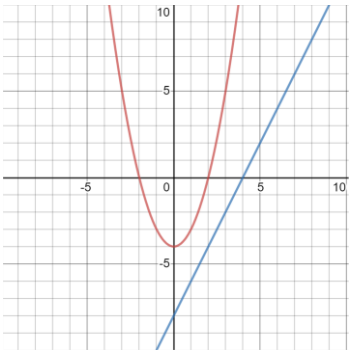
Section 5.1 – Graphing Non-Linear Systems of Equations

This Booklet Belongs to: _____ **Block:** _____

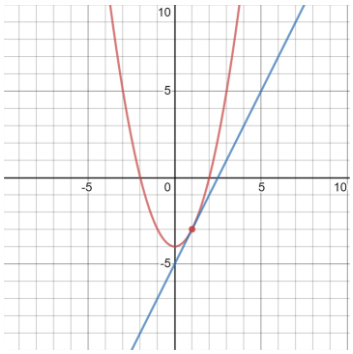
- In grade 10 we saw a system of equations for linear systems (two straight lines and where they intersect)
- What we are going to see this time around is a system of equations involving non-linear equations
- Either a straight line and curved line or two curve
- Non-linear equations have a degree of $2\ or\ more$, where a linear equation is of degree 1

Graphing a System: A Parabola and a Straight Line

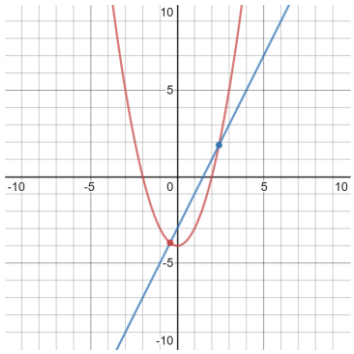
- In this scenario we will see either $0, 1, or\ 2$ solutions



$0\ Real\ Solutions$



$1\ Real\ Solutions$



$2\ Real\ Solutions$

- Be careful graphing these, as they will need to be very accurate to get an accurate solution

Example 1: Is $(3, -5)$ a solution to the non-linear system:

$$x^2 + y = 4 \text{ and } 2x + y = 1$$

Solution 1: Remember we are trying to determine if the (x, y) of the ordered pair can be replaced with our $(3, -5)$ and their equality remains true. It has to happen for both equations.

$$x^2 + y = 4 \qquad 2x + y = 1$$

$$(3)^2 + (-5) = 4 \qquad 2(3) + (-5) = 1$$

$$9 - 5 = 4 \qquad 6 - 5 = 1$$

$$\mathbf{4 = 4} \qquad \mathbf{1 = 1}$$

Both equations hold after the substitution so $(3, -5)$ is a solution.

Example 2: Is $(5, 7)$ a solution to the non-linear system:

$$3x^2 - 10y = 5 \text{ and } x - y = -3$$

Solution 2: Remember we are trying to determine if the (x, y) of the ordered pair can be replaced with our $(5, 7)$ and their equality remains true. It has to happen for both equations.

$3x^2 - 10y = 5$	$x - y = -3$
$3(5)^2 - 10(7) = 5$	$(5) - (7) = -3$
$75 - 70 = 5$	$5 - 7 = -3$
$\mathbf{5 = 5}$	$\mathbf{-2 \neq -3}$

Both equations DO NOT hold after the substitution so $(5, 7)$ is NOT a solution.

Example 3: Solve the system: $y = x^2 - 5x + 4 \quad and \quad x - y = 1$ by graphing

Solution 3: Graph the Parabola first, find the vertex and the $x - intercepts$ and $y - intercept$

$$y = x^2 - 5x + 4$$

Factor for the $x - intercepts$:

$$y = x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$\boldsymbol{x - intercepts}: (\mathbf{1, 0}) \boldsymbol{\ and\ } (\mathbf{4, 0})$$

Set $x = 0$ for $y - intercept$:

