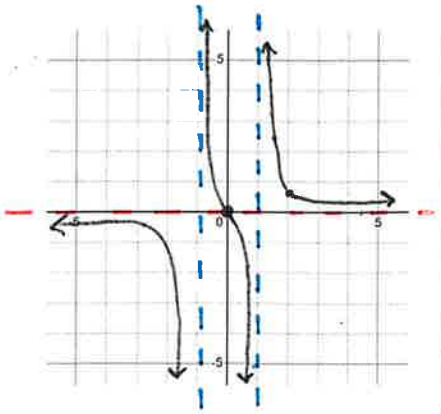
$$y = \frac{x}{x^2-1}$$

VA: $x = \pm 1$

HA: $y = 0$

y-int: $(0,0)$

x-int: $(0,0)$



f)

$$y = \frac{2x^2}{x^2+3x-4} = \frac{2x^2}{(x+4)(x-1)}$$

VA: $x = -4$

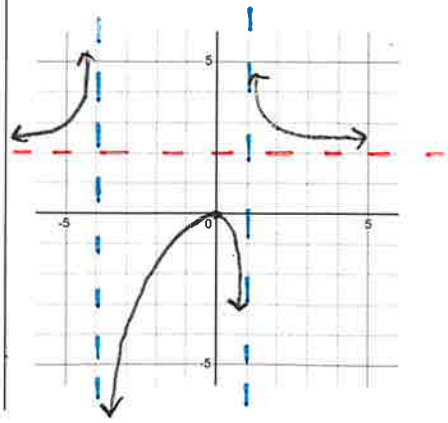
$x = 1$

HA: $y = 2$

$$\frac{2}{1} = 2$$

x-int: $(0,0)$

y-int: $(0,0)$



g) $y = \dfrac{x}{(x+2)^2}$

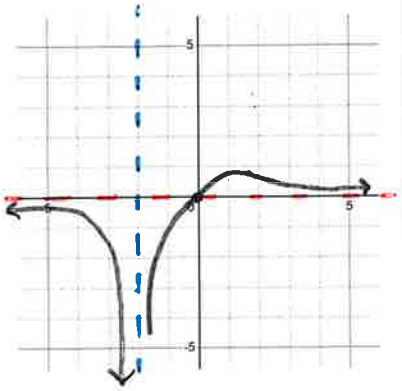
y-int: (0,0)

x-int: (0,0)

VA: $x = -2$

HA: $y = 0$

$x = 2$ $y = \frac{1}{8}$ $x = 1$ $y = \frac{1}{9}$



h) $y = \dfrac{x^2}{(x+2)^2}$

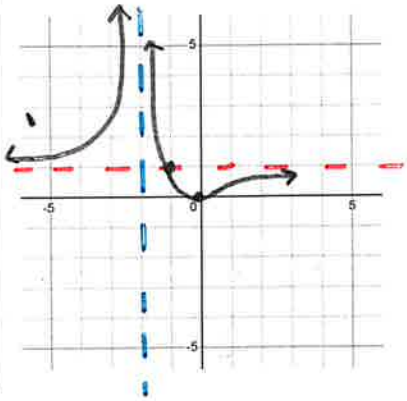
x-int: (0,0)

y-int: (0,0)

VA: $x = -2$

HA: $y = 1$

$x = -1$ $y = 1$ $x = 1$ $y = \frac{1}{9}$



5. Find the limit

a) $\lim_{x \to \infty} \sqrt{x}$

∞

b) $\lim_{x \to -\infty} x^5$

$-\infty$

c) $\lim_{x \to \infty} (x^3 - x^2)$

$x^2(x-1)$

∞

d) $\lim_{x \to -\infty} (x^3 - x^2)$

$x^2(x-1)$

\+ −

$-\infty$

e) $\lim_{x \to \infty} x^2(2x+1)(x-2)$

\+ \+ \+

∞

f) $\lim_{x \to \infty} (x+2)^4(3-x)$

\+ −

$-\infty$

6. Find the limits as $x \to \infty$ and $x \to -\infty$. Use this information, together with intercepts, to give a rough sketch of the graph.

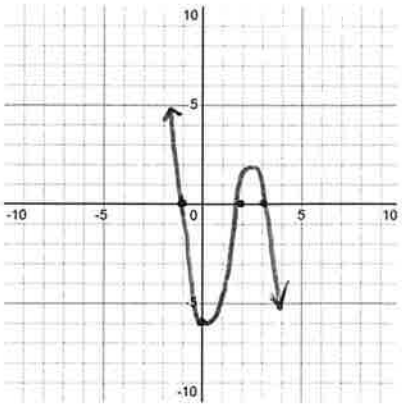
a) $y = (x+1)(x-2)(3-x)$

y-int: (0,-6)

x-int: (-1,0) (2,0) (3,0)

