4.5 Extreme Value Problems in Economics

We can now use the techniques we have learned in this chapter and apply it to a very important setting: business. Now we have the means to **minimizer and maximize average costs, revenue, and profits.**

In Section 3.4 we were introduced to the Cost Function $C(x)$, and the Marginal Cost Function $C'(x)$ which was the rate of change of C with respect to x (the Derivative). The **average cost function** however, is the **cost per unit when x units are produced** and is given by the function:

$$c(x) = \frac{C(x)}{x}$$

We want to minimize the average cost and we do so by locating the critical number of c. Using the Quotient Rule to differentiate the equation above we get:

$$c'(x) = \frac{xC'(x) - C(x)}{x^2} = 0$$

$$xC'(x) - C(x) = 0$$

$$xC'(x) = C(x)$$

$$C'(x) = \frac{C(x)}{x} = c(x)$$

What this demonstrates is that:

When the average cost is a minimum.
Marginal Cost = Average Cost

Ex 1: The cost in dollars, of producing x 5kg bags of flour is

$$C(x) = 140\,000 + 0.43x + 0.000\,001x^2$$

a) Find the average cost and marginal cost of producing 100 000 bags

Average Cost: $\frac{C(x)}{x} = \frac{140\,000 + 0.43x + 0.000\,001x^2}{x}$

$= \frac{140\,000}{x} + 0.43 + 0.000\,001x$ at $x = 100\,000$

$= \frac{140\,000}{100\,000} + 0.43 + 0.000\,001(100\,000)$

$= \boxed{\$1.93/\text{bag}}$

Marginal Cost

$C'(x) = 0.43 + 0.000\,002x$

at $x = 100\,000$

$= \boxed{\$0.63}$

b) At what production level will the average cost be smallest, and what is the average cost?

Differentiate the average cost and find a minimum.

$c'(x) = -\frac{140\,000}{x^2} + 0.000\,001$ minimum $c'(x) = 0$ possibly

$0 = -\frac{140\,000}{x^2} + 0.000\,001$

$\frac{140\,000}{x^2} = 0.000\,001$

$140\,000 = 0.000\,001x^2$

$14 \times 10^{10} = x^2$

$x = \sqrt{14} \times 10^5$

$\doteq 3.74 \times 10^5$

$c'(x) < 0$ for $x < \sqrt{14} \times 10^5$

$c'(x) > 0$ for $x > \sqrt{14} \times 10^5$

we have absolute minimum by First Derivative Test

Average Cost will be smallest when production is at 374 000 bags and the minimum average cost is $c(\sqrt{14} \times 10^5) \doteq \$1.18$/bag

Recall the relationship between the Demand Function $p(x)$ and the Revenue Function $R(x)$.

$$R(x) = xp(x)$$

The Marginal Revenue Function is: $R'(x)$

Recall that the Profit Function is:

$$P(x) = R(x) - C(x)$$

And to **maximize profit** we look for critical numbers of $P(x)$.

Where $\boldsymbol{P'(x) = 0}$ or in other words $R'(x) - C'(x) = 0 \rightarrow \boldsymbol{R'(x) = C'(x)}$

For Maximum Profit
Marginal Revenue = Marginal Cost

Ex 2: Recall in Section 3.4 Howard's Hamburgers had a yearly Demand Function of:

$$p(x) = \frac{800\,000 - x}{200\,000}$$

and a Cost Function

$$C(x) = 125\,000 + 0.42x$$

What level of sales will maximize profits?

$$R(x) = xp(x) = \frac{800\,000x - x^2}{200\,000} = \frac{1}{200\,000}(800\,000x - x^2)$$

$$R'(x) = \frac{1}{200\,000} \cdot (800\,000 - 2x) = \boxed{4 - \frac{x}{100\,000}}$$

$$\boxed{C'(x) = 0.42}$$

Max Profit: $R'(x) = C'(x)$

$$4 - \frac{x}{100\,000} = 0.42$$

$$3.58 = \frac{x}{100\,000} \rightarrow x = 358\,000$$

Sales of 358 000 will maximize profits.

Ex 3: A store has been selling 200 compact discs players a week a \$350 each. A market survey shows that for each \$10 rebate offered to the buyers, the number of sales increases by 20 a week.

a) Find the demand function and the revenue function
b) How large of a rebate would maximize the revenue?

a) Since we are writing a demand function.
Let x represent sales over 200

Weekly sales increase is : $(x-200)$
For each 20 sold rebate will decrease price by \$10 so each player represents $\$\frac{1}{20}$ so $\$\frac{1}{20}\cdot 10 = \frac{10}{20}$

$p(x) = 350 - \frac{10}{20}(x-200) \rightarrow 350 - \frac{1}{2}x + 100$

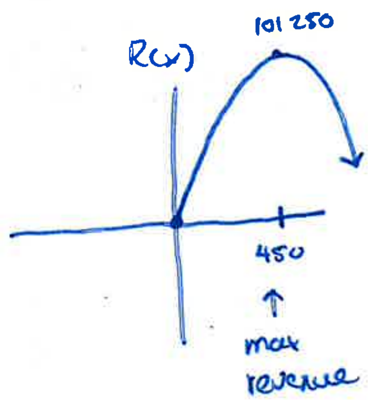
$p(x) = 450 - \frac{1}{2}x$

$R(x) = x\,p(x) = 450x - \frac{1}{2}x^2$ ← parabola opens down

$R'(x) = 450 - 1x$

$0 = 450 - x$

$x = 450$



Price is
b) $p(450) = 450 - \frac{1}{2}(450)$

$= 225$

$350 - 225 = \$125$

Rebate.

Homework Problems

- Section 4.5: #1 – 6