Review and Preview to Chapter 4

Intervals

In calculus sets of real numbers occur frequently and are called **intervals**. They often represent a range of x values over which you may be interested in knowing something about a function. There are two types of intervals, **open intervals,** and **closed intervals**.

- **Open Interval** – the open interval from a to b includes all numbers between a and b, excluding a and b. The interval can be written as

bracket difference. → $(a, b) = \{x | a < x < b\}$

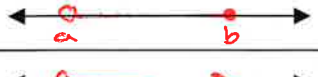
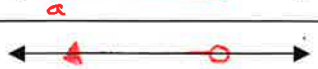
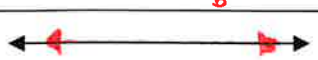
- **Closed Interval** – the closed interval from a to b includes all numbers between a and b, including a and b. The interval can be written as

$$[a, b] = \{x | a \leq x \leq b\}$$

	
The open interval (a, b)	The closed interval $[a, b]$

To write the interval all numbers greater than a we can write the following

$$(a, \infty) = \{x | x > a\}$$

Notation	Set Notation	Number Line
(a, b)	$\{x \mid x \in \mathbb{R}, a < x < b\}$	
$[a, b]$	$\{x \mid x \in \mathbb{R}, a \leq x \leq b\}$	
$[a, b)$	$\{x \mid x \in \mathbb{R}, a \leq x < b\}$	
$(a, b]$	$\{x \mid x \in \mathbb{R}, a < x \leq b\}$	
(a, ∞)	$\{x \mid x \in \mathbb{R}, x > a\}$	
$[a, \infty)$	$\{x \mid x \in \mathbb{R}, x \geq a\}$	
$(-\infty, b)$	$\{x \mid x \in \mathbb{R}, x < b\}$	
$(-\infty, b]$	$\{x \mid x \in \mathbb{R}, x \leq b\}$	
$(-\infty, \infty)$	$\{x \mid x \in \mathbb{R}\}$	

Ex. 1
Express the following intervals in terms of inequalities and graph the intervals on a number line.

(a) $\left[\frac{1}{2}, 4\right]$ (b) $[-2,1)$ (c) $(-4, \infty)$

$\{x \mid \frac{1}{2} \leq x \leq 4\}$ $\{x \mid -2 \leq x < 1\}$ $\{x \mid x > -4\}$

Inequalities

Rules for Inequalities (Applies to >, <, ≥, ≤)
1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and $c < d$, then $a + c < a + d$.
3. If $a < b$ and $c > 0$, then $ac < bc$.
4. If $a < b$ and $c < 0$, then $ac > bc$.
5. If $0 < a < b$, then $\frac{1}{a} > \frac{1}{b}$.

Ex. 2
Solve the inequality $1 + x < 6x - 4$.

$-x \quad -x$

$1 < 5x - 4 \rightarrow 5 < 5x \rightarrow 1 < x$

$x > 1$

Ex. 3
Solve $x^2 + 2x > 0$.

$x(x+2) > 0$ zero points at $x = 0$, $x = -2$

Test regions on either side

so to solve $x^2 + 2x > 0$

$\{x \mid x > 0 \text{ or } x < -2\}$

$(-\infty, -2) \cup (0, \infty)$

Ex. 4

(a) Solve the inequality $(x+1)(x-2)(x-3) < 0$.

(b) Solve the inequality $(x+1)(x-2)(x-3) > 0$

Interval	$(x+1)$	$(x-2)$	$(x-3)$	Product
$x < -1$	$-$	$-$	$-$	$-$
$-1 < x < 2$	$+$	$-$	$-$	$+$
$2 < x < 3$	$+$	$+$	$-$	$-$
$x > 3$	$+$	$+$	$+$	$+$

a) $\{x \mid -1 < x \text{ or } 2 < x < 3\} = (-\infty, -1) \cup (2, 3)$

b) $\{x \mid -1 < x < 2 \text{ or } x > 3\} = (-1, 2) \cup (3, \infty)$

Homework Assignment

- Exercise 1: #1 – 2
- Exercise 2: #1 – 2