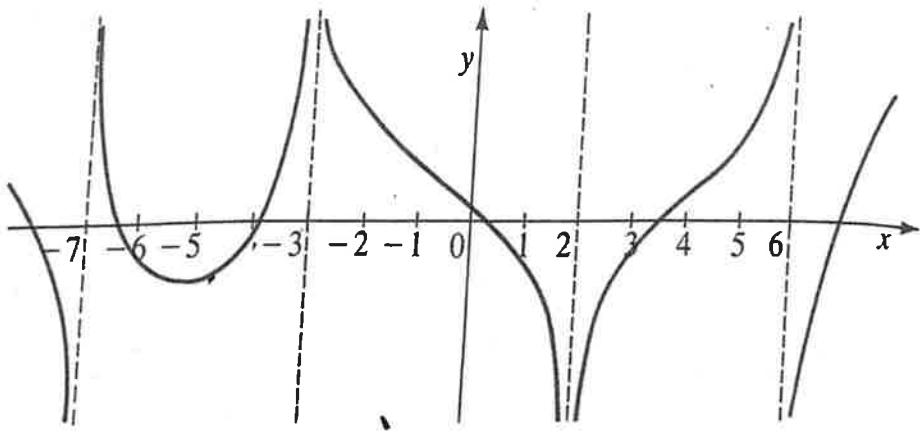
## Section 5.1 – Practice Problems

1. The graph of $f$ is given.




a) State the equations of the vertical asymptotes.

$x=-7$ $x=2$
$x=-3$ $x=6$

b) State the following.

| | |
|---|---|
| i. $\lim_{x\to -7^-} f(x)$ $-\infty$ | ii. $\lim_{x\to -7^+} f(x)$ $\infty$ |
| iii. $\lim_{x\to -3} f(x)$ $\infty$ | iv. $\lim_{x\to 2} f(x)$ $-\infty$ |
| v. $\lim_{x\to 6^-} f(x)$ $\infty$ | vi. $\lim_{x\to 6^+} f(x)$ $-\infty$ |

2. Find each limit.

all instances limits produce very small denominators.

a) $\lim_{x\to 8} \frac{1}{(x-8)^2}$ $\infty$

↑ always positive

b) $\lim_{x\to 1^-} \frac{3}{x-1}$ $\frac{3}{-}$ $-\infty$

c) $\lim_{x\to 1^+} \frac{3}{x-1}$ $\frac{3}{+}$ $\infty$

d) $\lim_{x\to -1} \frac{-2}{(x+1)^2}$ $\frac{-}{+}$ $-\infty$

e) $\lim_{x \to 2^+} \frac{x-4}{x-2}$ $\frac{-}{+}$ $-\infty$

f) $\lim_{x \to 2^-} \frac{x-4}{x-2}$ $\frac{-}{-}$ $\infty$

g) $\lim_{x \to -4} \left[1 + \frac{2x}{(x+4)^6}\right]$ $1 + -\infty$ $-\infty$ ↑ always positive

h) $\lim_{x \to 3^+} \left[x + \frac{2-x}{x-3}\right]$ $3 + \frac{-}{+}$ $3 + -\infty$ $-\infty$

i) $\lim_{x \to -2^+} \frac{x}{x^2-4}$ $\frac{-}{-}$ $\infty$

j) $\lim_{x \to -2^-} \frac{x}{x^2-4}$ $\frac{-}{+}$ $-\infty$ ↑ $-2.1$ as an example

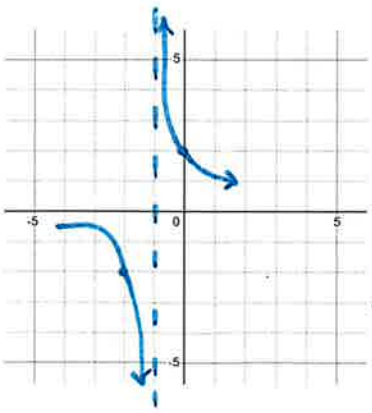
k) $\lim_{x \to 9^+} \frac{5-x}{\sqrt{x-9}}$ $\frac{-}{+}$ $-\infty$

l) $\lim_{x \to -3^+} \frac{10}{x^2-x-12}$ → $\frac{10}{(x-4)(x+3)}$ $-$ $+$ $-\infty$

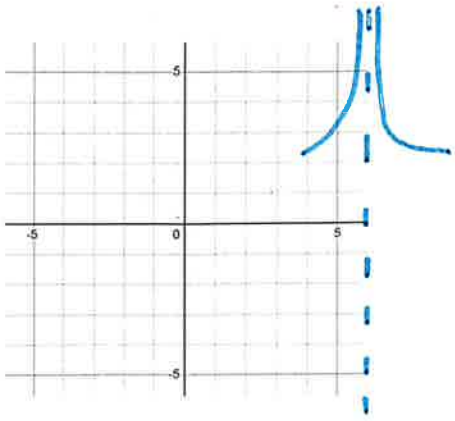
3. Find the vertical asymptotes and sketch the graph near the asymptotes.

