Section 5.1 – Review of Graphing and Linear Equations

- The graph of a a linear equation is **a straight line**
- We know two different equations of a straight line
  - **Standard Form:** $Ax + By = C$ *where* $A > 0$; *and a whole number*
  - **Slope-Intercept Form:** $y = mx + b$ *where* $m = SLOPE$ *and* $b = y - int$

- We can graph these equations by **plotting at least three points** of the grid
- Then we draw a line through the points, 3 assures a straight line

Graphing Standard Form – The EASY way #1

Step 1: To find $y - intercept$, set $x = 0$
To find $x - intercept$, set $y = 0$

Step 2: Then pick any value for x (switching the sign of the result for the $y - int$ works well)
Then solve for y

Step 3: Plot the 3 points from the first two steps, and draw a straight line through them

Note: When $x = 0$ we have the $y - intercept$, and when $y = 0$ we have the $x - intercept$

Example 1: *Graph* $3x + 2y = 6$

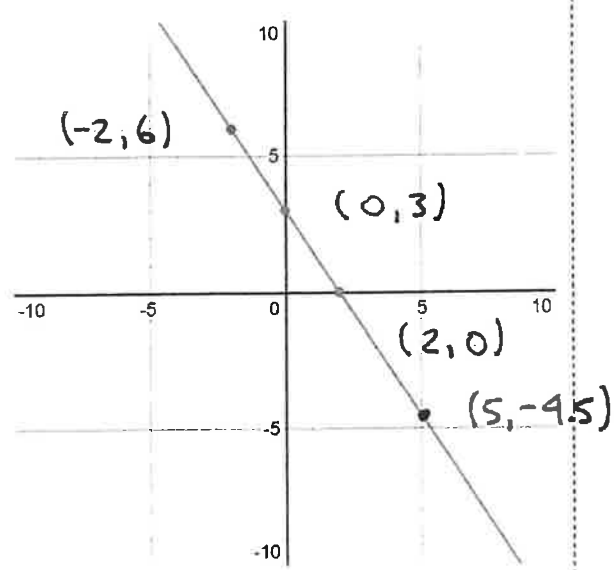
$3(0) + 2y = 6$
$2y = 6$
$y = 3$

$3x + 2(0) = 6$
$3x = 6$
$x = 2$

$3(-2) + 2y = 6$
$-6 + 2y = 6$
$2y = 12$
$y = 6$

$3(5) + 2y = 6$
$15 + 2y = 6$
$2y = -9$
$y = -\frac{9}{2}$
$= -4.5$

x	y
0	3
2	0
-2	6
5	-4.5



Graphing Standard Form – The EASY way #2 – Change it to Slope-Intercept Form

Step 1: Use algebra to transform the equation into $y = mx + b$ form

Step 2: Plot the $y - intercept$

Step 3: Map out the Slope.
Travel **up and to the right** (or down and to the left) if the **slope is positive**
Travel **down and to the right** (or up and to the left) if the **slope is negative**
Continue this trend, then connect the points

