## Section 4: Trigonometry

This booklet belongs to: $\qquad$ Block:

| Section | Due Date | How Did It Go? | Corrections Made <br> and Understood |
| :---: | :---: | :---: | :---: |
| 4.1 |  |  |  |
| 4.2 |  |  |  |
| 4.3 |  |  |  |

## Assessment Rubric

| Category | L-T Score | Learning Target Procedure | Algebraic/Arithmetic Procedure | Communication | Anecdotal Example |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Extending | 4 | Procedural context demonstrates a detailed understanding of the learning targets | Algebraic/Arithmetic process is error free, logic is clear and easy to follow | Written output is clear, easy to follow, and shows depth of understanding | "You could teach this" or "It's an answer key" |
|  | 3.5 | Procedural context demonstrates a thorough understanding of the learning targets | Algebraic/Arithmetic process contains very minor errors, logic is clear and easy to follow | Written output is clear, easy to follow, and shows depth of understanding | "Almost perfect, one or two little errors" |
| Proficient | 3 | Procedural context is clear, demonstrates sound reasoning and thought of the learning targets | Algebraic/Arithmetic process contains minor errors, logic is clear and easy to follow | Written output is clear and organized, and shows depth of understanding | "Good understanding with a few errors" |
| Developing | 2.5 | Procedural context is clear, contains errors but demonstrates sound reasoning and thought of the learning targets | Algebraic/Arithmetic process contains errors, logic is clear and easy to follow | Written output is difficult to follow, but shows an understanding of the task | "You know what to do bet not clear how to do it" |
| Developing | 2 | Procedural context contains errors. Understanding of the learning targets is developing | Algebraic/Arithmetic process contains numerous errors, difficult to follow | Written output is difficult to follow but shows an understanding of the task | "You are on the right track but key concepts are missing" |
| Emerging | 1 | Procedural context is not clear, demonstrates minimal understanding of the learning targets | Algebraic/Arithmetic process contains numerous errors, difficult to follow | Written output is difficult to follow, but shows an understanding of the task | "You have achieved the bare minimum to meet the learning outcome" |
| Not Yet <br> Meeting <br> Outcomes | IE | Procedural context is not clear, demonstrates minimal understanding of the learning targets | Algebraic/Arithmetic process contains numerous errors, difficult to follow | Written output is difficult to follow or completely absent and lacks clarity | "Learning outcomes are not met at this time" |

## Learning Targets and Self-Evaluation

| $\mathbf{L - T}$ | Description | Mark |
| :---: | :--- | :---: |
| $\mathbf{4 - 1}$ | $\bullet$ <br> - Understanding Trigonometric Ratios and their inverses <br> - Using calculator effectively in making angle calculations |  |
| $\mathbf{4 - 2}$ | - Solving proportions |  |
| $\mathbf{4 - 3}$ | Right angle triangle properties and trigonometric relationships |  |
| - Solving angles and sides using trigonometric properties Tan/Sine/Cosine |  |  |
| - Solving word problems involving angles and direction |  |  | |  |
| :--- |

## Comments:

## Competency Evaluation

A valuable aspect to the learning process involves self-reflection and efficacy. Research has shown that authentic self-reflection helps improve performance and effort, and can have a direct impact on the growth mindset of the individual. In order to grow and be a life-long learner we need to develop the capacity to monitor, evaluate, and know what and where we need to focus on improvement. Read the following list of Core Competency Outcomes and reflect on your behaviour, attitude, effort, and actions throughout this unit.

- Rank yourself on the left of each column: 4 (Excellent), 3 (Good), 2 (Satisfactory), 1 (Needs Improvement)



## Section 4.1 - Right Angle Triangle Trigonometry

- Trigonometry is the study of angles and the ratios that relate to them
- In the following sections will be only focus on Right Angle Triangle Trigonometry
- With the Pythagorean Theorem and 3 Trigonometric Functions we can solve triangles
- They are:
Sine
Tangent
Cosine


## Using your Calculator

- In my opinion, Trigonometry is the only topic that requires a calculator in this course
- This is because calculating the value of angles, given a specific Trig Function is very hard
- Also converting from the given ratios value back to the angle is just as tough
- In fact, I don't even know how to do it by hand!

