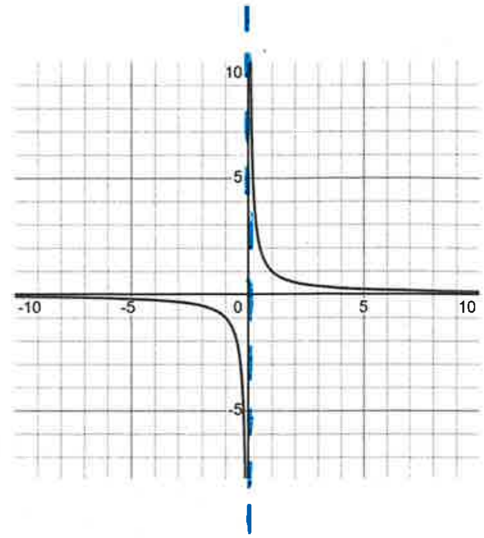
5.1 Vertical Asymptotes

We have seen this concept many times up to this point. Vertical asymptotes exist when our denominator is an undefined value, and is not cancelled out by the same factor in the numerator (this creates a Hole).

Let us examine the most basic function $f(x) = \frac{1}{x}$

We see the following behaviour.

x	$f(x)$
1	1
−1	−1
0.5	2
−0.5	−2
0.1	10
−0.1	−10
0.001	1000
−0.001	−1000



We see that as we approach the asymptote, our $f(x)$ values get exponentially large in either direction.

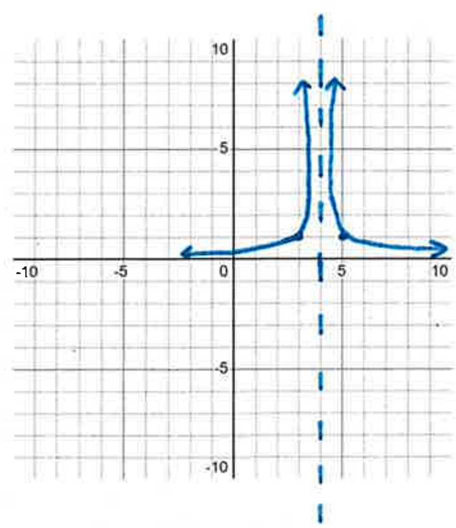
With respect to limits, we see the following behaviour:

$$\lim_{x \to 0^-} f(x) = -\infty \qquad \lim_{x \to 0^+} f(x) = \infty$$

From this behaviour, we say that the vertical asymptote of $f(x)$ is the line $x = 0$.

Ex 1. Consider the following function. Discuss the behaviour of the graph as we approach the vertical asymptote.

$$y = \frac{1}{(x-4)^2}$$



$\lim_{x \to 4^-} \left(\frac{1}{(3.999-4)^2} \right)$ ← denominator always positive

$\lim_{x \to 4^-} = \frac{1}{\text{very small}} = \infty$

$\lim_{x \to 4^+} = \frac{1}{\text{very small}} = \infty$

$x = 3 \quad y = 1$

$x = 5 \quad y = 1$

$x = 10 \quad y = \frac{1}{36}$

$x = -2 \quad y = \frac{1}{36}$

Ex. 2 Find

$$\lim_{x \to 6}\left[2 - \frac{5}{\underbrace{(x-6)^2}_{\text{always positive}}}\right]$$

so $\lim\limits_{x \to 6^-}$ and $\lim\limits_{x \to 6^+}$ is the same

very small positive

$$\lim_{x \to 6}\left[2 - \frac{5}{\text{very sm positive}}\right]$$

$$[2 - \infty]$$

$$\boxed{-\infty}$$

Ex. 3 Find the vertical asymptotes of the following function and graph the behaviour near the asymptotes.

$$y = \frac{x}{x^2 - x - 6} \rightarrow \frac{x}{(x-3)(x+2)}$$

VA at $x = 3$ $x = -2$

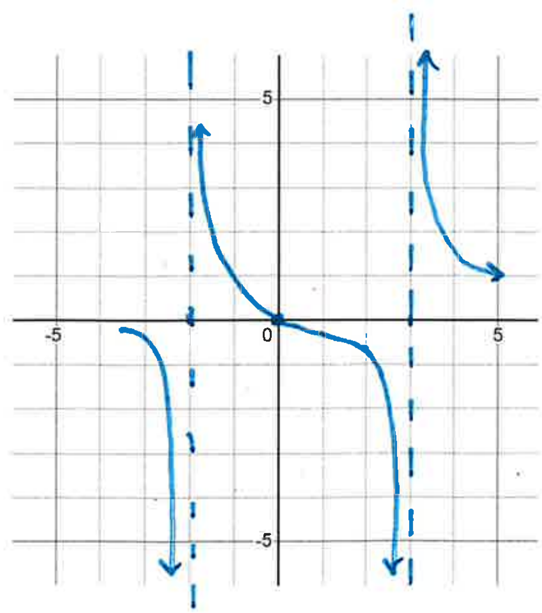
when $x = 0$ $y = 0$

$x = 1$ $y = -\frac{1}{6}$

$x = 2$ $y = -\frac{1}{2}$

$$\lim_{x \to -2^-}\left[\frac{-}{(-)(-)}\right] = -$$

$$\lim_{x \to 3^+}\left[\frac{+}{(+)(+)}\right] = +$$



Homework Questions

Practice Problems: #1, 2adgl, 3ac, 5