Section 5.1 – Practice Problems

1. Simplify the following statements

a) $\dfrac{\left(3^{\frac{1}{5}}\right)^{10} \cdot (3^{-3})}{9} \rightarrow \dfrac{3^{\frac{10}{5}} \cdot 3^{-3}}{3^2}$

$\dfrac{3^2 \cdot 3^{-3}}{3^2} \rightarrow \dfrac{3^{-1}}{3^2} \rightarrow 3^{-3} = \dfrac{1}{3^3} \rightarrow \boxed{\dfrac{1}{27}}$

b) $\dfrac{(-4x^2y^{-2})^{-3}}{x^{-1}y^2} \rightarrow \dfrac{-4^{-3}x^{-6}y^{6}}{x^{-1}y^{2}}$

$-4^{-3}x^{-5}y^{4} \rightarrow -\dfrac{y^4}{4^3x^5} \rightarrow \boxed{-\dfrac{y^4}{64x^5}}$

c) $\dfrac{125^{3x-1} \cdot 25^{1-2x}}{\left(\frac{1}{5}\right)^{2x-3}} \rightarrow \dfrac{(5^3)^{3x-1} \cdot 5^{2(1-2x)}}{(5^{-1})^{2x-3}}$

$\dfrac{5^{9x-3} \cdot 5^{2-4x}}{5^{-2x+3}} \rightarrow \dfrac{5^{5x-1}}{5^{-2x+3}}$

$\boxed{5^{7x-4}}$

d) $\dfrac{2x^4 \cdot 3^{5x} - 4x^3 \cdot 3^{5x}}{x^3 - 2x^2} \rightarrow \dfrac{3^{5x}(2x^4 - 4x^3)}{x^3 - 2x^2}$

$\dfrac{3^{5x} \cdot 2x^2(x^2 - 2x)}{x(x^2-2x)} \rightarrow \dfrac{3^{5x} \cdot 2x^2(x^2-2x)}{x(x^2-2x)}$

$\boxed{3^{5x} \cdot 2x}$

e) $(4^{-x} \cdot 8^x)^2$

$\left((2^2)^{-x}(2^3)^x\right)^2$

$(2^{-2x} \cdot 2^{3x})^2 \rightarrow (2^x)^2$

$2^{2x} \rightarrow (2^2)^x = \boxed{4^x}$

f) $\dfrac{2^x(2^x + 2^{-x}) - 2^x(2^x - 2^{-x})}{2^{-2}}$

$\dfrac{(2^{x+x} + 2^{x-x}) - (2^{x+x} - 2^{x-x})}{2^{-2}}$

$\dfrac{(2^{2x} + 2^0) - (2^{2x} - 2^0)}{2^{-2}} \rightarrow \dfrac{(2^{2x}+1) - (2^{2x}-1)}{2^{-2}}$

$\dfrac{2^{2x} + 1 - 2^{2x} + 1}{2^{-2}} \rightarrow \dfrac{2}{2^{-2}} \rightarrow 2^3 = \boxed{8}$

2. Solve for x

a) $4^{x^2-x} = 1$

$1 = 4^0$

$4^{x^2-x} = 4^0$

base equal means exponents equal

$x^2 - x = 0$

$x(x-1) = 0$

$\boxed{x = 0 \quad x = 1}$

b) $3^{x^2} = 9 \cdot 3^{-x}$

$3^{x^2} = 3^2 \cdot 3^{-x} \rightarrow 3^{x^2} = 3^{2-x}$

$x^2 = 2 - x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\boxed{x = -2 \quad x = 1}$

c) $4^{\sqrt{x+1}} = 2^{3x-2}$

* check solutions for extraneous roots

$2^{2(\sqrt{x+1})} = 2^{3x-2}$

Divide by 2

$2\sqrt{x+1} = 3x - 2 \rightarrow \sqrt{x+1} = \frac{3}{2}x - 1$

$x+1 = \left(\frac{3}{2}x - 1\right)^2 \rightarrow x+1 = \left(\frac{3}{2}x - 1\right)\left(\frac{3}{2}x - 1\right)$

$x+1 = \frac{9}{4}x^2 - \frac{6}{2}x + 1 \rightarrow x+1 = \frac{9}{4}x^2 - 3x + 1$