The very first thing you want to check:
Make sure your Calculator is in DEGREE MODE, you should see a little D or DEG on the top

## Second:

There are three buttons we will be using on our calculators they are:

| $\operatorname{Tan}^{-1}$ | $\operatorname{Sin}^{-1}$ |
| :--- | :--- |
| $\operatorname{Sin}^{-1}$ | $\operatorname{Cos}^{-1}$ <br> $\operatorname{Cos}$ |

- We use Tan, Sin, Cos when we are trying to get the decimal value of a given angle
- Or their INVERSE buttons:
- We use $\boldsymbol{T a n}^{-1}, \boldsymbol{\operatorname { S i n }}^{-\mathbf{1}}, \boldsymbol{C o s}^{-\mathbf{1}}$ when we are trying to get the angle value of a given decimal - You will need to hit the $2^{\text {nd }}$ function button to access the INVERSE buttons

Third:
Depending on the calculator you have you will either be pressing:

- The Trig button first then the angle/ratio
or
- The angle/ratio first and then the Trig button


## Solving for Values

$>$ When solving an angle and getting it's decimal expansion, round to 4 decimal places

| Example: | Solve the following: | $\operatorname{Tan} 60^{\circ}, \operatorname{Sin} 60^{\circ}, \operatorname{Cos} 60^{\circ}$ |  |
| :---: | :---: | :---: | :---: |
| Solution: | $\operatorname{Tan} 60^{\circ}=1.732$ | $\operatorname{Sin} 60^{\circ}=0.8660$ | $\operatorname{Cos} 60^{\circ}=0.5$ |
| Example: | Solve the following: | $\operatorname{Tan} 30^{\circ}, \operatorname{Sin} 30^{\circ}, \mathrm{Cos}$ |  |
| Solution: | $\operatorname{Tan} 30^{\circ}=0.5774$ | $\operatorname{Sin} 30^{\circ}=0.5$ | $\operatorname{Cos} 30^{\circ}=0.8660$ |
| Example: | Solve the following: | $\operatorname{Tan} 0^{\circ}, \operatorname{Sin} 0^{\circ}, \operatorname{Cos}$ |  |
| Solution: | $\operatorname{Tan} 0^{\circ}=0$ | $\operatorname{Sin} 0^{\circ}=0$ | $\operatorname{Cos} 0^{\circ}=1$ |
| Example: | Solve the following: | $\operatorname{Tan} 34^{\circ}, \operatorname{Sin} 57^{\circ}$, |  |
| Solution: | $\operatorname{Tan} 34^{\circ}=0.675$ | $\operatorname{Sin} 57^{\circ}=0.8387$ | $\operatorname{Cos} 102^{\circ}=-0.208$ |

When converting a decimal expansion to an angle, round to 1 decimal place
Example: Convert 0.8660 to angles of all three trigonometric functions
Solution: $\quad \operatorname{Tan}^{-1}(0.8660)=40.9^{\circ} \quad \operatorname{Sin}^{-1}(0.8660)=60.0^{\circ} \quad \operatorname{Cos}^{-1}(0.8660)=30.0^{\circ}$

Example: Convert 1.0 to angles of all three trigonometric functions
Solution: $\quad \operatorname{Tan}^{-1}(1.0)=45.0^{\circ} \quad \operatorname{Sin}^{-1}(1.0)=90.0^{\circ} \quad \operatorname{Cos}^{-1}(1.0)=0.0^{\circ}$

Example: Convert 0.7002 to angles of all three trigonometric functions
Solution: $\quad \operatorname{Tan}^{-1}(0.7002)=35.0^{\circ} \quad \operatorname{Sin}^{-1}(0.7002) \approx 44.4^{\circ} \quad \operatorname{Cos}^{-1}(0.7002) \approx 45.6^{\circ}$

Next we will see how we use these Trigonometric Functions to Solve for missing information

## Solving Proportions

Solving trigonometry problems is just solving a proportion.