Example 2: $Graph\ 2x + 3y = 12$

$2x + 3y = 12$

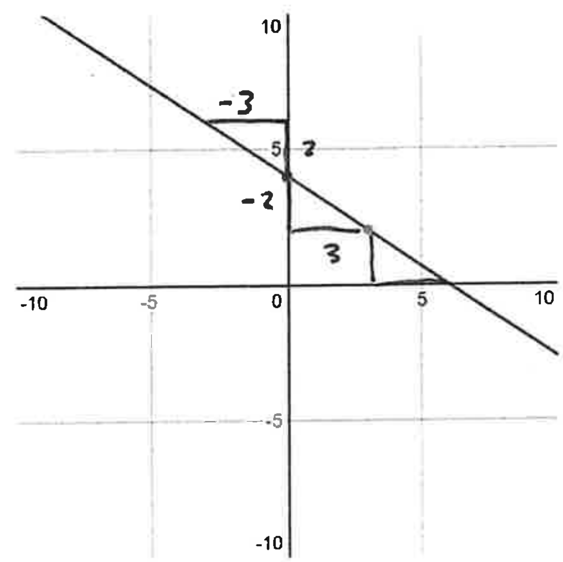
$3y = -2x + 12$

$\frac{3y}{3} = -\frac{2x}{3} + \frac{12}{3}$

$y = -\frac{2}{3}x + 4$

y-int, $b = 4$

slope, $m = \frac{-2}{3} = \frac{2}{-3}$



Summary

To Graph a Linear Equation

- **Step 1:** Find at least three ordered pairs that are solutions to the linear equations. The third point provides a check, if the line doesn't connect all three, there has been an error
- **Step 2:** Plot the corresponding points and connect them with a straight line

Or

- Convert to Slope-Intercept Form and map out the Slope from the y-intercept

Practice Problems: 1,3,5,6,7-10,12,13,16,17

Solving Linear Systems Algebraically

- **Two or more equations** viewed together are called a **system of equations**
- The **ordered pair or pairs** that are common to all the linear equations is the **solution**

Example 1: Is $(2, -2)$ a solution to $2x - y = 6$ and $-x + 3y = -8$?

$2x - y = 6$

$2(2) - (-2) \stackrel{?}{=} 6$

$4 + 2 \stackrel{?}{=} 6$

$6 = 6$

$-x + 3y = -8$

$-(2) + 3(-2) \stackrel{?}{=} -8$

$-2 - 6 \stackrel{?}{=} -8$

$-8 = -8$

Since $(2, -2)$ is a **solution to both equations, it is a solution** to the system

Example 2: Is $(-1, 3)$ a solution to $2x + y = 1$ and $x - 3y = -9$?

$2x + y = 1$

$2(-1) + (3) \stackrel{?}{=} 1$

$-2 + 3 \stackrel{?}{=} 1$

$1 = 1$ True

$x - 3y = -9$

$(-1) - 3(3) \stackrel{?}{=} -9$

$-1 - 9 \stackrel{?}{=} -9$

$-10 \neq -9$ False

Since $(-1, 3)$ is a **solution to only one equation,** it is **not a solution** to the system

- There are **two very effective methods** we can use **to solve a system** of equations
- They contain two variables, so we **eliminate one, through addition**
- **Or substitute** in so we are **only ever dealing with one** at a time

Elimination Methods

Addition Method

- **Step 1:** Write the equation of the system in **Standard Form** $Ax + By = C$
- **Step 2:** Multiply the terms of one or both of the equations by a constant so that the **coefficients of** x *or* y are **different only in there sign**
- **Step 3: Add the equations** and solve the remaining equation
- **Step 4: Substitute the value** you solved for in Step 3 **into either of the original equations**, and solve for the other variable
- **Step 5:** Take the solution of each variable from Step 3 and 4, substitute them into the **equation not used** in step 4 to check the solution

Example 1: Solve $2x - 3y = 2$ and $x + 2y = 8$

Solution 1: To eliminate x, multiply the second equation by -2. Add the results.

$2x - 3y = 2$ ...1

$(x + 2y = 8)(-2)$ ...2

$$\begin{array}{rrcr} & 2x - 3y & = & 2 \\ + & -2x - 4y & = & -16 \\ \hline & -7y & = & -14 \\ & y & = & 2 \end{array}$$

Solution $(4, 2)$

Now sub $y = 2$ into either original equs and solve for x

$x + 2y = 8$

$x + 2(2) = 8$

$x + 4 = 8$

$x = 4$

Example 2: Solve $4x + 3y = 5$ and $3x - 2y = 8$

Solution 2: To eliminate y, multiply equation one by 3, and equation two by 3, then add the results.

$(4x + 3y = 5)\,2 \quad \ldots 1$

$(3x - 2y = 8)\,3 \quad \ldots 2$

$$\begin{array}{r} 8x + 6y = 10 \\ + \quad 9x - 6y = 24 \\ \hline 17x = 34 \end{array}$$

$x = 2$

Sub into either eqn

$4x + 3y = 5$

$4(2) + 3y = 5$

$8 + 3y = 5$

$3y = -3$

$y = -1$

Soln $(2, -1)$

Example 3: Solve $3x - 2y = 1$ and $-6x + 4y = 3$

Solution 3: To eliminate x, multiply the equation one by 2. Add the results.