$0 = \frac{9}{4}x^2 - 4x \rightarrow 0 = 9x^2 - 16x \rightarrow 0 = x(9x - 16)$

$x = 0$ ← reject

* $\boxed{x = 16/9}$

d) $4^{-|x+1|} = \frac{1}{16} \rightarrow 4^{-|x+1|} = 16^{-1}$

$4^{-|x+1|} = 4^{-2} \rightarrow -|x+1| = -2$

so $x + 1 = 2$ or $x + 1 = -2$

$\boxed{x = 1}$ $\boxed{x = -3}$

e) $4^{-2x+1} = 8^{x-4}$

$2^{2(-2x+1)} = 2^{3(x-4)}$

$2^{-4x+2} = 2^{3x-12}$

$-4x + 2 = 3x - 12$

$-7x = -14$

$\boxed{x = 2}$

f) $9^{2x-1} = \left(\frac{1}{27}\right)^{x+2}$

$3^{2(2x-1)} = 27^{-1(x+2)} \rightarrow 3^{4x-2} = 3^{-3(x+2)}$

$3^{4x-2} = 3^{-3x-6}$

$4x - 2 = -3x - 6$

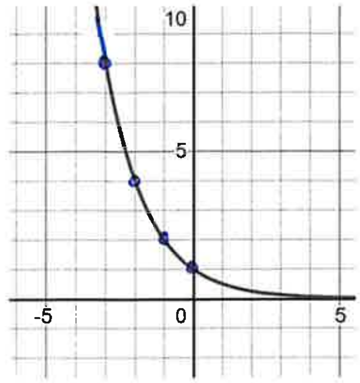
$7x = -4$

$\boxed{x = -4/7}$

3. If $y = ab^x$ is defined by the graph below, what is the shape of the following:

↓

$f(x) = ab^x$



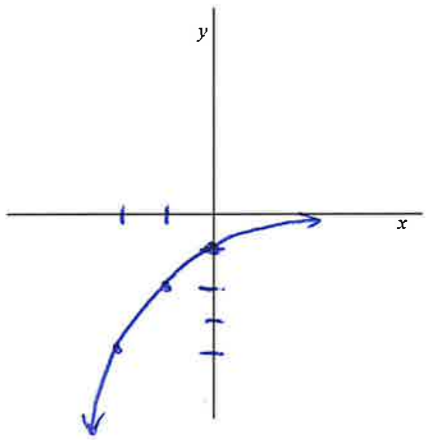
Horizontal Asymptote (HA)
$y=0$ (the x-axis)

a) $y = -ab^x$

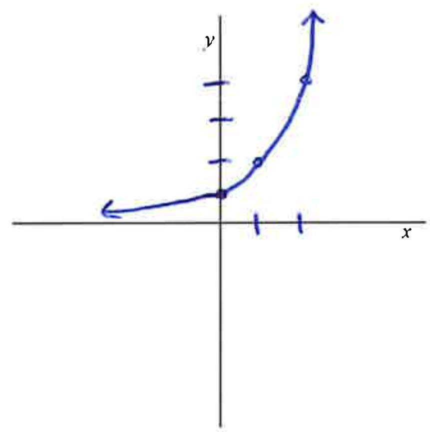
↑

$-f(x)$

reflected y-value



b) $y = ab^{-x}$ ✓ $f(-x)$ reflected x-values

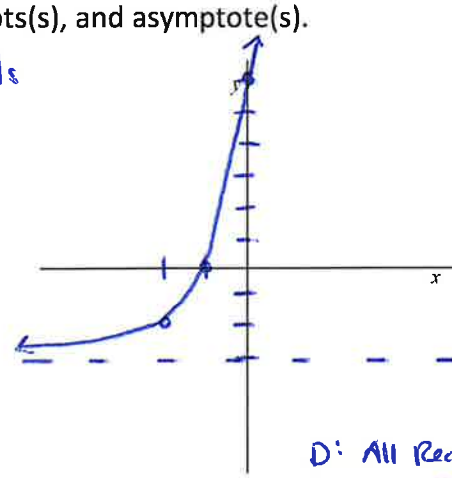


4. Explain the transformation listed in the equations below of the basic equation $y = 3^x$. Graph the transformation, identify the Domain, Range, intercepts(s), and asymptote(s).