- A proportion is when we have two things equal to one another and one piece of information is unknown, ALGEBRA all over again


## Example:

Solve the following proportions for $a$

1. $a b=c \quad \rightarrow \quad \frac{a b}{b}=\frac{c}{b} \quad \rightarrow \quad a=\frac{c}{b}$

Divide both sides by $b$
2. $a b c=d \quad \rightarrow \quad \frac{a b c}{b c}=\frac{d}{b c} \quad \rightarrow \quad \boldsymbol{a}=\frac{\boldsymbol{d}}{\boldsymbol{b c}}$

Divide both sides by bc
3. $\frac{a b}{c}=d \rightarrow \quad \rightarrow * \frac{a b}{c}=d * c \quad \rightarrow \quad a b=d c \quad \rightarrow \quad \frac{a b}{b}=\frac{d c}{b} \quad \rightarrow \quad \boldsymbol{a}=\frac{\boldsymbol{d} \boldsymbol{c}}{\boldsymbol{b}}$

Multiply both sides by c
Divide both sides by $b$
4. $\frac{a+b}{c}-d=e \rightarrow \frac{a+b}{c}-d+d=e+d \quad \rightarrow \quad \frac{a+b}{c}=e+d \quad \rightarrow \quad c * \frac{a+b}{c}=(e+d) c \quad \rightarrow$
$a+b=(e+d) c \quad \rightarrow \quad a+b-b=(e+d) c-b \quad \rightarrow \quad a=c(e+d)-\boldsymbol{b}$
Subtract b from both sides

## Section 4.1 - Practice Problems

Solve for the following Trigonometric Ratios. (Round to 4 decimals)

| 1. $\operatorname{Sin} 12^{\circ}=$ | 2. $\operatorname{Tan} 57^{\circ}=$ | 3. $\operatorname{Cos} 123^{\circ}=$ |
| :--- | :--- | :--- |
| 4. $\operatorname{Cos} 34^{\circ}=$ | 5. $\operatorname{Sin} 360^{\circ}=$ | 6. $\operatorname{Tan} 270^{\circ}=$ |
| 7. $\operatorname{Sin} 234^{\circ}=$ | 8. $\operatorname{Tan} 2^{\circ}=$ | 9. $\operatorname{Cos} 180^{\circ}=$ |
| 10. $\operatorname{Tan} 45^{\circ}=$ | 11. $\operatorname{Sin} 45^{\circ}=$ | 12. $\operatorname{Cos} 45^{\circ}=$ |

Solve for the following angles. (Round to 1 decimal)

| 13. $\operatorname{Sin}^{-1}(0.8660)=$ | 14. $\operatorname{Tan}^{-1}(0.2354)=$ | $\operatorname{Cos}^{-1}(0.6775)=$ |
| :---: | :---: | :---: |
| 16. $\operatorname{Cos}^{-1}(0.1111)=$ | 17. $\operatorname{Sin}^{-1}(0.9999)=$ | 18. $\operatorname{Tan}^{-1}(1.234)=$ |
| 19. $\operatorname{Sin}^{-1}(0.5628)=$ | 20. $\operatorname{Tan}^{-1}(0.5555)=$ | 21. $\operatorname{Cos}^{-1}(0.6258)=$ |
| 22. $\operatorname{Tan}^{-1}(1.879)=$ | $23 . \operatorname{Sin}^{-1}(0.1111)=$ | $24 . \operatorname{Cos}^{-1}(0.0001)=$ |