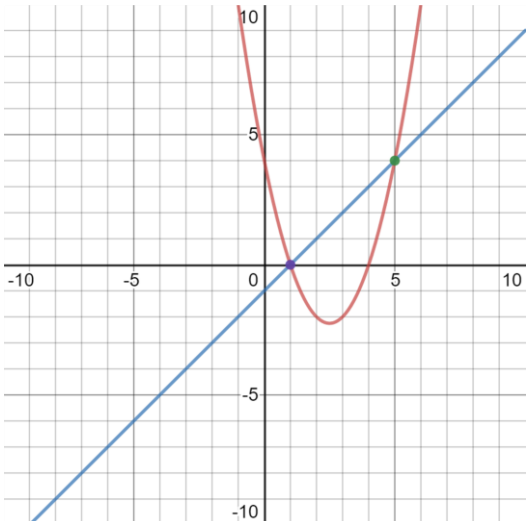
$$y = x^2 - 5x + 4 \quad \rightarrow \quad y = (0)^2 - 5(0) + 4$$

$$\boldsymbol{y = 4} \qquad \boldsymbol{y - intercept}: (\mathbf{0, 4})$$

Complete the Square for the Vertex:

$$y = x^2 - 5x + 4 \quad \rightarrow \quad \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + 4$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4} \qquad \boldsymbol{Vertex\ is}: \left(\mathbf{\frac{5}{2}, -\frac{9}{4}}\right)$$



Linear equation in Slope –Intercept Form:

$$y = x - 1$$

So a $\boldsymbol{Slope\ of\ 1}$

and $\boldsymbol{y - intercept\ of\ (0, -1)}$

The Graphs intersect at two points: $(\mathbf{1, 0}) \ \boldsymbol{and}\ (\mathbf{5, 4})$

Example 4: Solve the system: $y = x^2 - 4 \quad and \quad 2x - y = 5$ by graphing

Solution 4: Graph the Parabola first, find the vertex and the $x - intercepts$ and $y - intercept$

$$y = x^2 - 4$$

Factor for the $x - intercepts$:

$$y = x^2 - 4 = (x - 2)(x + 2)$$

$$\boldsymbol{x - intercepts}: (\mathbf{2}, \mathbf{0}) \ \boldsymbol{and} \ (\mathbf{-2}, \mathbf{0})$$

Set $x = 0$ for $y - intercept$:

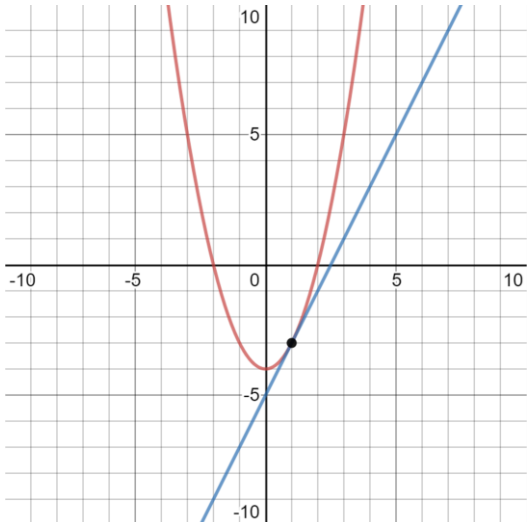
$$y = x^2 - 4 \quad \rightarrow \quad y = (0)^2 - 4$$

$$\boldsymbol{y = -4} \qquad \boldsymbol{y - intercept}: (\mathbf{0}, \mathbf{-4})$$

The Vertex is easily discernible since there is no b term:

$$y = x^2 - 4 \quad \rightarrow \quad \left(x^2 + \frac{0}{4}\right) - \frac{0}{4} - 4$$

$$y = (x)^2 - 4 \qquad \boldsymbol{Vertex\ is}: (\mathbf{0}, \mathbf{-4})$$



Linear equation in Slope –Intercept Form:

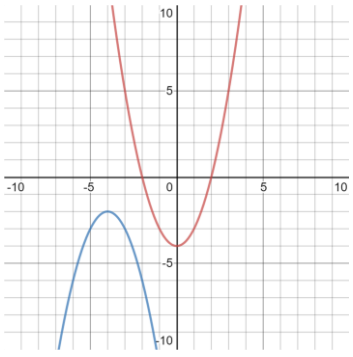
$$y = 2x - 5$$

So a $\boldsymbol{Slope\ of\ 2}$

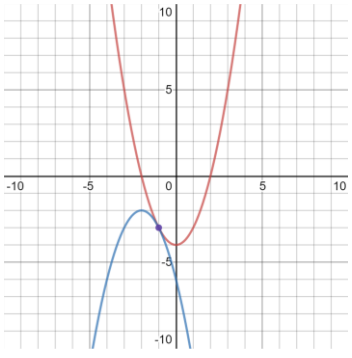
and $\boldsymbol{y - intercept\ of\ (0, -5)}$