a) $y = \frac{2}{x+1}$

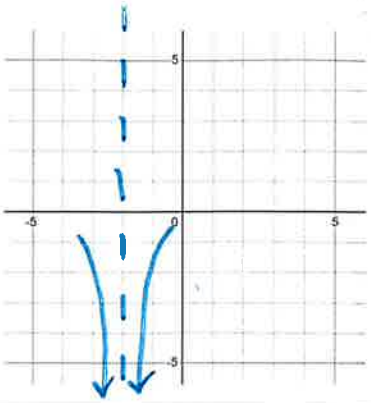
VA: $x = -1$



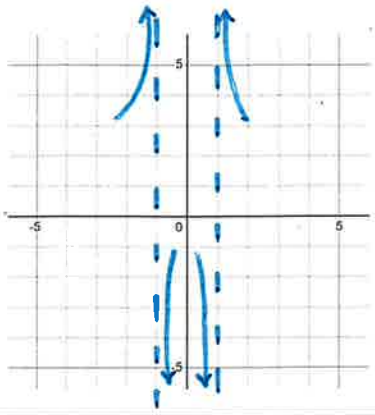

b) $y = \frac{3}{(x-6)^2}$



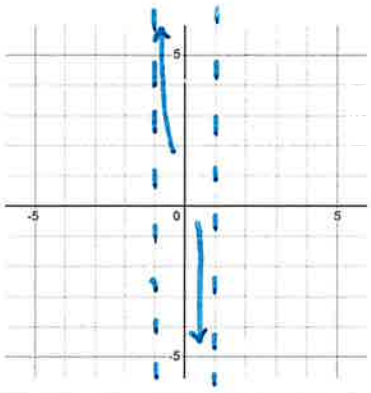
c) $y = \dfrac{x}{(x+3)^2}$



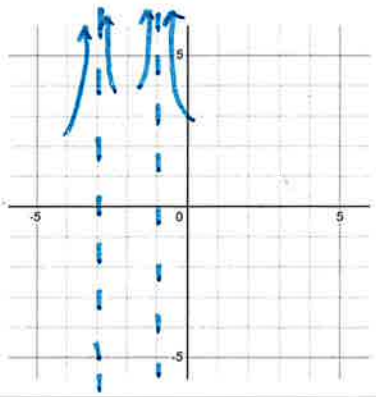
d) $y = \dfrac{1}{x^2-1}$



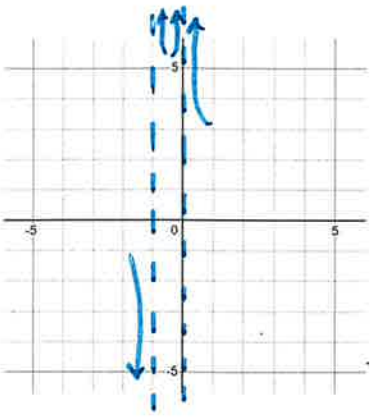
e) $y = \dfrac{x}{x^2-1}$



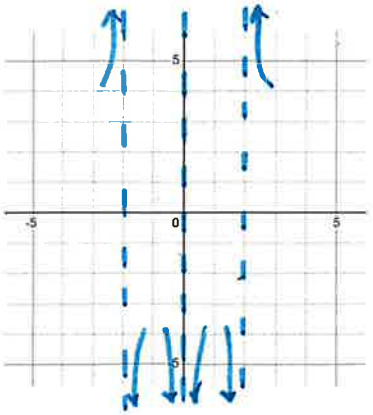
f) $y = \dfrac{6x^3}{x^2+4x+3} = \dfrac{6x^3}{(x+1)(x+3)}$



g) $y = \dfrac{1}{x^2(x+1)}$



h) $y = \dfrac{1}{x^4-4x^2} \rightarrow \dfrac{1}{x^2(x^2-4)} = \dfrac{1}{x^2(x-2)(x+2)}$



4. Find

$$\lim_{x \to 0^+} \left( \frac{5}{x} - \frac{2}{x^2} \right)$$

$\lim_{x \to 0^+} \left( \frac{5x-2}{x^2} \right) \rightarrow \frac{-2}{+} \rightarrow -\infty$

very small

5. How small do we have to take $x$ so that: $\frac{1}{x^4} > 100\,000\,000$?

$\frac{1}{100\,000\,000} > x^4$

$\sqrt[4]{\frac{1}{100\,000\,000}} > |x|$

$\frac{1}{100} > |x|$