$(3x - 2y = 1)\,2 \quad \ldots 1$

$-6x + 4y = 3 \quad \ldots 2$

$$\begin{array}{r} 6x - 4y = 2 \\ + \quad -6x + 4y = 3 \\ \hline 0 = 5 \end{array}$$

Not true

Therefore, there is no solution

The graph will be parallel lines

Example 4: Solve $2x + 5y = 2$ and $-4x - 10y = -4$

Solution 4:

$(2x + 5y = 2)\,2 \quad \ldots 1$

$-4x - 10y = -4 \quad \ldots 2$

$$\begin{array}{r} 4x + 10y = 4 \\ -4x - 10y = -4 \\ \hline 0 = 0 \end{array}$$

True

Therefore there are infinite solutions

The graph will be coincident lines

Therefore, there are **infinite solutions**, both equations are the **same line**

Practice Problems #18-25

Substitution Method

- **Step 1:** Solve one equation for one of its variables in terms of the other one
- **Step 2:** Substitute the equation from Step 1 into the other equation and solve it
- **Step 3:** Take the value solved for in Step 2 and substitute the value into any equation containing both variables
- **Step 4:** Check your solution by taking the values you got from Step 2 and Step 3, and put them into the equation you **did not use** in Step 3

Example 1: Solve for $2x + 3y = 1$ and $3x - y = 7$

Solution 1: Look at both equations. Pick the one that is easiest to change into $\boldsymbol{x =}$ ***or*** $\boldsymbol{y =}$

Start by **solving for y** in equation two. **Substitute this expression for y** into equation one and **solve for x**

$2x + 3y = 1 \quad \ldots 1$

$3x - y = 7 \quad \ldots 2$

$3x - y = 7 \quad \ldots 2$

$3x - 7 = y$

sub into (1)

$2x + 3y = 1 \quad \ldots 1$

$2x + 3(3x - 7) = 1$

$2x + 9x - 21 = 1$

$11x = 22$

$x = 2$

sub $x = 2$ into (2).

$y = 3x - 7 \quad \ldots 2$

$y = 3(2) - 7$

$y = 6 - 7$

$y = -1$

Soln $(2, -1)$

Example 2: Solve for $4x - y = 2$ and $x - 3y = -5$

Solution 2: Solve for x in equation two, and substitute the x value into equation one

$4x - y = 2 \quad \ldots 1$

$x - 3y = -5 \quad \ldots 2$

solve (2) for x

$x - 3y = -5$

$x = 3y - 5$

sub into (1)

$4x - y = 2$

$4(3y-5) - y = 2$

$12y - 20 - y = 2$

$11y = 22$

$y = 2$

sub $y = 2$ into (2)

$x = 3y - 5$

$x = 3(2) - 5$

$x = 6 - 5$

$x = 1$

soln $(1, 2)$

Practice Problems #26-33

Problem Solving

- Don't be scared about problem solving, use these tips to help

Steps to follow Problem Solving

Step 1: Read the problem carefully, what are you trying to solve?

Step 2: Let *two variables* be your unknowns (x, y) (r, s) (l, w) it's your choice

Step 3: If you can, draw a diagram, make a table, organize the data

Step 4: Express all equations in terms of your two variables

Step 5: Use an elimination method to solve for one of the unknowns

Step 6: Check your answer to make sure your solution makes sense

Example 1: Adult tickets for the school play are $12.00 and children's tickets are $8.00. If a theatre holds 300 seats and the sold out performance brings in $3280.00, how many children and adults attended the play?