The Graphs intersect at two points: $\boldsymbol{(1, -3)\ only}$

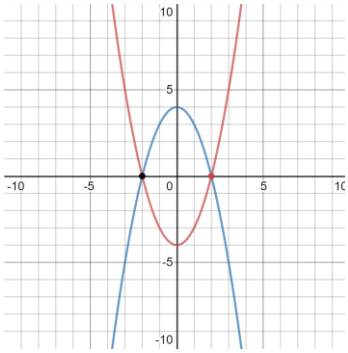
Graphing a System: Two Parabolas



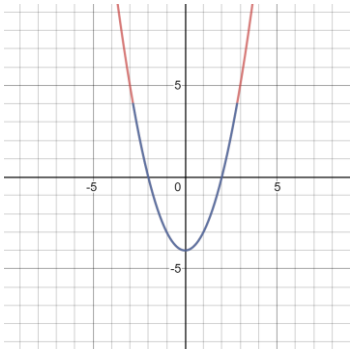
0 Real Solutions



1 Real Solutions



2 Real Solutions



Infinite Real Solutions

Identical Graph

Example 5: Solve the system $y = -(x-2)^2 + 3$ and $y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$ by graphing.

Solution 5: Both are in Vertex Form, so just need to find $x\ and\ y-intercepts$

$$y = -(x-2)^2 + 3$$

For the $x-intercepts$ use the Square Root Method:

$$0 = -(x-2)^2 + 3 \quad \rightarrow \quad (x-2)^2 = 3$$

$$x = 2 \pm \sqrt{3}$$

$$\boldsymbol{x-intercepts}: (\mathbf{0.27}, \mathbf{0})\ \boldsymbol{and}\ (\mathbf{3.73}, \mathbf{0})$$

Set $x = 0$ for $y-intercept$:

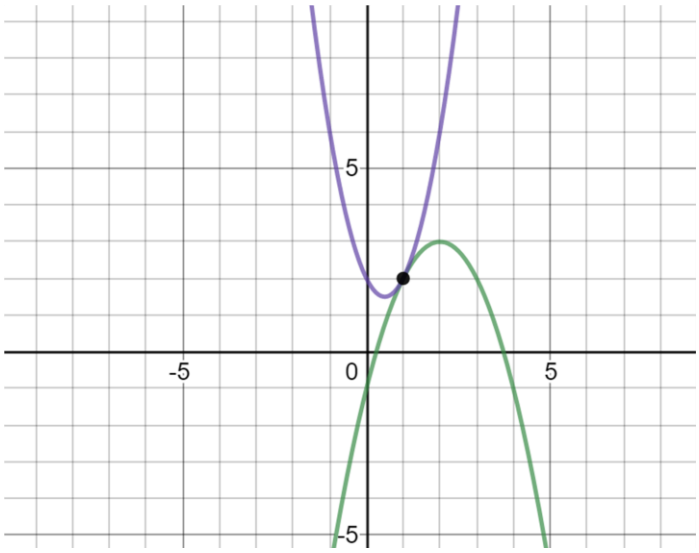
$$y = -(x-2)^2 + 3 \quad \rightarrow \quad y = -(0-2)^2 + 3$$

$$\boldsymbol{y = -1} \qquad \boldsymbol{y-intercept}: (\mathbf{0}, \mathbf{-1})$$

The Vertex:

$$y = -(x-2)^2 + 3$$

$$\boldsymbol{Vertex\ is}: (\mathbf{2}, \mathbf{3})$$



$$y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$$

For the $x-intercepts$ use the Square Root Method:

$$y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2} \quad \rightarrow \quad -\frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$Cannot\ Square\ Root\ a\ Negative$$

Could also notice that the Parabola opens up: $a = 2$ and has a vertex above the $x-axis$

$$\boldsymbol{x-intercepts}: \boldsymbol{None}$$

Set $x = 0$ for $y-intercept$:

$$y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2} \quad \rightarrow \quad y = 2\left(0 - \frac{1}{2}\right)^2 + \frac{3}{2}$$

$$\boldsymbol{y = 2} \qquad \boldsymbol{y-intercept}: (\mathbf{0}, \mathbf{2})$$

The Vertex:

$$y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2} \qquad \boldsymbol{Vertex\ is}: \left(\mathbf{\frac{1}{2}}, \mathbf{\frac{3}{2}}\right)$$