a) $y = 3^{x+2} - 3$ ⟵ vertical down 3 units (asymptote too)

↑

Horizontal left 2 units



original $f(x) = 3^x$

$(0,1) \rightarrow (-2,-2)$

$(1,3) \rightarrow (-1,0)$ ⟵ x-int

$(2,9) \rightarrow (0,6)$ ⟵ y-int

D: All Real #'s

R: $y > -3$

y-int: $(0,6)$

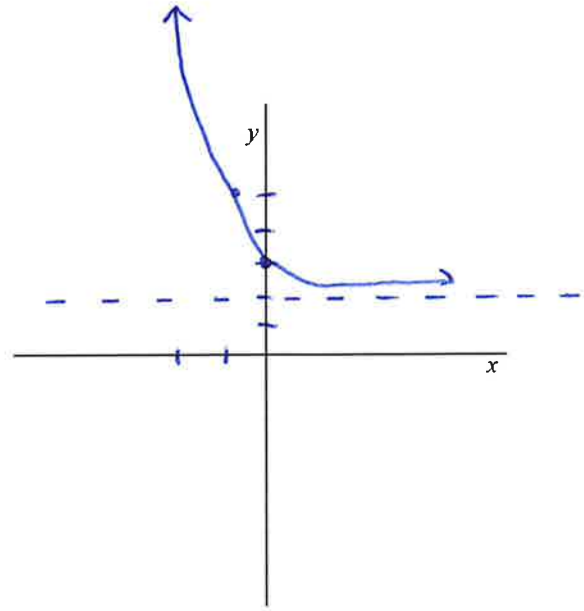
x-int: $(-1,0)$

HA: $y = -3$

b) $y = \left(\frac{1}{3}\right)^x + 2$

Reflection in y-axis

$f(x) = 3^{-x} + 2$ ← vertical shift up 2



original

$f(x) = 3^x$

$(0,1) \longrightarrow (0,3)$

$(1,3) \longrightarrow (-1,5)$

$(2,9) \longrightarrow (-2,11)$

D: All Real #'s

R: $y > 2$

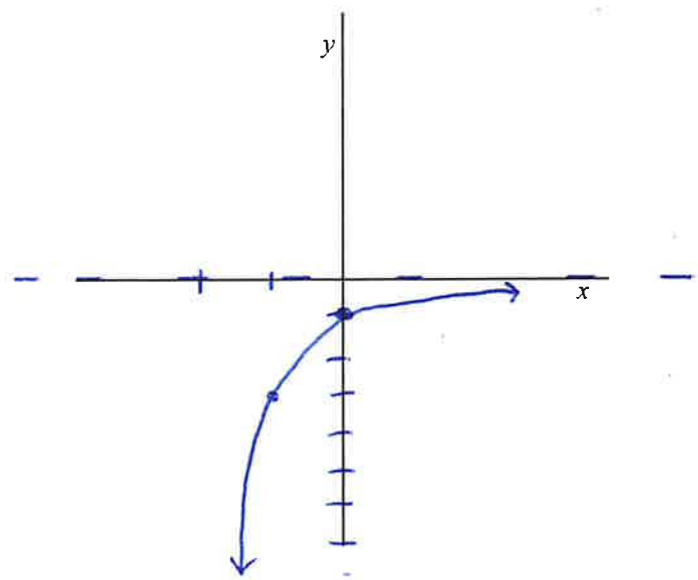
HA: $y = 2$

y-int: $(0,3)$

x-int: NONE

c) $y = -3^{-x}$ ← reflect in y-axis

reflect in x-axis



original $f(x) = 3^x$

$(0,1) \rightarrow (0,-1)$

$(1,3) \rightarrow (-1,-3)$

$(2,9) \rightarrow (-2,-9)$

HA: $y = 0$

D: All Real Numbers

R: $y < 0$

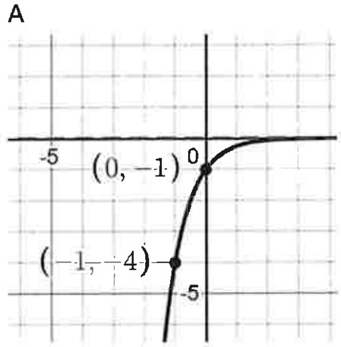
x-int: NONE

y-int: $(0,-1)$

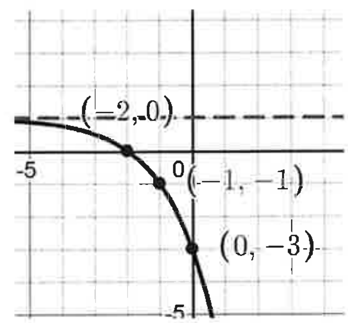
5. Match the Equation to the Graph

a) $y = 4^{-x}$	D
b) $y = 3^{-x+1}$	C
c) $y = 3^{-x} + 1$	F
d) $y = -4^{-x}$	A
e) $y = 2^{x+1} - 2$	E
f) $y = -2^{x+2} + 1$	B