Solution 1: Let $x = adult\ tickets$ and $y = child\ tickets$. Then one equation has to deal with the **number** of tickets and the other equation must deal with **revenue** of the tickets

	Adult Tickets	Children Tickets	Total
# of Tickets	x	y	300
Revenue $	$12x$	$8y$	3280

So $x + y = 300 \ldots 1$

$12x + 8y = 3280 \ldots 2$

choose eqn (1) and solve for y

$x + y = 300 \ \ldots 1$

$y = -x + 300$

sub into (2)

$12x + 8y = 3280$

$12x + 8(-x + 300) = 3280$

$12x - 8x + 2400 = 3280$

$4x + 2400 = 3280$

$4x = 880$

$x = 220$

Now, back to (1)

$y = -x + 300$

$y = -(220) + 300$

$y = 80$

Soln $(220, 80)$

Therefore, 220 adult tickets and 80 children's tickets were sold.

Example 2: A small airplane makes a 2400 km trip in 7 and a half hours, and makes the return trip in 6 hours. If the plane travels at a constant speed, and the wind blows at a constant rate, find the airplane's airspeed and the speed of the wind.

Solution 2: Let $x = speed\ of\ the\ airplane$ and $y = speed\ of\ the\ wind$.

	Speed (km/h)	Time (hr)	Distance (km)
With Wind	$x + y$	6	2400
Against Wind	$x - y$	7.5	2400

So
$x + y$ *is speed of plane with the wind*
$x - y$ *is speed of plane going against the wind*

$distance = speed \cdot time$

speed · time = distance

$(x+y)\,6 = 2400 \quad \ldots 1$

$(x-y)\,7.5 = 2400 \quad \ldots 2$

$$\begin{array}{r} x+y = 400 \\ + \quad x-y = 320 \\ \hline 2x = 720 \\ x = 360 \end{array}$$

Now, $x + y = 400$

$360 + y = 400$

$y = 40$

Soln $(360, 40)$

Therefore, speed of airplane is 360km/h, and wind speed is 40km/h

Example 3: A chemist has two acid solutions in stock: one that is a 50% solution and the other an 80% solution. How much of each solution should be mixed to obtain 100 millilitres of a 68% solution?

Solution 3: Let $x = \#\ of\ millilitres\ of\ 50\%\ solution$

and $y = \#\ of\ millilitres\ of\ 80\%\ solution$

	50% Solution	80% Solution	Total
Acid (mL)	x	y	100
Strength	$0.5x$	$0.8y$	$0.68(100)$

$0.5(x + y = 100) \quad \ldots 1$

$0.5x + 0.8y = 68 \quad \ldots 2$

$$\begin{array}{r} 0.5x + 0.5y = 50 \\ - \quad 0.5x + 0.8y = 68 \\ \hline -0.3y = -18 \\ y = 60 \end{array}$$

$x + y = 100 \quad \ldots 1$

$x + 60 = 100$

$x = 40$

Soln $(40, 60)$

Therefore, we need 40 millilitres of the 50% solution, and 60 millilitres of the 80% solution

Practice Problems #34-39

Section 5.1 – Practice Problems

Determine whether the given ordered pair is a solution to the following equation:

1. $(2,3); 3x - 5y = -9$

2. $(0,4); y = -\frac{1}{3}x + 4$

3. $(1,-1);\ 3y = 5 - 2x$

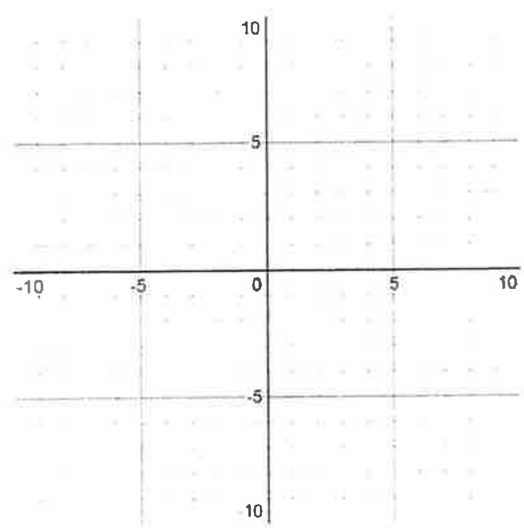
4. $(6,8);\ \frac{1}{3}x - \frac{1}{4}y = 4$