Solve the following proportions for the variable $a$.
25. $b=\frac{a}{c} \quad\left\{\begin{array}{c:c}\text { 26. } b=\frac{c}{a} \\ \text { 27. } c=\frac{b}{a+d} & 28 . d=a b-a c \\ \hline \text { 29. } a b=a c+d & \text { 30. } b=\frac{a c}{d}\end{array}\right.$

## Section 4.2 - Solving Triangles

- Solving triangles involves solving for all three angles and all three sides
- A quick reminder, all three angles in a triangle add up to $\mathbf{1 8 0}^{\circ}$
- At this level we only discuss RIGHT ANGLE triangles so we already know one angle is $90^{\circ}$
- So the other two must also add to $\mathbf{9 0}^{\circ}$


## Solving Triangles

- Whenever we are solving triangle we need at least 2 pieces of information
- Either $\mathbf{2}$ sides or $\mathbf{1}$ side and $\mathbf{1}$ angle
- From there we can then solve for everything else

All of the rest of the information comes from working with ratios
To help remember these ratios we think about these three words

## SOH CAH TOA

They stand for:

$$
\text { Sine } \theta=\frac{\text { Opposite }}{\text { Hypotenuse }} \quad \text { Cosine } \theta=\frac{\text { Adjacent }}{\text { Hypotenuse }} \quad \text { Tangent } \theta=\frac{\text { opposite }}{\text { Adjacent }}
$$

- With a right angle triangle, depending on what angle you want, the sides get named differently



## Different Solving Scenarios

## An unknown Side - Using Tangent

- If you look at the triangle we have an angle and the opposite side
- We want the adjacent side
- So we have two letters of TOA, so were using TANGENT

- Using the TOA Ratio from before:
$\operatorname{Tan}(\theta)=\frac{\text { Opposite }}{\text { Adjacent }} \rightarrow \operatorname{Tan}(30)=\frac{7}{t}$
- Doing some Algebra:
$(t) \operatorname{Tan}(30)=7 \quad \rightarrow \quad t=\frac{7}{\operatorname{Tan}(30)}$
- Then we solve, you can simplify the $\operatorname{Tan}(30) 1^{\text {st }}$
$t=\frac{7}{0.5774} \quad \rightarrow \quad t=12.1$


## An unknown Side - Using Tangent

- If you look at the triangle we have an angle and the adjacent side
- We want the opposite side
- So we have two letters of TOA, so were using TANGENT



## An unknown Side - Using Sine

- If you look at the triangle we have an angle and the opposite side
- We want the hypotenuse
- So we have two letters of SOH, so were using SINE

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- Using the SOH Ratio from before:
$\operatorname{Sin}(\theta)=\frac{\text { Opposite }}{\text { Hypotenuse }} \rightarrow \operatorname{Sin}(30)=\frac{7}{t}$
- Doing some Algebra:
$(t) \operatorname{Sin}(30)=7 \quad \rightarrow \quad t=\frac{7}{\operatorname{Sin}(30)}$
- Then we solve, you can simplify the $\operatorname{Sin}(30) 1^{\text {st }}$
$t=\frac{7}{0.5} \quad \rightarrow \quad t=\mathbf{1 4}$


## An unknown Side - Using Sine

- If you look at the triangle we have an angle and the hypotenuse
- We want the opposite side
- So we have two letters of SOH, so were using SINE



## An unknown Side - Using Cosine

- If you look at the triangle we have an angle and the adjacent side
- We want the hypotenuse
- So we have two letters of CAH, so were using COSINE



## An unknown Side - Using Cosine

- If you look at the triangle we have an angle and the hypotenuse
- We want the adjacent side
- So we have two letters of CAH, so were using COSINE



## Trigonometry of Right Angle Triangles - SOH CAH TOA

Solve for the missing side of the right angle triangle, round to the nearest tenth.