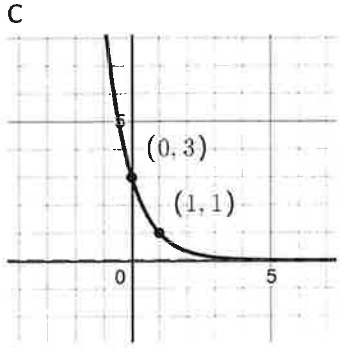
A



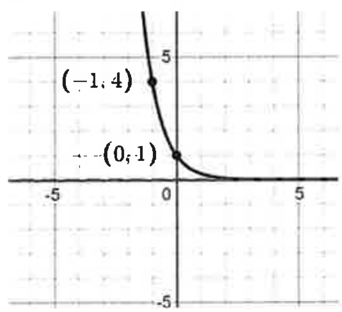
B



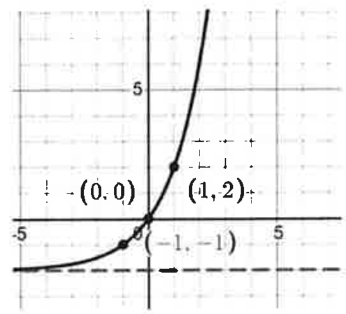
C



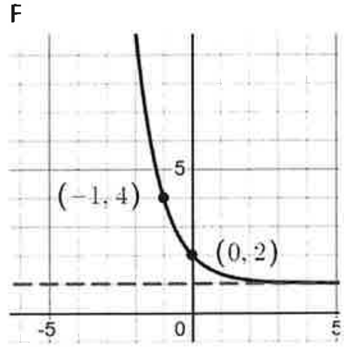
D



E



F



6. Find the base in the exponential function $y = b^x$ that contains the given point.

a) $(-1, 3)$

$y = b^x$

$3 = b^{-1} \rightarrow 3 = \frac{1}{b}$

$\boxed{b = \frac{1}{3}}$

b) $(\frac{3}{2}, 27)$

$y = b^x$

$27 = b^{3/2} \rightarrow 27^{2/3} = (b^{3/2})^{2/3}$

$\rightarrow (\sqrt[3]{27})^2 = b$

$b = 3^2$

$\boxed{b = 9}$

c) $(-\frac{2}{3}, \frac{1}{9})$

$y = b^x$

$\frac{1}{9} = b^{-2/3}$

$\frac{1}{9} = \frac{1}{b^{2/3}}$

$9 = b^{2/3}$

$9^{3/2} = (b^{2/3})^{3/2}$

$(\sqrt{9})^3 = b$

$3^3 = b$

$\boxed{b = 27}$

7. Find the exponential function in the form $c \cdot 2^{kx}$ that passes through $(0, 4)$ and $(12, 256)$.

$y = c \cdot 2^{kx}$ when $(0,4)$

$4 = c2^{k(0)} \rightarrow 4 = c2^0$

$4 = c$

when $(12, 256)$

$256 = 4 \cdot 2^{k(12)}$

$\frac{256}{4} = \frac{4 \cdot 2^{12k}}{4}$

$64 = 2^{12k}$

$2^6 = 2^{12k}$

$12k = 6$

$k = \frac{1}{2}$

$y = 4 \cdot 2^{\frac{1}{2}x}$

$\downarrow$

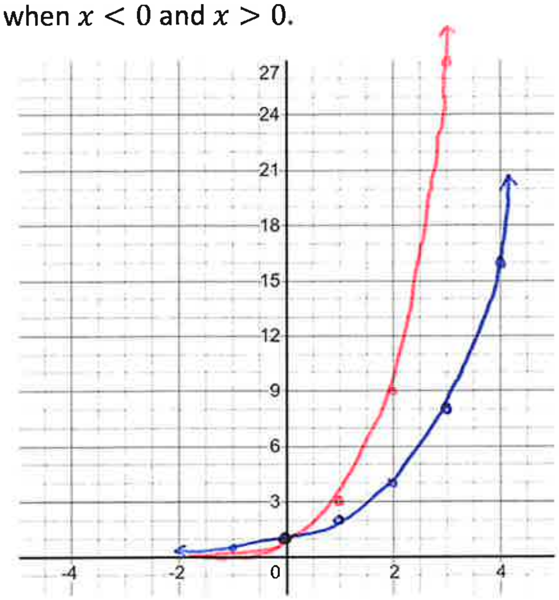
$y = 2^2 \cdot 2^{\frac{1}{2}x}$

$\boxed{y = 2^{\frac{1}{2}x + 2}}$

8. Sketch the graph of $y = 2^x$ *and* $y = 3^x$ on the same grid.

x	-3	-2	-1	0	1	2	3	
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	Blue
3^x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	Red

Discuss the behaviour of the two functions when $x < 0$ and $x > 0$.



2^x increases slower than 3^x

as $x \rightarrow \infty$ $3^x > 2^x$

2^x approaches 0 (on the y-axis) slower than 3^x

as $x \rightarrow -\infty$ $2^x > 3^x$

9. Solve the given scenarios.

a) In 1876, an earthquake in Chile measured 8.9 on the Richter scale. How many times more powerful was this earthquake compared to the one in Utah that measured 6.4 on the Richter scale. (The Richter scale is a power of ten scale)

Radio of Chile to Utah

$$\frac{10^{8.9}}{10^{6.4}} = 10^{2.5}$$

$$10^{2.5} = 316$$