5. $(4,2); x = 4$

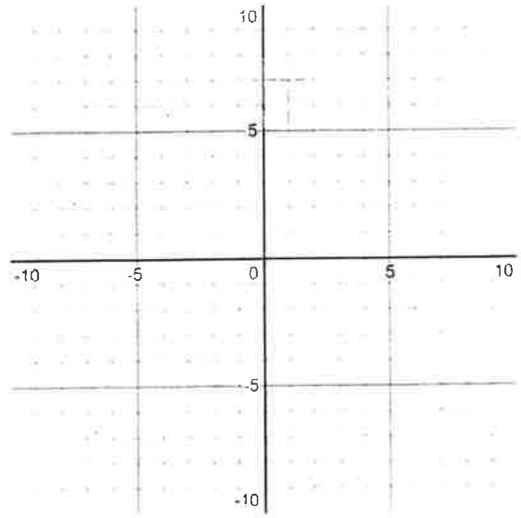
6. $(-1,3); y = -1$

Graph the following equations

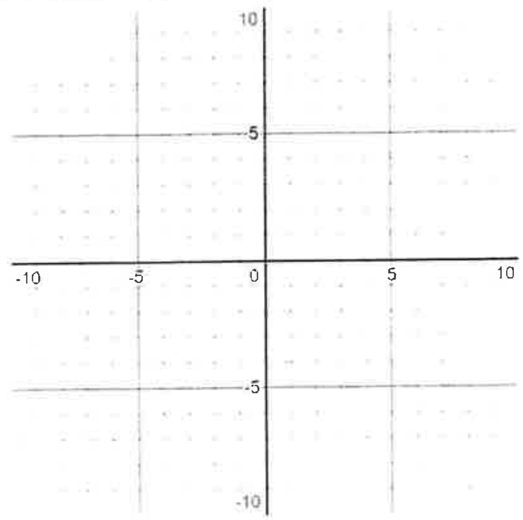
7. $2x + 3y = 6$



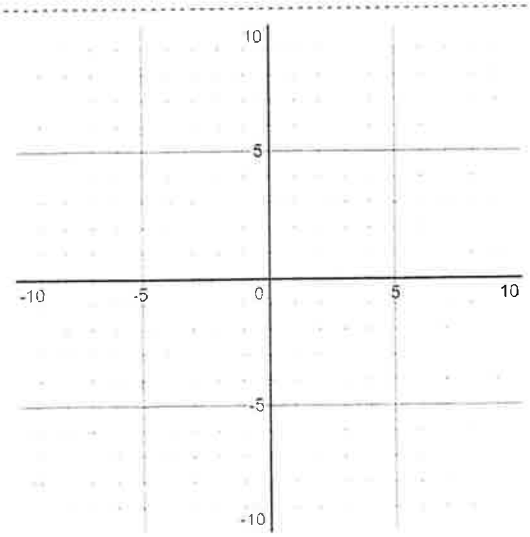
8. $2x + y = -4$



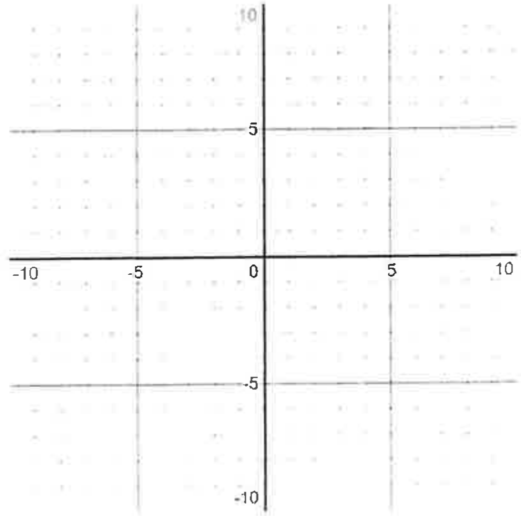
9. $2x - \frac{1}{2}y = 2$



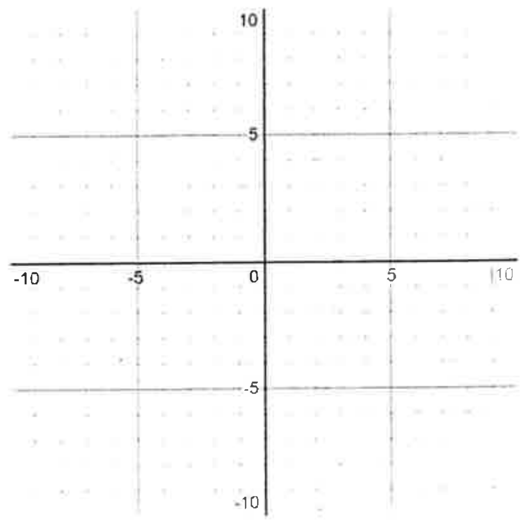
10. $3x + 2y = 5$



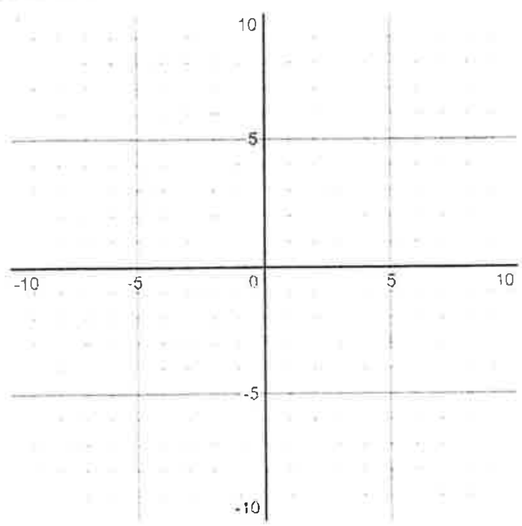
11. $\frac{2}{3}x - 0.4y = 2$



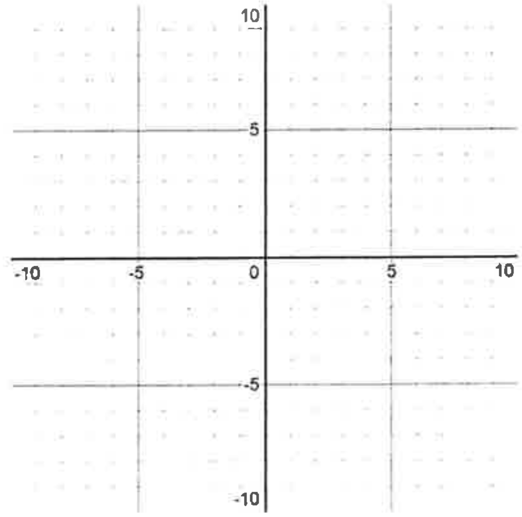
12. $y = -2x - 1$



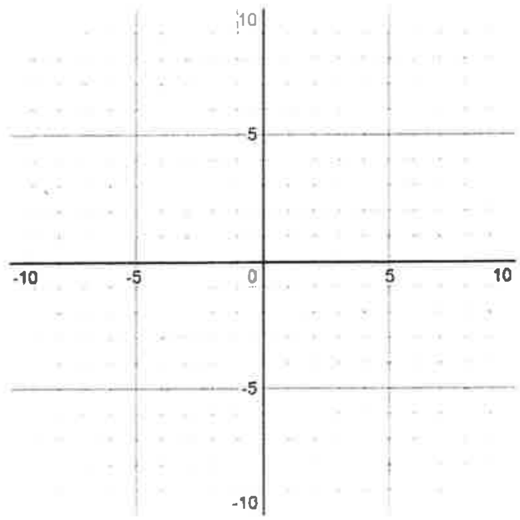
13. $y = -\frac{3}{4}x + 1$



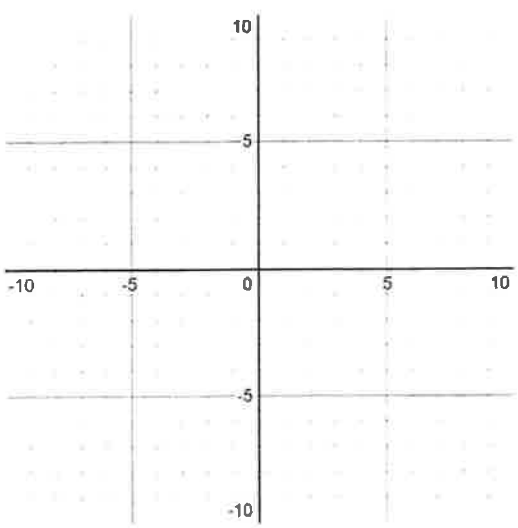
14. $x = 2(y - 1) + 1$



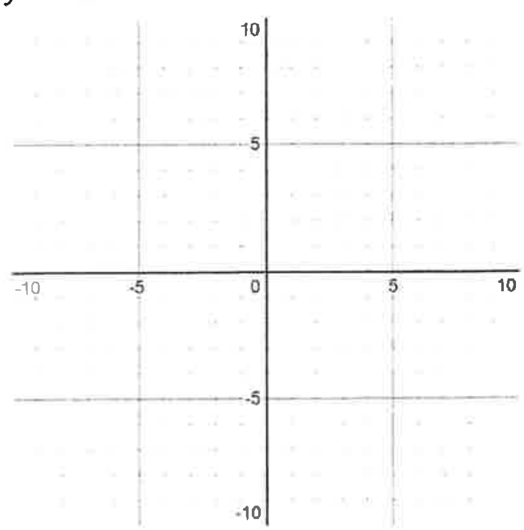
15. $2(x - y) + 6 = 0$



16. $x = 3$



17. $y = -2$



Solve the Linear Systems Algebraically Using the Addition Method

18. $3x + 5y = 17$
    $4x - y = -8$

19. $4x + 3y = 1$
    $3x + 2y = 2$

