Section 4.5 – Practice Problems

1. A company determines that the cost, in dollars, of producing x items is:

$$C(x) = 280\,000 + 12.5x + 0.07x^2$$

a) Find the average cost and marginal cost of producing 1000 items

Average Cost $= c(x) = \frac{C(x)}{x}$

$\frac{280\,000}{x} + 12.5 + 0.07x$

$\boxed{c(1000) = \$362.50/\text{item}}$

Marginal Cost $C'(x) = 12.5 + 0.14x$

$\boxed{C'(1000) = \$152.50/\text{item}}$

b) At what production level will the average cost be least.

$c'(x) = -\frac{280\,000}{x^2} + 0.07$

$0 = \frac{-280\,000 + 0.07x^2}{x^2}$

$280\,000 = 0.07x^2$

$4\,000\,000 = x^2$

$x = 2000$

min: $-$ | $+$ at 2000

$\boxed{\text{average cost at least when producing 2000 items}}$

c) What is the minimum average cost?

$c(2000) = \frac{280\,000}{2000} + 12.5 + 0.07(2000)$

$\boxed{\$292.50/\text{unit}}$

2. The cost, in dollars, for the production of x units of a commodity is:

$$C(x) = 6400 + \frac{x}{10} + \frac{x^2}{1000}$$

a) Find the average cost and marginal cost of producing 3000 units

$c(x) = \frac{6400}{x} + \frac{1}{10} + \frac{x}{1000}$

$c(3000) = \boxed{\$5.23/\text{unit}}$

$C'(x) = \frac{1}{10} + \frac{1}{500}x$

$C'(3000) = \boxed{\$6.10/\text{unit}}$

b) At what production level will the average cost be minimized.

$c'(x) = -\frac{6400}{x^2} + \frac{1}{1000}$

$0 = -\frac{6400}{x^2} + \frac{1}{1000} \rightarrow -6\,400\,000 + x^2 = 0$

$x^2 = 6\,400\,000$

$x \doteq 2529.8$

$\doteq \boxed{2530}$

c) What is the smallest average cost?

$c(2530) = \frac{6400}{2530} + \frac{1}{10} + \frac{2530}{1000} = \boxed{\$5.16/\text{unit}}$

3. The Parker Soup Company estimates that the cost, in dollars, of making x cans of pea soup is:

$$C(x) = 48\,000 + 0.28x + 0.000\,01x^2$$

and the Revenue is:

$$R(x) = 0.68x - 0.000\,01x^2$$

In order to maximize profits, how many cans of soup should be sold.

$R'(x) = 0.68 - 0.000\,02x$

$C'(x) = 0.28 + 0.000\,02x$

$0.68 - 0.000\,02x = 0.28 + 0.000\,02x$

$0.40 = 0.000\,04x$

$x = 10\,000$

Max profits occur when 10000 cans are sold.

4. Korzeniowski's Submarines has found that the monthly demand for subs is given by:

$$p(x) = \frac{30\,000 - x}{10\,000}$$

And cost of making x subs is:

$$C(x) = 6000 + 0.8x$$

What level of sales will maximize profit.

$R(x) = x\,p(x) \rightarrow \dfrac{30\,000x - x^2}{10\,000} \rightarrow 3x - \dfrac{x^2}{10\,000}$

$R'(x) = 3 - \dfrac{x}{5000} = \boxed{\dfrac{15000 - x}{5000}}$

$C'(x) = 0.8$

$R'(x) = C'(x)$

$\dfrac{15000 - x}{5000} = 0.8 \rightarrow 15000 - x = 4000$

$x = 11\,000$

$\boxed{11\,000 \text{ subs sold}}$

5. The Blue Jays play in a stadium that holds 52 000 fans. Average attendance at a game was 27 000 with tickets priced at $10. When ticket prices were lowered to $8 the average attendance was 33 000.

a) Find the demand function, assuming the trend is linear.

$$\frac{y_2-y_1}{x_2-x_1} = \frac{10-8}{27000-33000} = \frac{2}{-6000}$$

$$p(x) = 10 - \frac{2}{6000}(x - 27000)$$

$$= 10 - \frac{1}{3000}x + 9 \rightarrow \boxed{19 - \frac{x}{3000}}$$

b) How should the owners set ticket prices so as to maximize revenue?

$$R(x) = x(p(x))$$

$$= 19x - \frac{x^2}{3000}$$

open down parabola

$$R'(x) = 19 - \frac{x}{1500}$$

$$0 = 19 - \frac{x}{1500}$$

$$\frac{x}{1500} = 19 \rightarrow x = 28500$$

maximum at $x = 28500$

$$p(28500) = 10 - \frac{1}{3000}(1500)$$

$$= \boxed{\$9.50/\text{ticket}}$$

6. A chain of stores has been selling a line of cameras for $50 each and has been averaging sales of 8000 cameras a month. They decide to increase the price, but market research shows that for each $1 increase in price, sales will fall by 100 cameras.

a) Find the demand function. Let x be the sales less than 8000

$$p(x) = 50 + \frac{1}{100}(8000 - x)$$

$$p(x) = 50 + 80 - \frac{x}{100} \rightarrow \boxed{p(x) = 130 - \frac{x}{100}}$$

b) Find the price that will maximize revenue.

$$R(x) = xp(x)$$

$$= 130x - \frac{x^2}{100}$$

$$R'(x) = 130 - \frac{x}{50}$$

$$0 = 130 - \frac{x}{50}$$

$$\frac{x}{50} = 130 \rightarrow x = 6500$$

$$p(6500) = 50 + \frac{1}{100}(1500)$$

$$\boxed{= \$65/\text{camera}}$$

7. The manager of a 120-unit apartment complex knows from experience that all units will be occupied if the rent is $400 a month (not in Victoria). Research suggests that for every $10 rent is increased, one additional unit will remain vacant. What rent should she charge to maximize revenue?

Let x represent decrease in rentals

$p(x) = 400 + \frac{10}{1}(120 - x)$

$R(x) = 400x + 1200x - 10x^2$

$= 1600x - 10x^2$ ← open down parabola

$R'(x) = 1600 - 20x$

$0 = 1600 - 20x$

$20x = 1600$

$\boxed{x = 80}$

$p(80) = 400 + 10(40)$

$= 400 + 400$

$= \boxed{\$800 \text{/apartment}}$

8. Out of this World Travel advertises a package plan for a Maui vacation. The fare for the flight is $400/person plus $8/person for each unsold seat on the plane. The plane holds 120 passengers and the flight will be cancelled if there are fewer than 50 passengers. What number of passengers will maximize revenue?

Let x represent # of booked tickets

$p(x) = 400 + 8(120 - x)$

$= 400 + 960 - 8x$

$= 1360 - 8x$

$R(x) = 1360x - 8x^2$

$R'(x) = 1360 - 16x$

$0 = 1360 - 16x$

$16x = 1360$

$x = 85$

Revenue is maximized when there are 85 passengers