## An unknown Angle - Using Tangent

- If you look at the triangle we have the opposite and adjacent sides
- We want the angle
- So we have two letters of TOA, so were using TANGENT

- Using the TOA Ratio from before:
$\operatorname{Tan}(\theta)=\frac{\text { Opposite }}{\text { Adjacent }} \rightarrow \operatorname{Tan}(\theta)=\frac{7}{12}$
- Then we can simplify the $\frac{7}{12} 1^{\text {st }}$
$\operatorname{Tan}(\theta)=0.5833$
- Then we use the inverse Tangent button
$\theta=\operatorname{Tan}^{-1}(0.5833) \quad \rightarrow \quad 30.3^{\circ}$


## An unknown Angle - Using Sine

- If you look at the triangle we have the opposite side and hypotenuse
- We want the angle
- So we have two letters of SOH, so were using SINE



## An unknown Angle - Using Cosine

- If you look at the triangle we have the adjacent side and hypotenuse
- We want the angle
- So we have two letters of CAH, so were using COSINE



## Advanced Trigonometry of Right Angle Triangles

Solve for the desired side of the figure below $(x)$, round to the nearest tenth.


$$
\begin{aligned}
& \cos \theta=\frac{a d_{j}}{h y p} \\
& \cos 35^{\circ}=\frac{z}{11}
\end{aligned}
$$

$\sin \theta=\frac{\text { opp }}{h y p}$

$$
z=11 \cos 35^{\circ}
$$

$$
\sin 73^{\circ}=\frac{7}{23}
$$

$$
z=9.011 \mathrm{~m}
$$

$$
\begin{aligned}
& y=23 \sin 73^{\circ} \\
& y=21.995 \mathrm{~m}
\end{aligned}
$$

$$
x=y-z
$$

$$
=21.995-9.011
$$

$$
=12.934=13.0 \mathrm{~m}
$$

Solve for the desired side of the figure below $(x)$, round to the nearest tenth.

$\sin 61^{\circ}=\frac{h}{44}$
$\sin 25^{\circ}=\frac{38.453}{x}$
$h=44 \sin 61^{\circ}$

$$
x=\frac{38.483}{\sin 25^{\circ}}
$$

$$
=38.483 \mathrm{~m}
$$

$$
x=91.1 \mathrm{~m}
$$

Find the area of the following triangles, round answers to the nearest tenth.


$$
\begin{aligned}
& \sin 70^{\circ}=\frac{h}{23} \\
& \cos 70^{\circ}=\frac{y}{23} \\
& h=23 \sin 70^{\circ} \\
& y=23 \cos 70^{\circ} \\
& h=21.613 \times 25 \\
& y=7.866 \mathrm{yds} \\
& \tan 10^{\circ}=\frac{21.613}{x} \\
& b=x+y \\
& =122.573+7.866 \\
& x=\frac{21.613}{\tan 10^{\circ}} \\
& =130.439 \\
& x=122.573 \mathrm{yds} \\
& \begin{aligned}
A & =\frac{1}{2} \mathrm{bh} \\
& =\frac{1}{2}(130.439)(21.613)
\end{aligned}
\end{aligned}
$$

## Section 4.2 - Practice Problems

Use your Trigonometric Ratios to solve for the desired side.

1. $r=$ $\qquad$

2. $\mathrm{u}=$ $\qquad$
3. $\mathrm{c}=$ $\qquad$

4. $\mathrm{m}=$ $\qquad$


## Workplace Math 10

Find the measure of the indicated angle, to the nearest tenth of a degree.


## Workplace Math 10

Find the length of the side denoted by a variable. Round answers to the nearest tenth.


Find the area of the triangle. Round answers to the nearest tenth.
17. Area $=$ $\qquad$ $\mathrm{mm}^{2}$

18.

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## Section 4.3 - Applications of Trigonometric Concepts

- As we move into more complex problems and word problems we need some terminology
- Trigonometry is used to calculate heights and distances that are difficult or impossible to measure by ordinary methods, these concepts will help

Terminology

Angle of Elevation


Angle of Depression


- Trigonometry was used primarily for Navigation in the past
- Bearings represent direction of travel and have to be read a certain way
- Bearings measure the movement East and West from a fixed North-South line


## Bearings



- With the following tools and the information from the previous sections, we should now be able to tackle any questions involving triangles formed with a RIGHT ANGLE

Example: A pilot has to approach Vancouver airport at an $8^{\circ}$ angle of descent. IF the plane is travelling at an altitude of 10000 ft , what is the horizontal distance that the airplane should be at to begin its descent?