Chile was 316 times stronger than Utah.

b) If an earthquake in Alberta had an amplitude 1000 times larger than an earthquake that measured 4.9 on the Richter scale. What would the Alberta earthquake measure?

$$1000 \cdot 10^{4.9} = 10^x$$

$$10^3 \cdot 10^{4.9} = 10^x$$

$$10^{7.9} = 10^x$$

$$x = 7.9$$

Alberta Quake Measured 7.9

c) If you invest \$1000 at an annual interest rate of 6% compounded quarterly, what is the amount you would have in the account after 8 *years*? (No withdrawals made)

$$A = P(1 + \tfrac{r}{n})^{n \cdot t}$$

$P = 1000$

$r = 0.06$

$n = 4$

$t = 8$

$$A = 1000(1 + \tfrac{0.06}{4})^{4 \cdot 8}$$

$$\boxed{A = \$1610.32}$$

d) In the wasteland of the Commonwealth in Fallout 4, the safest locations to live are surrounded by radioactive isotopes with a half-life of 4 *minutes*. This means half the amount of the isotope decays every four minutes. If there was 84 grams of the isotope initially, how much is remaining after 23 minutes?

$$A = A_0 x^{\frac{t}{T}}$$

$A_0 = 84$

$x = \frac{1}{2}$

$T = 4 \text{ mins}$

$t = 23 \text{ mins}$

$$A = 84(\tfrac{1}{2})^{23/4}$$

$$\boxed{A = 1.56\text{ g}}$$

e) My brother managed to get me an investment account that pays 9.6% interest, compounded monthly. If I invested \$12 250, how much money would I have after 10 years.

$$A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$

$P = 12\,250$

$r = 0.096$

$n = 12$

$t = 10$

$$A = 12250\left(1+\frac{0.096}{12}\right)^{10 \cdot 12}$$

$$\boxed{A = \$31\,871.31}$$

f) If the population of Sweden is around 30 000 000 people in the year 2040, and the population continues to grow at 1.9% compounded yearly, what will the population be in the year 2072?

$$A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$

$P = 30\,000\,000$

$r = 0.019$

$n = 1$

$t = 32$

in millions

$$A = 3\left(1+\frac{0.019}{1}\right)^{32}$$

$$A = 5.478922318$$

$\times 1000000$

$$\boxed{A = 54\,789\,223.18 \text{ people}}$$

See Website for Detailed Answer Key

Extra Work Space