$\lim_{x \to -\infty}$ ∞

$\lim_{x \to \infty}$ $-\infty$



bounces off

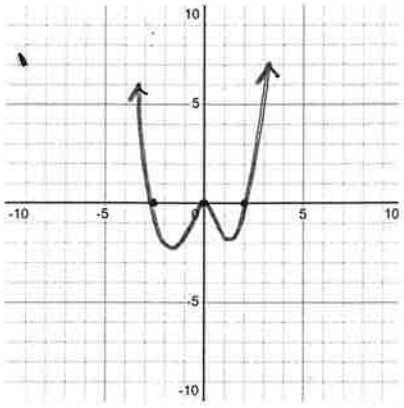
b) $y = x^2(x-2)(2x+5)$

$\lim_{x \to \infty}$ ∞

$\lim_{x \to -\infty}$ ∞

x-int: (0,0) (2,0) $(-5/2, 0)$

y-int: (0,0)



bounces off

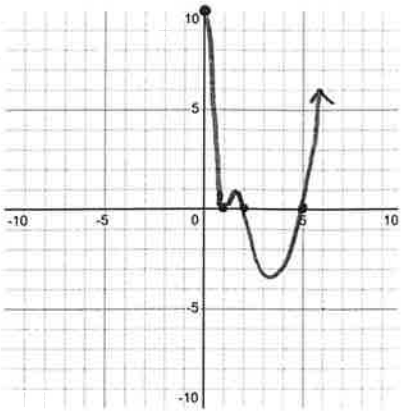
c) $y = (1-x)^2(2-x)(5-x)$

x-int: (1,0) (2,0) (5,0)

y-int: (0,10)

$\lim_{x \to \infty}$ ∞

$\lim_{x \to -\infty}$ ∞



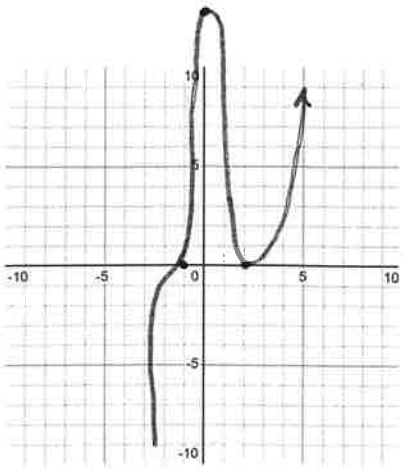
d) $y = (x+1)^3(x-2)^4$

x-int: (-1,0) (2,0)

y-int: (0,16)

$\lim_{x \to -\infty}$ $-\infty$

$\lim_{x \to \infty}$ ∞



7. Find the horizontal asymptote of

$$y = \frac{x}{|x| + 1}$$

$\lim_{x \to \infty} \frac{x}{x+1} \to \frac{\frac{x}{x}}{\frac{x}{x} + \frac{1}{x}} \to \frac{1}{1+0} = 1$

$\lim_{x \to -\infty} \frac{x}{-x+1} \to \frac{1}{-1+0} = -1$

HA: $y = 1$, $y = -1$

8. Find

$$\lim_{x \to \infty} \frac{\sqrt{4x^2 + 1}}{2x - 3}$$

(Hint: Divide numerator and denominator by x)

$\lim_{x \to \infty} \frac{\frac{\sqrt{4x^2+1}}{x}}{\frac{2x-3}{x}} \to \frac{\sqrt{\frac{4x^2}{x^2} + \frac{1}{x^2}}}{\frac{2x}{x} - \frac{3}{x}} \to \frac{\sqrt{4+0}}{2 - \frac{3}{x}} \lim_{x \to \infty}$

$\frac{\sqrt{4}}{2} = 1$

9. Find

$$\lim_{x \to -\infty} \frac{3x}{\sqrt{x^2 + 6}}$$

(Hint: Note that $\sqrt{x^2} = -x$ when $x < 0$)

$\frac{\frac{3x}{x}}{\frac{\sqrt{x^2+6}}{x}} \to \frac{3}{-\sqrt{\frac{x^2}{x^2} + \frac{6}{x^2}}} \to \frac{3}{-\sqrt{1+0}} = \frac{3}{-1} = -3$

neg, can't go in to root

10. Find

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 5x + 1} - x \right)$$

(Hint: Rationalize)

$$\lim_{x \to \infty} \left(\sqrt{x^2+5x+1} - x \right)\left(\sqrt{x^2+5x+1} + x \right) \rightarrow \frac{x^2+5x+1-x^2}{\sqrt{x^2+5x+1}+x}$$

$$\lim_{x \to \infty} \frac{5x+1}{\sqrt{x^2+5x+1}+x} \rightarrow \frac{\frac{5x}{x}+\frac{1}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{5x}{x^2}+\frac{1}{x^2}}+\frac{x}{x}} \rightarrow \frac{5+0}{\sqrt{1+\frac{5}{\infty}+\frac{1}{\infty}}+1} \rightarrow \boxed{\frac{5}{2}}$$

11. Find

$$\lim_{x \to -\infty} \frac{x^{10} + 6x^6 - 3}{x^5 + 2x}$$

(Hint: Divide numerator and denominator by x^5)

$$\lim_{x \to -\infty} \frac{\frac{x^{10}}{x^5}+\frac{6x^6}{x^5}-\frac{3}{x^5}}{\frac{x^5}{x^5}+\frac{2x}{x^5}} \rightarrow \frac{x^5+6x+\frac{3}{x^5}}{1+\frac{2}{x^4}} \lim_{x \to -\infty} \rightarrow \frac{-\infty}{1} = \boxed{-\infty}$$

12. How large do we have to take x so that $\frac{1}{x^2} < 0.000\,001$?

$$\frac{1}{x^2} < 0.000\,001$$

$$\frac{1}{0.000\,001} < x^2$$

$$\sqrt{\frac{1}{0.000\,001}} < |x|$$

$$\sqrt{1\,000\,000} < |x|$$

$$\boxed{1000 < |x|}$$