20. $7x - 3y = -5$
    $3x + 5y = -21$

21. $5x + 2y = 8$
    $3x + 5y = 20$

22. $5x - 3y = \frac{21}{2}$
    $2x + 5y = -2$

23. $3x - 2y = 6$
    $-6x + 4y = -6$

24. $3x - 2y = 6$
$-6x + 4y = -12$

25. $\frac{x}{3} + \frac{y}{4} = 1$

$\frac{x}{2} - \frac{y}{8} = \frac{7}{2}$

Solve Using The Substitution Method

26. $y = 3x + 4$
$2x - 3y = 2$

27. $y = -2x$
$x + 4y = 21$

28. $6x - y = 0$
$8x - 3y = 25$

29. $2s + t = -3$
$3s + 2t = -4$

30. $y = \frac{1}{3}x + 2$
$2x - 6y = -12$

31. $2x = 3y + 4$
$6x = 9y + 8$

32. $-3a + 2b = 4$
$5a - 3b = 1$

33. $\frac{x}{3} - \frac{y}{4} = \frac{1}{12}$

$\frac{x}{6} - \frac{3y}{2} = -\frac{7}{8}$

Problem Solving with 2 and 3 variables

34. Jean has \$50 000 to invest. He invests some in the stock market which earns 8%, and some in bonds that earns 6% on his investments. If the total interest earned was \$3500, how much did Jean invest in stock and how much in bonds?

35. Stephanie has 80 coins, consisting of dimes and quarters. If the total value of her coins is \$15.20, how many dimes does she have?

36. A barrel of wine has 8% alcohol, another barrel has 15% alcohol. How much of each must be mixed to have 100 liters of 12.2% alcohol wine?

37. A plane travels $2835km$ in $7\ hours$ with a tailwind, but only $1827km$ with a headwind in the same time. Find the speed of the plane, and the speed of the wind.

38. $50-50$ Tickets for the Royals game are $\$2.00$ each or $3\ for\ \$5.00$. The girls Volleyball team sold 300 tickets and the total amount of money taken in was $\$528.00$. How many people bought only a single ticket?

39. A car travels $480km$ in the same time that a truck travels $400km$. The speed of the car is $16km/h$ faster than the truck. Determine the speed of the car and truck respectively.