Check:

$$y = -(x-2)^2 + 3$$

$$2 = -(1-2)^2 + 3$$

$$2 = 2$$

$$y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$$

$$2 = 2\left(1 - \frac{1}{2}\right)^2 + \frac{3}{2}$$

$$2 = 2$$

The Graphs intersect at: $(\mathbf{1}, \mathbf{2})$

Example 6: Solve the system: $y - x^2 + 4 = 0$ and $-2y + 2x^2 - 8 = 0$

Solution 6: Simplify the equations first

$y - x^2 + 4 = 0$ $y = x^2 - 4$	$-2y + 2x^2 - 8 = 0$ $-y + x^2 - 4 = 0$ $y = x^2 - 4$

Since both equations are the same, they are **the same graph** and have an **infinite number of solutions**

Using Technology to Graph

- It is possible to using graphing technology to solve more complex systems
- We will look at the **DESMOS graphing app** to play around with graphing and solving in class.

Section 5.1 – Practice Problems

Determine if the given ordered pair is a solution to the non-linear system.

1. $2x^2 - 3y = 2$ $(2, 2)$

   $x - 2y = -2$

2. $2x^2 - 3y = 2$ $(-1.25, 0.375)$

   $x - 2y = -2$

3. $x^2 + 2y = -2$ $(-2, -3)$

   $-2x + y = 2$

4. $y = x^2 + 2x - 1$ $(-3, 2)$

   $y = x^2 + 4x + 5$

5. $y = x^2 - 3x - 2$ $(-\frac{3}{5}, \frac{4}{25})$

   $y = x^2 + 2x + 1$

6. $y = x^2 - 3x - 4$ $(3, -4)$

   $y = -x^2 - 13$

Determine all possible values for k, so that the graph of the equation contains the given point

7. $y = 2(x-k)^2 + 7; \quad (3, 57)$

8. $y = -3(x-k)^2 + 6; \quad (2, -42)$

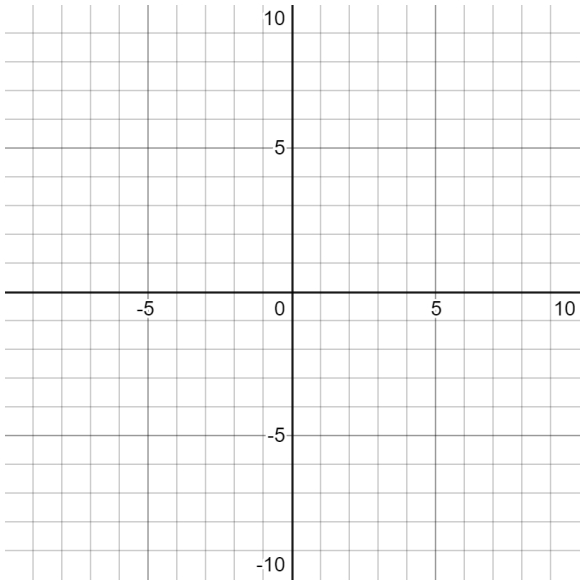
9. $y = -\frac{1}{2}(x-k)^2 + 8; \quad (-1, -10)$