## Solution:



- $\quad \operatorname{Tan} 82^{\circ}=\frac{x}{10000}$
- $10000\left(\right.$ Tan $\left.82^{\circ}\right)=x$
- $x=71154 f t$

- The pilot should start descending 71154 ft from the airport.

Example: The equal sides of an isosceles triangle are 30 cm , and the third side is 36 cm . Determine the measure of the interior angles of the training.

## Solution:

- $\operatorname{Cos} \theta=\frac{18}{30}$
- $\theta=\operatorname{Cos}^{-1}\left(\frac{18}{30}\right)$
- $\theta=53.1^{\circ}$



## Section 4.3 - Practice Problems

For the following questions, include a drawing of the concept being discussed. (Round to the tenth if necessary)

1. The area of a right triangle is 50 . One of the angles is $45^{\circ}$. Find the lengths of the legs and the hypotenuse of the triangle.
2. From the top of a 200 meters high building, the angle of depression to the bottom of a second building is 20 degrees. From the bottom of the building, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.
3. Karla is riding vertically in a hot air balloon, directly over a point $P$ on the ground. Karla spots a parked car on the ground at an angle of depression of $30^{\circ}$. The balloon rises 50 metres. Now the angle of depression to the car is 35 degrees. How far is the car from point P?
4. After an hour of flying, a jet has covered 300 km , but winds have blown it off course. The instruments on the plane show that it is 48 km West of the planned flight path. By how many degrees if the plane off course?
5. Two boats take-off from the same spot. One travels due North for 320 km , and the other due East for 450 km . If the boat travelling East wanted to turn to travel to where the other boat stopped, what angle North of East should it turn?
6. Butch, who is 1.6 m tall and works for the forestry department is tasked to measure the approximate height of some trees. The angle of inclination from his head to the tip of the tree is $48^{\circ}$, and he is 7.2 m from the tree. How tall is the tree?

## Answer Key

## Section 4.1

1. 0.2079
2. 1.5399
3. -0.5446
4. 0.8290
5. 0
6. No Solution
7. -0.8090
8. 0.0349
9. -1
10. 1
11. 0.7071
12. 0.7071
13. $60^{\circ}$
14. $13.2^{\circ}$
15. $47.4^{\circ}$
16. $83.6^{\circ}$
17. $89.2^{\circ}$
18. $51.0^{\circ}$
19. $34.2^{\circ}$
20. $29.1^{\circ}$
21. $51.3^{\circ}$
22. $62.0^{\circ}$
23. $6.4^{\circ}$
24. $90.0^{\circ}$
25. $a=b c$
26. $a=\frac{c}{b}$
27. $a=\frac{b-c d}{c}$
28. $a=\frac{{ }_{d}^{c}}{(b-c)}$
29. $a=\frac{d}{(b-c)}$
30. $a=\frac{b d}{c}$

## Section 4.2

1. 3.86
2. $\quad 11.47$
3. 23.81
4. 4.53
5. $\quad 16.48$
6. 9.33
7. $64.9^{\circ}$
8. $43.6^{\circ}$
9. $48.9^{\circ}$
10. $30.5^{\circ}$
11. $53.1^{\circ}$
12. $12.7^{\circ}$
13. 49.1 in
14. 94.9 cm
15. 15.1 ft
16. 8.1 in
17. $924.8 \mathrm{~mm}^{2}$
18. $3729.9 y d^{2}$

## Section 4.3

1. $H=10, L=10$
2. 96.9 m
3. 235 m
4. $9.2^{\circ}$
5. $35.4^{\circ}$
6. 9.6 m