10. $y = \frac{1}{3}(x-k)^2 - 4; \quad (-2, 71)$

Solve the System by Graphing

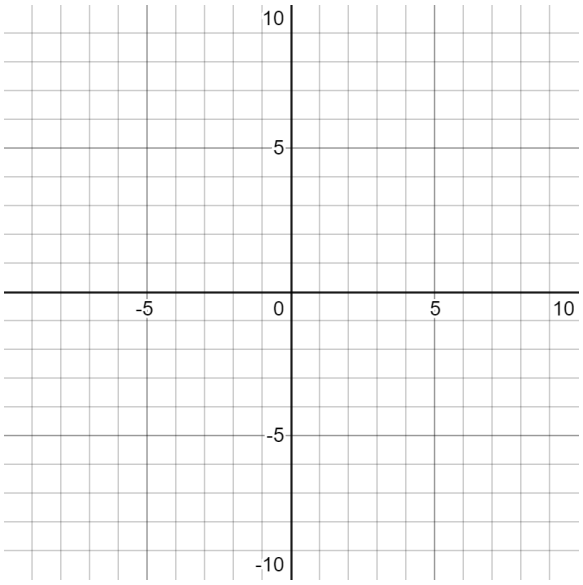
11. $y = 4 - x^2$

$y = x + 2$



12. $y = x^2 - 1$

$y = 2x - 2$



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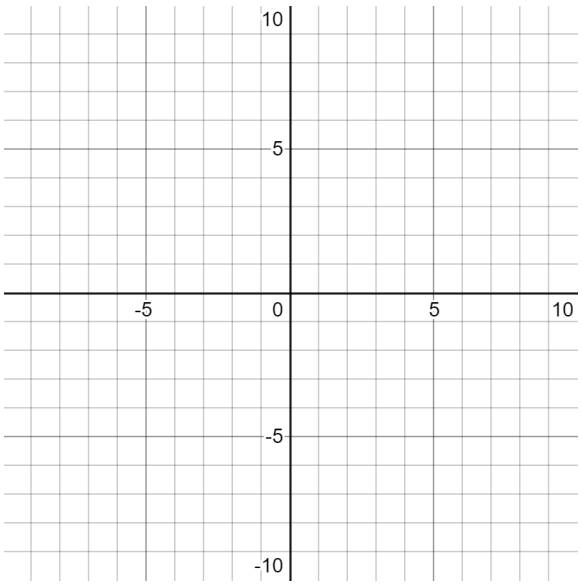
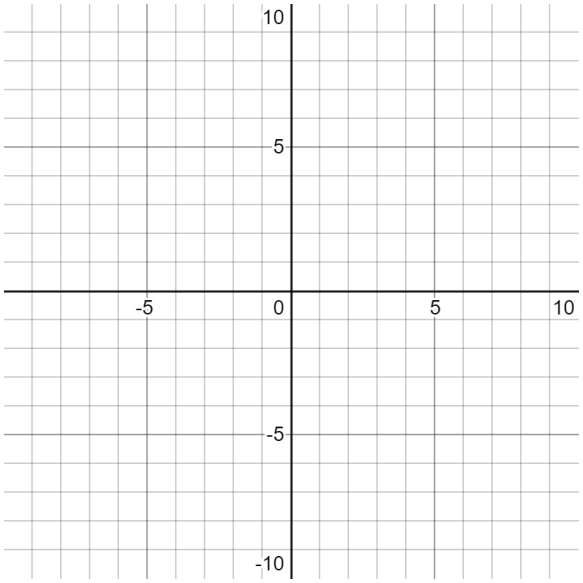
13. $y = -x^2 + 1$

$x + y = 2$



14. $y = \frac{1}{2}(x-1)^2 - 4$

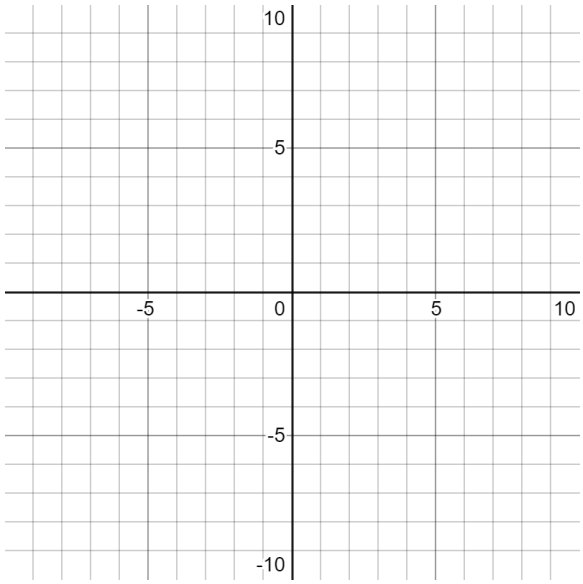
$2x + y = -4$



Solve the following systems by graphing

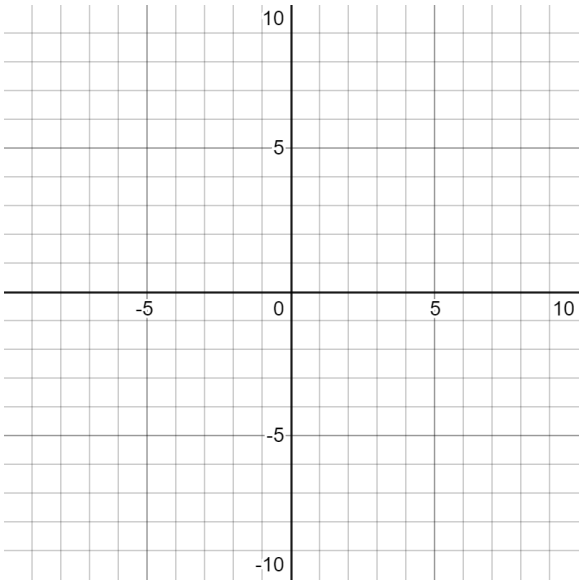
15. $y = x^2 - 4x + 4$

$y = x^2$



16. $y = -x^2 + 3$

$y = x^2 + 1$



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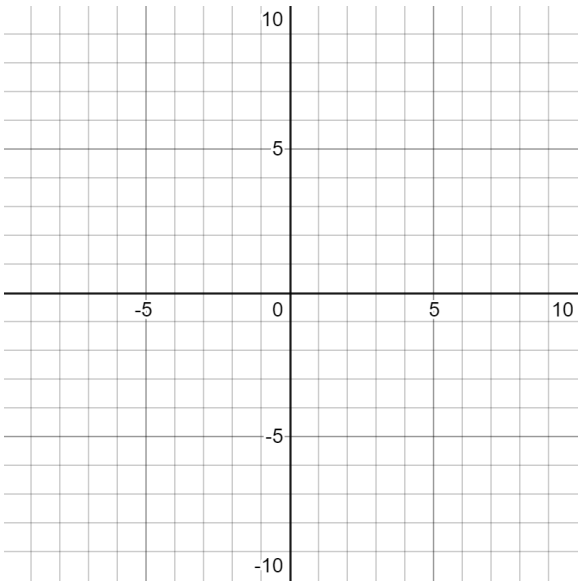
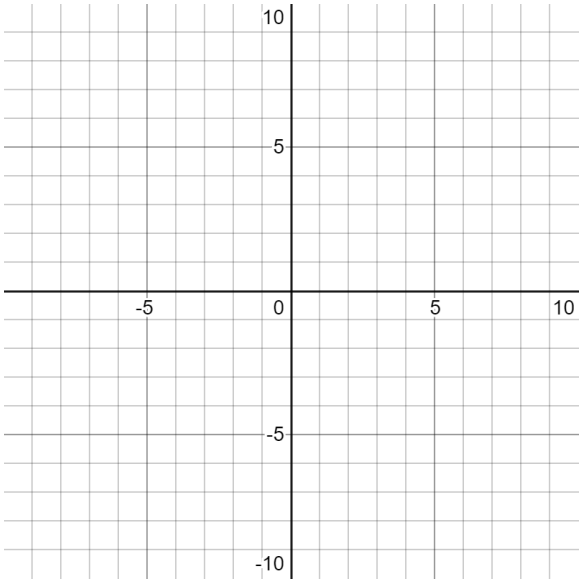
17. $y = x^2 + 2$

$y = 2x^2 + 1$



18. $y = 3x^2 + x - 4$

$y = 3x^2 - 8x + 5$



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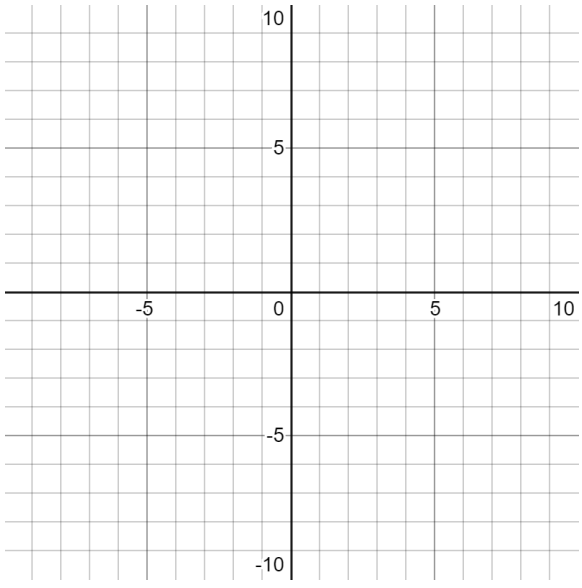
19. $y = -x^2 + x - 1$

$y = x^2 + 2x - 2$



Answer Key – Section 5.1

1. Yes
2. Yes
3. No
4. Yes
5. Yes
6. No
7. $-2\ or\ 8$
8. $-2\ or\ 6$
9. $-7\ or\ 5$
10. $13\ or\ -17$
11. $(-2,0)\ and\ (1,3)$
12. $(1,0)$
13. No Solution, Do Not Intersect
14. $(-1,-2)$
15. $(1,1)$
16. $(-1,2)\ and\ (1,2)$
17. $(-1,3)\ and\ (1,3)$
18. $(1,0)$
19. $(-1,-3)\ and\ (0.5,-0.75)$

Extra Work Space